On the intraday return curves of Bitcoin: predictability and trading opportunities

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Abstract

Motivated by the potential inferences from intraday price data in the controversial Bitcoin market, we apply functional data analysis techniques to study cumulative intraday return (CIDR) curves. First, we indicate that Bitcoin CIDR curves are stationary, non-normal, uncorrelated, but exhibit conditional heteroscedastic, although we find that the projection scores of CIDR curves could be serially correlated during some certain periods. Second, we show the possibility of predicting the CIDR curves of Bitcoins based on the projection scores and then assess the forecasting performance. Finally, we utilize the functional forecasting methods to explore the intraday trading opportunities of Bitcoins and the results provide evidence of profitable trading opportunities based on intraday trading strategies, which confronts the efficient market hypothesis.

Keywords: Bitcoin; cumulative intraday return (CIDR) curves; predictability; efficiency; trading opportunities

1. Introduction

Being the first decentralized cryptocurrency and the most popular vehicle in the emerging era of digital financial investments, Bitcoin continues to evolve and attract much interest and debate in digital media technology, financial press, academia, regulatory bodies, and the investment community (Atsalakis et al., 2019). Many studies cover issues related to its blockchain technology, regulatory and legal aspects (Mansfield-Devine, 2017), function and relevance in the financial system (Fajri and Yamin, 2019), and relationship with conventional assets (Ji et al., 2018; Bouri et al., 2018; Shahzad et al., 2019)[1]. One particular area of research concerns price characteristics and predictability of Bitcoin, which confronts the efficient market hypothesis (EMH) documented in Malkiel and Fama (1970) and poses significant challenges for investors and regulators. However, the existing literature is split. Some academic studies show evidence of inefficiency and predictability (Urquhart, 2016; Nadarajah and Chu, 2017; Tiwari et al., 2018; Atsalakis et al., 2019), whereas others indicate that the Bitcoin market has become more informationally efficient (Al-Yahyaee et al., 2018; Vidal-Tomás and Ibañez, 2018; Sensoy, 2019). The related academic debate intensifies as sophisticated statistical methods and techniques are being applied, which include econophysics methods (e.g., fractality (Alvarez-Ramirez et al., 2018), multifractality (Al-Yahyaee et al., 2018; Takaishi and Adachi, 2019; Kristjanpoller and Bouri, 2019)) and computational and machine learning techniques (e.g., artificial neural networks (Nakano et al., 2018), neuro-fuzzy (Atsalakis et al., 2019), and support vector machines (Mallqui and Fernandes, 2019)).

With the availability of intraday data on Bitcoin emerges the need to use suitable methods to characterize the properties and dynamics of intraday price data and make inferences regarding market efficiency and trading schemes. This is important as intraday price data have properties that are different than those of daily closing prices (Tsay, 2005; Kokoszka et al., 2015). Urquhart and Zhang (2019) show that Bitcoin can be used as an intraday hedge against the risk of sovereign currencies. Wang and Ngene (2020) apply Granger causality models and bivariate GARCH-based models to intraday data on Bitcoin and six other leading cryptocurrencies (Ripple, Ethereum, EOS, Litecoin, Ethereum Classic, and Zcash). They indicate the importance of the dynamics of the Bitcoin market for predicting the performance of other cryptocurrencies. Petukhina et al. (2020) use intraday data to examine trading patterns related to returns, volumes, volatility periodicity, and provide evidence supporting the presence

[1] More details about the role of Bitcoin as an asset are given in Corbet et al. (2019a).

of intraday momentum of trading patterns in the cryptocurrency market. Notably, studying the predictability of Bitcoin intraday returns provides important implications regarding intraday trading opportunities, which matters to the efficiency market hypothesis. Considering market efficiency, Naeem et al. (2020) use hourly price data on Bitcoin and three other cryptocurrencies (Ethereum, Litecoin, and Ripple) within a multifractality model and indicate evidence of time variation in market efficiency as reflected by the effect of the COVID-19 on market efficiency of leading cryptocurrencies. In fact, studying the efficiency of the Bitcoin market has been an appealing research topic although some of the results drawn so far are mixed (Urquhart, 2016; Nadarajah and Chu, 2017; Tiwari et al., 2018; Atsalakis et al., 2019; Al-Yahyaee et al., 2018; Vidal-Tomás and Ibañez, 2018; Sensoy, 2019). Evidence suggests that Bitcoin price changes are not random and thus abnormal returns can be earned from exploiting the dynamics of some trading patterns and the information flow from other markets. However, the efficiency of Bitcoin has been on the rise given indication that the Bitcoin market is maturing and gaining more liquidity and participants covering institutional investors.

While several methodological techniques have been applied to study the price dynamics of Bitcoin, functional time series, which represents a subfield of functional data analysis, is a promising and suitable technique and has not been applied so far to the Bitcoin market. Especially, cumulative intraday return (CIDR) curves (Gabrys et al., 2010) are an informative way of transforming daily price curves into a stationary functional time series[2]. CIDR curves reveal how the return evolves during a trading day and give more relevant information than daily closing prices. It is therefore relevant and necessary to use CIDR while trying to predict intraday prices in the Bitcoin market and propose trading schemes for the sake of investors and traders.

Given the rise in high-frequency trading, it becomes more important to capture the price movement of Bitcoin over the course of a trading day. Accordingly, Bitcoin traders are very interested in a modelling framework involving an intraday trading strategy capable of generating abnormal returns. Only using the intraday price data, traders can implement the trading strategies and capitalize on short-term price fluctuations, while daily data only contains the opening/closing price, which ignores the price movement information during the trading day. Day traders intend to make profit and take advantage of small price fluctuations within a

[2] Kokoszka et al. (2015) argue that most of financial data take the form of curves (e.g., intraday price curves) observed repeatedly over time.

single day's trading and this trading strategy is increasingly popular for at-home traders due to advent of electronic trading of Bitcoin.

In fact, the high volatility of Bitcoin of intraday prices attracts speculative interest and investors because it provides traders with great profit-earning opportunities. The decentralized Bitcoin market is available 24 hours a day, 7 days a week, and it allows trading activities taking place between day traders in different locations across the world. Various trading schemes has been advocated, and they can be categorized as fundamental, technical, and quantitative trading strategies. Many researchers have focused on technical trading strategies in cryptocurrency markets. Examples of this stream of research include Turtle Soup pattern strategy (TradingstrategyGuides, 2019a); Nem (XEM) strategy (TradingstrategyGuides, 2019b); Amazing Gann Box strategy (TradingstrategyGuides, 2019c); Busted Double Top Pattern strategy (TradingstrategyGuides, 2019d), and Bottom Rotation Trading strategy (TradingstrategyGuides, 2019e), but these so called strategies lack of econometrics foundations. More complex trading strategies include the use of machine learning models involving high frequency data (see for examples, Nakano et al. 2018; Sun et al.2019; Zhengy et al. 2019). Another stream of study uses econometrics model, to estimate and predict the movement of Bitcoin daily price, including copula quantile causality approach (Bouri et al., 2019), dynamic equicorrelation model (Bouri et al., 2020), GARCH-MIDAS model (Conrad et al. 2018), GARCH-type family models (Katsiampa 2019; Corbet et al. 2018). However, these approaches maintain some econometrics theory, but provide no evidence on the utility of applying those methods to generate abnormal profits.

Therefore, we follow the new line of studies dealing with functional time series (Kokoszka et al., 2017; Kearney and Shang, 2019; Horvath et al., 2020), which have sound econometrics foundation and examine the properties of Bitcoin CIDR curves by applying the recent developed hypothesis testing methods in the functional data setting to assess the properties of stationarity (Horvath et al. 2014), serial correlation (Kokoszka et al. 2017), conditional heteroscedasticity (Rice et al. 2020), and normality (Gorecki et al. 2018). Notably, we propose profitable trading opportunities with intraday trading strategies based on the functional forecasting methods. Based on the functional data analyses, we develop our major research question consists of examining whether we can utilize the functional forecasting methods to assess the intraday trading opportunities of Bitcoins by relying on our proposed intraday trading strategies. To the best of our knowledge, we are the first to analyse these issues in the controversial Bitcoin market.

We introduce significant contributions to the related literature dealing with price prediction and market efficiency. Firstly, we move the debate regarding Bitcoin price dynamics to the intraday level, extending studies such as Eross et al. (2019) that assess intraday returns, volume, volatility and liquidity, and Hu et al. (2019) that focus on price clustering[3]. Secondly, we examine for the first time interesting properties of Bitcoin (CIDR) curves related to a stationarity (Horvath et al. 2014), serial correlation (Kokoszka et al. 2017), conditional heteroscedasticity (Rice et al. 2020), and normality (Gorecki et al. 2018), extending the academic debate beyond conventional assets (e.g., Kearney and Shang, 2019). Thirdly, we propose profitable trading opportunities with intraday trading strategies based on the functional forecasting methods, and then evaluate their performance. The results show the positive performance of such strategies, which is somewhat comparable to recent evidence on the profitability of technical trading rules in the Bitcoin market (e.g., Nakano et al., 2018; Corbet et al., 2019b; Gerritsen, et al., 2019). Fourthly, we provide overall results that point to the inefficiency of Bitcoin by showing how it is possible to develop a profitable trading strategy based on historical intraday prices.

This remainder of this paper is given in five sections. Section 2 describes the data and considers the functional time series on the property of Bitcoin cumulative intraday return. Section 3 explains the methods we use in forecasting CIDR curves. Section 4 presents the results obtained from forecasting the one-day-ahead Bitcoin CIDR curve. Section 5 offers a practical application from our forecasting methods through the design and test of an intraday trading strategy for Bitcoin CIDR. Section 6 summarizes the findings and provide new paths for future research.

2. Data

We download the intraday price data (in the currency of US dollars) of Bitcoin at 5-minute frequency from https://www.kaggle.com/mczielinski/bitcoin-historical-data, which collects the high frequency data from the Bitstamp exchange. The intraday price data at 5-minute frequency is used due to the consideration of the trade-off between incorporating informative signals and effect of market microstructure errors (Barndorff-Nielsen and Shephard, 2002). The sample ranges from 01-November-2014 to 10-August-2019, covering $T = 1367$ days. The period between 05-January and 10-January-2015 is eliminated from our sample because

[3] Shi and Shi (2019) show that South Korea's Bitcoin futures ban has an impact on the intraday spot price dynamics of Bitcoin.

of the hacked exchange event. Since Bitcoin is traded over the clock, there are 290 observations across the intraday grids. Denote the intraday price curves $P_t(u)$, for the daily time index $1 \leq t \leq T$ and intraday index $u \in [0,1]$. Following Gabrys et al. (2010), the CIDR curve at day t is defined as,

$$y_t(u) = 100 \times (\log P_t(u) - \log P_t(0)), 1 \leq t \leq T, u \in [0,1], \quad (1)$$

where $P_t(0)$ is the opening price at day t. The CIDR curves are of interest not only because they depict the entire intraday return movements but also because the natural logarithm smoothes the return data, making the Bitcoin return curves more suitable for a functional time series analysis. Figure 1 shows an example plot of five days intraday price and CIDR curves from 06-August to 10-August-2019.

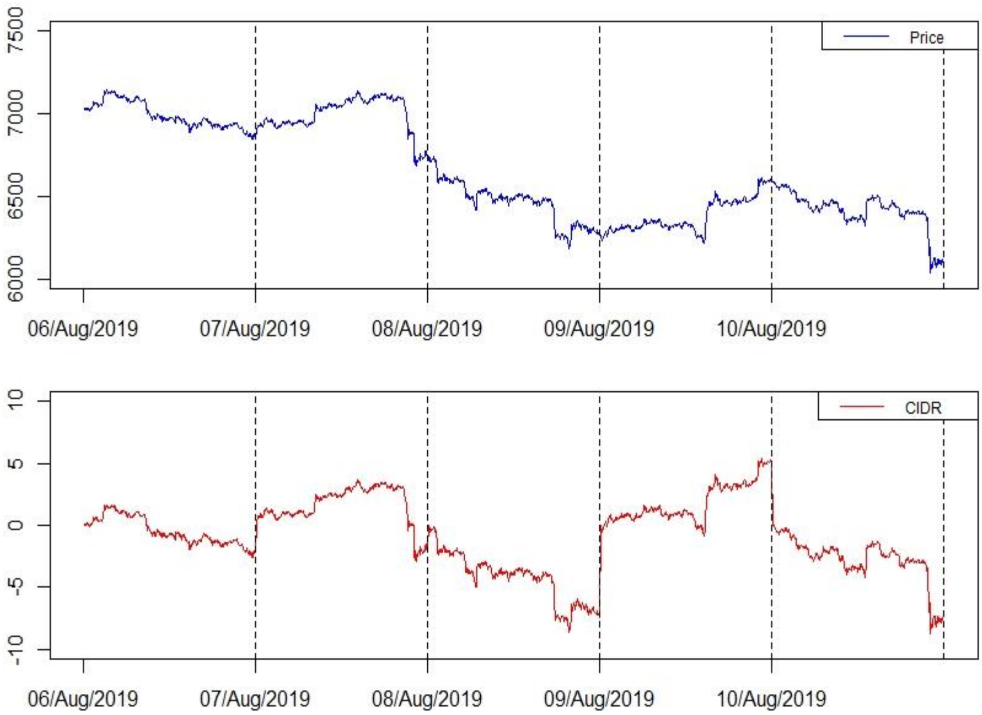

Figure 1. Plots of Bitcoin intraday prices and CIDR curves in August 2019

So far, the properties of Bitcoin CIDR curves are still not investigated in the academic literature. Treating Bitcoin CIDR curves as functional time series sequence, we need to test their dependence structures via functional data-type hypothesis testing.[4] We therefore apply the recent developed hypothesis testing methods in the functional data setting to assess the

[4] The conventional stationarity, normality and heteroskedasticity tests are only applicable to the scalar time series, but not applicable on the functional time series (i.e. CIDR curves in this paper).

properties of stationarity ($H_0\colon stationary$, Horvath et al. 2014), serial correlation ($H_0\colon no\ serial\ correlation$, Kokoszka et al. 2017), conditional heteroscedasticity ($H_0\colon independence$, Rice et al. 2020), and normality ($H_0\colon normality$, Gorecki et al. 2018).

Before presenting the summary statistics, we firstly review some key development in the Bitcoin market during our sample period. In fact, the tendency of the number of financial institutions including cryptocurrencies in their portfolios have eventually accelerated over the past years. Various events have occurred during 2014 - 2019, which were closely associated with Bitcoin price fluctuations. In 2013, the payment processor "BitInstant" and the largest exchange platform MtGox were suffered from delays of a half an hour on sales orders. During 2014 - 2015, the price of Bitcoin crashed more than 70%, leading to a bear market. During 2015-2016, Bitcoin exchanges suffered from stronger regulatory scrutiny and major hacks. In 2016, the Bitcoin skyrocket into the thousands driven by the depreciation in the Chinese Renminbi. In 2017, the Bitcoin price run close to $20,000, but then dropped to below $4,000 at the end of 2018. Still, the Bitcoin market experienced solid growth during that year along with more coins and tokens in the crypto markets, and Bitcoin cash solved the scaling issue by increasing the block size. In 2018, Bitcoin price dropped from $20,000 down to below $8,000, and some positive developments emerged in the year 2019 attributed to the hope of institutionalization of the Bitcoin market. For example, the BTC-settled Bitcoin futures contract was launched by Bakkt and the commodity-backed VanEck/SolidX BTC ETF proposal was initiated. To convey more concrete information to the readers, we evaluate the performance of our proposed trading scheme year by year to reflect account for the occurrence of various market events. Our empirical results show that the performance of our proposed trade scheme is satisfactory in different years.

Table 1 documents the basic statistic summary and the results of these tests. Considering that the Bitcoin market goes through the above key developments during the period studied, we separate the entire sample into 6 yearly sub-samples. Year 2015 to Year 2018 sub-samples start from the 1st of January and end at 31st of December, while Year 2014 and Year 2019 sub-samples contain fewer observations given the sample start and termination dates were included in these two years. Panel A describes the statistical summary of the scalar intraday return observations. We observe that Bitcoin returns experience smaller intraday variation in the year 2016 and become more volatile after 2017. Panel B exhibits the *p*-values of the hypothesis testing on the CIDR curves. The serial correlation and conditional heteroscedasticity tests are performed at lag lengths (1, 5, 10, 20) to check the robustness of dependence structure.

According to the results, we find that similar to the properties of intraday returns from the equity market (Rice et al. 2019). Specifically, the Bitcoin CIDR curves are stationary, non-normal, uncorrelated but conditional heteroscedastic. This finding is robust over yearly sub-samples. Additionally, Figure 2 displays the functional autocorrelation functions of Bitcoin CIDR and its squared curves. Consistent with the results discussed in Table 1. We observe that the CIDR curves are uncorrelated but show some dependence at the second moment. Thus, it is unlikely to forecast the CIDR curves via functional autoregressive-type models. Instead, we focus on the predictability in the CIDR curves via the forecasted projection scores, which will be elaborated in Section 3.

Table 1. Statistical summary and P values of hypothesis testing

	Panel A: Statistic summary of scalar intraday return observations									
	Mean		Min		Max	Median		Standard deviation		
Entire sample	0.0841		-36.2775		30.4314	0.0408		2.8076		
2014	-0.0196		-10.6377		16.0243	0.0476		2.3181		
2015	0.0504		-36.2775		30.4314	0.0164		2.6321		
2016	0.1136		-22.4513		17.8361	0.0000		1.8007		
2017	0.3502		-32.5082		22.2349	0.2920		3.5198		
2018	-0.2744		-28.8199		15.6812	-0.0672		3.0869		
2019	0.2295		-22.4213		19.1223	0.0968		2.7248		
	Panel B: Hypothesis testing on CIDR curves									
	Station.	Serial correlation				Heteroscedasticity				Normality
Lag		1	5	10	20	1	5	10	20	
Entire sample	0.62	0.35	0.47	0.28	0.21	0.00	0.00	0.00	0.00	0.00
2014	0.08	0.42	0.60	0.62	0.42	0.00	0.00	0.00	0.00	0.00
2015	0.12	0.57	0.33	0.42	0.37	0.00	0.00	0.00	0.00	0.00
2016	0.39	0.46	0.23	0.32	0.36	0.00	0.00	0.01	0.27	0.00
2017	0.69	0.29	0.86	0.60	0.72	0.00	0.00	0.00	0.00	0.00
2018	0.42	0.39	0.57	0.68	0.78	0.00	0.00	0.00	0.00	0.00
2019	0.38	0.48	0.84	0.75	0.78	0.00	0.00	0.00	0.07	0.00

Notes: "Station." denotes the of functional stationarity test ($H_0: stationary$, Horvath et al. 2014), "Serial correlation" means the functional serial correlation test ($H_0: no\ serial\ correlation$, Kokoszka et al. 2017), "Heteroscedasticity" stands for the functional conditional heteroscedasticity test ($H_0: independence$, Rice et al. 2019), and "Normality" is the functional normality test ($H_0: normality$, Gorecki et al. 2018).

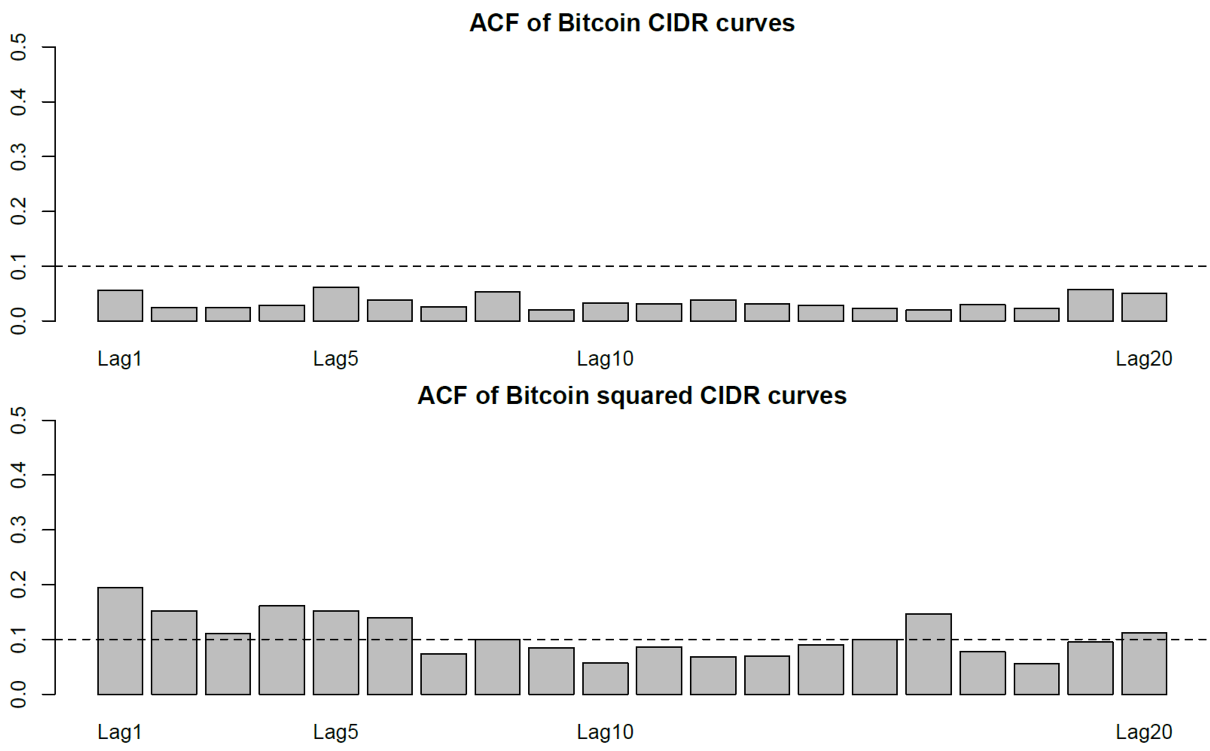

Figure 2. Functional autocorrelation function of Bitcoin CIDR curves

3. Methodology

In this section, we describe the main methods used to forecast CIDR curves. Usually, the next day CIDR curve can be predicted by using functional autoregressive-typed models if the curves are serially correlated (see, Bosq, 2000; Kokoszka and Reimherr, 2013; Horvath et al., 2020). However, from Table 1 we have known that CIDR curves are uncorrelated, thus it is inappropriate to fit these curves with functional autoregressive-typed models. As an alternative solution, we first project the CIDR curves into a finite number of data-drive bases, and then obtain the predicted curves by forecasting the scalar projecting scores. This method works because the uncorrelated functional curves do not imply that the projection score processes are also uncorrelated during the whole sample. A similar approach has been implemented by Kearney and Shang (2019), who obtain the forecasts of the crude oil forward curves by fitting and predicting the projection scores with an exponential smoothing model.

We define the CIDR curve $y_t(u)$ as a stochastic process with sample path on the interval $[0,1]$, and $y_t(u)$ are square integrable with the condition $E\|y_t(u)\|^2 = \int_0^1 y_t^2(u)du < \infty$. Taking the sample mean of these curves, we obtain the functional mean $\mu(u)$, so that the sample nonnegative-definite covariance operator can be written as,

$$\hat{C}(u,v) = Cov\big(y_t(u), y_t(v)\big) = T^{-1}\sum_{t=1}^{T}\big[\big(y_t(u) - \mu(u)\big)\big(y_t(v) - \mu(v)\big)\big]. \quad (2)$$

According to Mercer's theorem (Horvath and Kokoszka, 2012), we are able to project the CIDR curves into K basis functions with a Karhunen-Loeve expression,

$$y_t(u) = \mu(u) + \sum_{j=1}^{\infty}\theta_{j,t}\varphi_j(u) \approx \mu(u) + \sum_{j=1}^{K}\theta_{j,t}\varphi_j(u), \quad (3)$$

where $\theta_{j,t}$, $1 \le j \le K$, is the projection scores on the jth given basis $\varphi_j(u)$. Then, it is not hard to deduce that the forecasts of $y_{t+1}(u)$ can be predicted if we can forecast the one-step ahead scores,

$$\hat{y}_{t+1}(u) = \mu(u) + \sum_{j=1}^{K}\hat{\theta}_{j,t+1}\,\varphi_j(u), \quad (4)$$

where $\hat{\theta}_{j,t+1}$ is the prediction of $\theta_{j,t}$ at day $t+1$. It is necessary to mention that the selection of the given function $\varphi_j(u)$ can be critical, because it determines the value of score processes. Technically, it is possible to choose $\varphi_j(u)$ from any linearly independent functions, such as, exponential polynomials, B-spline, and Fourier bases. Here, in order to find bases that can maximally explain the total variation from the CIDR curves, we use empirical functional principal components derived by the functional principal component analysis - FPCA. Suppose there exists a set of orthonormal functions $\{\hat{\varphi}_1(u), \hat{\varphi}_2(u), \cdots\}$ with corresponding to eigenvalues that $\hat{\lambda}_1 > \hat{\lambda}_2 > \cdots$, such that the sample covariance operator can be decomposed by the FPCA method as,

$$\hat{C}(u,v) \approx \sum_{j=1}^{K}\hat{\lambda}_j\,\hat{\varphi}_j(u)\hat{\varphi}_j(v), \quad (5)$$

where empirical principal components $\hat{\varphi}_j(v)$ are computed by solving an optimization problem that $\hat{\varphi}_1(u)$ explains the largest proportion of the total variations in $y_t(u)$, and $\hat{\varphi}_2(u)$ explains the second largest proportion, so on and so forth (see Ramsay and Silverman (2007) for more technique details). There are several options to select the dimension K, for examples, using the information criteria, the cross-validation, the bootstrapping, and the total variation explanation (Kearney and Shang, 2019). Here, we apply the total variation explanation and choose the number of K that is enough to explain 90% of the total variation from the CIDR curves.

Let us now focus on the procedure to forecast the projection scores. According to Equation (4), we obtain the of projection scores, $\hat{\theta}_{j,t}$, by projecting the CIDR curves onto K number of orthonormal basis functions. Then, we deploy two different (scalar) time series models to forecast the projection scores. In the first method, we consider the method applied in Kearney and Shang (2019) and forecast the scores by using the exponential smoothing model. In order to find a better exponential smoothing model to fit the scores, we apply the "ets" function in the R package "forecast" (Hyndman et al., 2019) that automatically choose the optimal model from available ones, including, simple exponential smoothing, Holt's method, exponential trend, damped trend, and damped exponential trend method. We denote this method as *FPES* which is based on exponential smoothing to forecast one-day-ahead scores.

In the second method, the underlying principle is that the projection scores can be predictable if they are serial correlated during specific period. Thus, we first test the null hypothesis of independence on these scores by using the classic Box and Ljung test (Ljung and Box, 1978) given a sample period. The scores would be fitted and predicted by (scalar) time series models if the null hypothesis was rejected at 95% significance level. Otherwise, we retain the best prediction of scores as zero, resulting the best prediction of intraday return as the functional mean $\mu(u)$. Specifically, we stick on AR(1) model to forecast the scores if there exists a serial correlation. This AR(1) model is preferred because it is parsimonious and can reasonably well-performed in our dataset. Eventually, the next day CIDR curve $\hat{y}_{t+1}(u)$ can be obtained by substituting the predicted scores into Equation (3). We use the acronym *FPAR* to represent this method which is based on AR(1) to forecast one-day-ahead scores.

As a benchmark, we choose to use the sample mean curve $\mu(u)$ as the forecast, and this benchmark method is denoted as *Fmean*. In terms of evaluating the accuracy of the predictions, we use integral-typed measurements to assess the forecasting errors, because in our context the predictors and observations are functional curves. Following Didericksen et al. (2012) and Kokoszka et al. (2014), we calculate mean squares distance (MSD) and relative predictive efficiency (RPE) for $\hat{y}_{t+1}(u)$ as,

$$MSD = \int_0^1 (\hat{y}_{t+1}(u) - y_{t+1}(u))^2 du, \quad (6)$$

$$RPE = 100\left(\frac{MSD_1}{MSD_2} - 1\right), \quad (7)$$

where, MSD_1 is the *MSD* of the benchmark method *Fmean*, and MSD_2 is the *MSD* of either *FPES* or *FPAR*.

4. Forecasting Results

We now apply the proposed methods to forecast the one-day-ahead Bitcoin CIDR curves. Given a sample with size T, we follow a three-step forecasting procedure with a rolling window approach: first, setting a training sample with S observations, which delivers $T - S$ observations in out-of-sample; second, applying the FPCA method the training sample in order to extract data-driven bases that explain a minimum 90% of the total variation, and obtaining the corresponding scores $\theta_{j,t}$; third, using the ES model to forecast one-day-ahead scores, or assessing the dependence of the score sequences and then apply AR(1) model to forecast, and then moving to the next window. Figure 3 displays the data-driven basis functions and the functional mean computed from the entire sample. From the left sub-figure, we can see that the first two bases are enough to explain 90.23% of the total variation, and the right sub-figure illustrates that the functional mean is an upward trend during the entire sample from 2014 to 2019. The first principal component explains the variation of the functional mean, and the second principal component describes a mean-reversion mechanism of return curves. The third and the fourth account for very small proportions of the total variation and difficult to interpret, so that we treat them as noises. Note that these analyses are conducted in each training sample, and we skip these results for saving space. Besides, since the Bitcoin market develops in a rapid and unpredicted pace, we also consider yearly sub-samples as we did in Section 2. For the same reason, we employ mid-ranged training sample and avoid using long-term historical information. In specific, we set window length $S = 182$, and 365 for the entire sample.

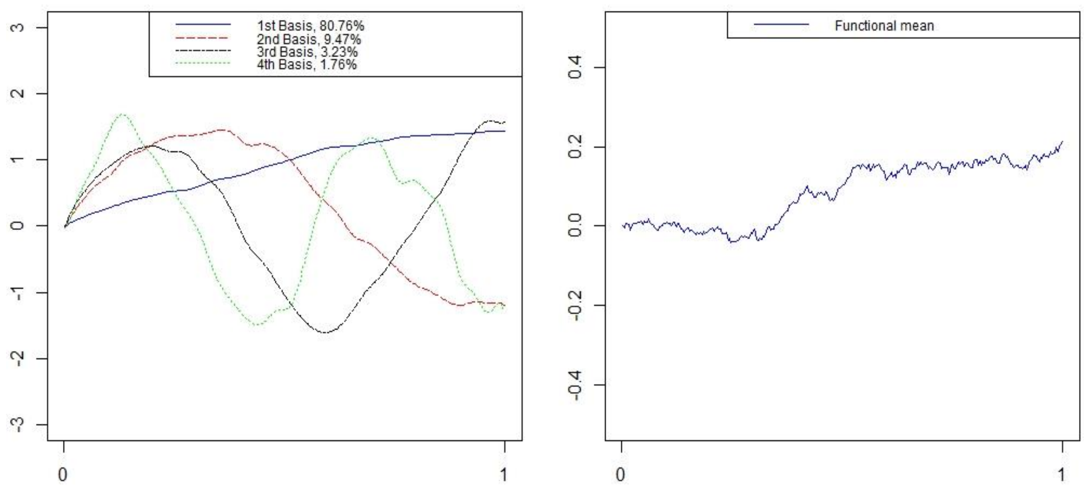

Figure 3. The first four functional principal components (bases) and the functional mean derived from the entire sample

Table 2 documents the forecasting errors of each model in yearly sub-samples from 2016 to 2019. Table 3 reports the errors from the entire sample and two selected periods: (1) the sample period where either the first or second scores are serially correlated, (2) the sample period where both the first and second scores are serially correlated. Both Table 2 and 3 indicate that, in general, the *FPES* and *FPAR* model underperform the benchmark *Fmean* model. However, *FPES* and *FPAR* show superior forecasting performance if we only evaluate the out-of-sample period when the first and second scores are serially correlated. This is a sensible result as the *FPES* and *FPAR* improve the predictability of Bitcoin CIDR curves by capturing the dependence structure of the scores. Note that the magnitude of our forecasting errors is larger than the commonly used mean absolute error or root mean square error for univariate variable forecasting. These results are not too surprising considering that the forecasting error measurements are taking integral over the entire intraday interval.

Table 2. Forecasting errors of the models from the out-of-samples - yearly sub-samples

	2016			
	S=182		**S=365**	
	MSD	RPE	MSD	RPE
Fmean	1.1566	0.0000	1.1491	0.0000
FPES	1.1898	-2.7935	1.1714	-1.9064
FPAR	1.1582	-0.1409	1.1520	-0.2515
	2017			
Fmean	2.7056	0.0000	2.7034	0.0000
FPES	2.7470	-1.5057	2.7163	-0.4728
FPAR	2.7060	-0.0133	2.7035	-0.0005
	2018			
Fmean	2.3573	0.0000	2.3467	0.0000
FPES	2.4204	-2.6063	2.3806	-1.4215
FPAR	2.3607	-0.1430	2.3480	-0.0556
	2019			
Fmean	1.9417	0.0000	1.9451	0.0000
FPES	1.9711	-1.4952	1.9731	-1.4190
FPAR	1.9451	-0.1789	1.9434	0.0897

Notes: *Fmean* stands for functional mean; *FPES* represents the method based on exponential smoothing to forecast one-day-ahead scores; and *FPAR* is the method based on AR(1) to forecast one-day-ahead scores. The parameter S is length of rolling window.

Table 3. Forecasting errors of the models from the out-of-samples – entire and selected sub-samples

	Entire sample			
	S=182		**S=365**	
	MSD	RPE	MSD	RPE
Fmean	1.9568	0.0000	2.0409	0.0000
FPES	1.9991	-2.1141	2.0644	-1.1380
FPAR	1.9615	-0.2372	2.0419	-0.0468
	The sample period where the first or second scores are serial correlated			
Fmean	1.8738	0.0000	2.0067	0.0000
FPES	1.9002	-1.3911	2.0212	-0.7133
FPAR	1.8904	-0.8778	2.0085	-0.0861
	The sample period where the first and second scores are serial correlated			
Fmean	1.7454	0.0000	2.2456	0.0000
FPES	1.7300	0.8890	2.2322	0.5971
FPAR	1.7116	1.9766	2.2342	0.5049

Notes: *Fmean* stands for functional mean; *FPES* represents the method based on exponential smoothing to forecast one-day-ahead scores; and *FPAR* is the method based on AR(1) to forecast one-day-ahead scores. The parameter S is length of rolling window.

Recently, there is a raising interesting in the CRptocurrency IndeX (CRIX), which is a market index for the cryptocurrency market. The details on the methodology of CRIX is provided by Trimborn and Hardle (2015). We implement the same forecasting methods to the CIDR curves of CRIX, which are derived from CRIX 5-minute frequency returns.[5] The sample period of CIDR is ranged from 01-July-2016 to 16-August-2020, with first S dates as in-sample and the remaining $T - S$ dates as out-of-sample. Table A1 in the appendix reports the forecasting errors on the CIRX. The results show that it is more challenging to find the predictability of CRIX, which potentially implies a weak serially correlated structure of the CRIX diluted by including many different cryptocurrencies. This is expected as a market index is generally more efficient than individual financial assets.

5. Intraday Trading of Bitcoin

One natural application of our forecasting methods is to design intraday trading strategy for Bitcoin CIDR. An intraday trading strategy is a short-term investment that typically holds a long or short position in a trading day, while it does not hold any position after the closure of the exchange (i.e. no overnight return). There are two specific issues related to the intraday

[5] We thank Simon Trimborn for kindly providing us the intraday data of CRIX. We used a shape-preserving piecewise cubic interpolation to fill the missing data.

trading of Bitcoin. Firstly, there is no easy way to short-sell Bitcoins in the major Bitcoin exchanges[6]. Therefore, we mainly consider long position in our trading strategy. Secondly, almost all Bitcoin exchanges are continuously trading in 24 hours and 7 days. Therefore, we will restrict our strategy not holding the position across different trading days as the convention of intraday trading. To be specific, once we open a position in a trading day, the position must be cleared before the midnight.

Based on the forecasting methods (*Fmean*, *FPES*, or *FPAR*), the procedure of our trading strategy in day $t+1$ is elaborated as follows[7]:

Step 1. Estimate the model parameters from the data in day $t-w+1$ and t.

Step 2. Use the estimated parameters to make one-day-ahead forecasting CIDR, $\hat{y}_{t+1}(u)$.

Step 3. Find the timing of minimum in the forecasted curve $\hat{y}_{t+1}(u)$, and denote it as u^{min}. This will be regarded as the timing of generating a long position.

Step 4. Due to the short-sell constraint, we truncate the forecasted curve $\hat{y}_{t+1}(u)$ before the timing u^{min}, and then find the timing of maximum in the forecasted curve $\hat{y}_{t+1}(u)$, which is denoted as u^{max}. This will be the timing that we clear our position.

Step 5. On the day $t+1$, we buy Bitcoin at time u^{min} and sell it at time u^{max}.

The return of the trading strategy on day $t+1$ can be calculated[8] as

$$R_{t+1} = y_{t+1}(u^{max}) - y_{t+1}(u^{min}) \tag{7}$$

where $y_{t+1}(\cdot)$ is the actual CIDR on day $t+1$ and $\{u^{min}, u^{max}\}$ are determined by the forecasted curve $\hat{y}_{t+1}(u)$.

We demonstrate our trading strategy in Figure 4. Before the trading day, we obtain the forecasted CIDR from the *FPAR* method, shown in the upper panel of Figure 4. Based on the forecasted CIDR, we can determine the time points u^{min} (5:45:00) and u^{max} (23:45:00), when the minimum and the maximum are predicted to occur. In this particular case, we make

[6] Investor can short-sell bitcoin via the act of borrowing bitcoins, but this is not commonly available. Otherwise, investor can short-sell bitcoin futures or bitcoin ETF.

[7] Such procedure of trading strategy can be implemented in practice. This is due to the computational time of our forecasting methods is typically within several seconds for a one-day-ahead forecasting.

[8] Our calculation on the return of the trading strategy is without transaction cost.

the trading plan for 23-May-2016 as "buy Bitcoin at 5:45:00 and sell it at 23:45:00". We will execute the trading plan on that day, and the lower panel of Figure 4 shows the actual CIDR on for 23-May-2016 and the timing when we buy and sell. Our return of such trading plan is $0.82\% - 0.34\% = 0.48\%$. It is worth noting that u^{max} is after u^{min} in this demonstration and thus there is no impact from the short-selling constraint on this trading day.

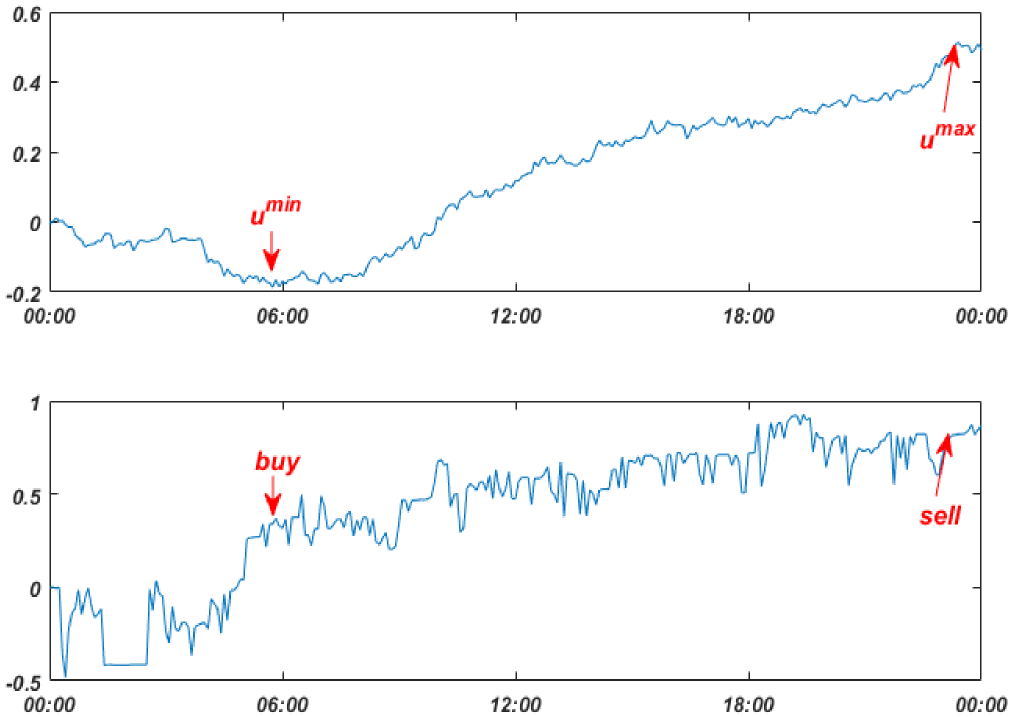

Figure 4. The demonstration of our intraday trading strategy. Upper panel: forecasted CIDR for 23-May-2016 from the *FPAR* method. Lower Panel: Actual CIDR on 23-May-2016. We make the planning of buy and sell based on the u^{min} and u^{max} before the trading day, and we execute the trading plan in the trading day.

The trading of Bitcoin is subject to the short-selling constraint. We may encounter the situation where the maximum can happen before the minimum in our forecasted CIDR. To better illustrate, the forecasted CIDR by the method of *FPAR* for 06-August-2018 is displayed in upper panel of Figure 5. We can observe that the global maximum (00:50:00) is occurred before u^{min} (10:45:00). We cannot practically short-sell at the timing of global maximum, and it is only feasible to sell after we have open the long position. This is the reason that we need to truncate the forecasted curve $\hat{y}_{t+1}(u)$ before the timing u^{min} in Step 4 of our trading procedure, and the practical selling time can only be after u^{min}. In this particular case, u^{max} happens at 14:15:00. Therefore, our trading plan for 06-August-2018 is "buy at 10:45:00 and sell at 14:15:00", and the return of this trading plan is $(-1.19\%) - (-0.61\%) = -0.57\%$.

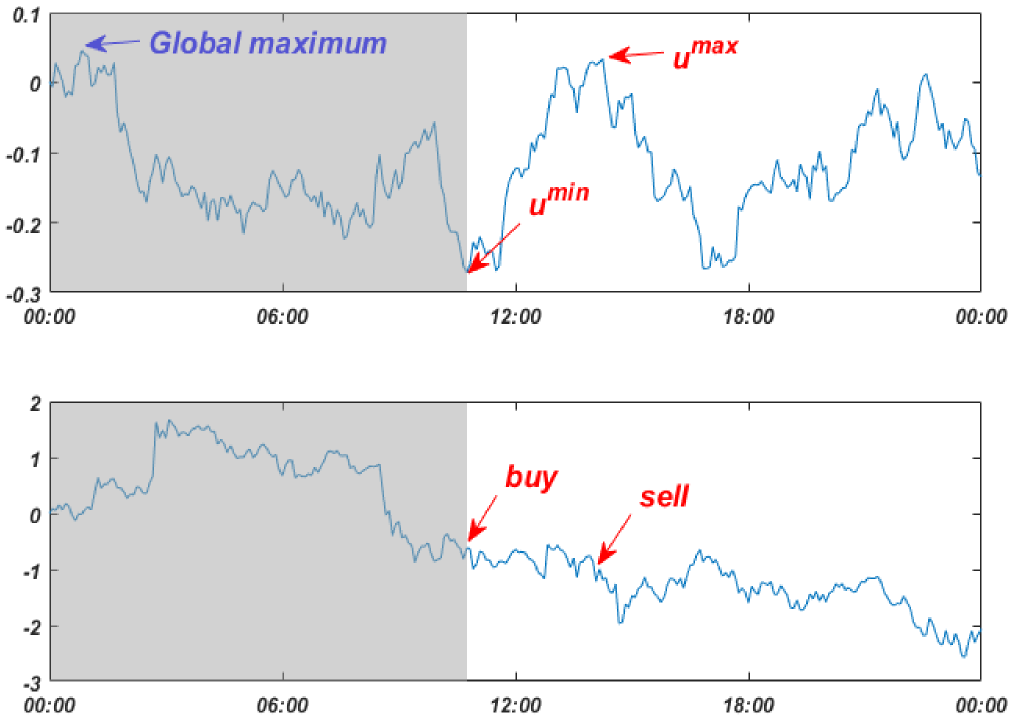

Figure 5. The demonstration of impact for no short-selling constraint. Upper panel: forecasted CIDR for 06-August-2018 from the *FPAR* method. Lower Panel: Actual CIDR on 06-August-2018. We make the planning of buy and sell based on the u^{min} and u^{max} before the trading day, and we execute the trading plan in the trading day.

We collect the daily return of the trading strategies based on the three functional forecasting methods. Their daily trading returns are used to calculate four common trading performance evaluation measures, including annualised return, annualised volatility, Sharpe ratio, and maximum drawdown, which are reported in the upper panel of Table 4. Firstly, the trading strategy based on *FPAR* with $S = 182$ is superior to the other two because of its higher annualised return, lower annualised volatility, and higher Sharpe ratio, though its maximum drawdown[9] is higher. The superiority of *FPAR* is consistent if $S = 365$. Secondly, *FPES* can outperform the *Fmean* in terms of a higher Sharpe ratio if $S = 182$, while this is not consistent if $S = 365$. Overall, the developed three trading strategies can exploit the trading opportunities

[9] Note that the maximum drawdown can below -100% because our calculation is based on the log returns.

in the intraday market as measured by more than 1 of Sharpe ratio, while the risk level is also high indicated by their maximum drawdown.

To reveal more insights, we restrict the trading only in the sample that the first or second scores are serially correlated. In other words, we only trade if we find there is evidence of dependence in the times series of FPC scores. This setting is motivated by the EMH (i.e., "prices reflect all information"), and the existence of dependence in the historical data indicates the market inefficiency. Four performance measures for such a setting are reported in the middle panel of Table 4. The trading strategy based on $FPAR$ is superior to the other two in terms of all measures, and it is consistent with both of $S = 182$ and 365. Another important observation is that the risk level measured by the maximum drawdown is dramatically reduced.

We are also interested in the sub-sample where the first and second scores both serially correlated. The trading performance of three trading strategies is reported in the lower panel of Table 4. The result is sensitive to the parameter S. We have a positive Sharpe ratio if $S = 182$, while a negative Sharpe ratio if $S = 365$. This is not a surprising result because there is a small number of days that first and second scores are both serially correlated. Thus, the result in this setting is subject to small sample issue.

Although there is no easy way to short-sell Bitcoins, investors and readers may also be interested in the performance of our trading strategies if we assume no short-selling constraint.[10] The results of performance in the same three settings are reported in Table A2 of the Appendix. The *FPAR* is still superior to the other measures, indicating the robustness of such a strategy.

Transaction costs could have an impact on the revealed trading opportunities. The Bitstamp exchange has a complicated tier structure for trading fee which depends on the total trading volume over the past 30 days up to the trading time.[11] To provide an estimated cost, we have assumed that the fee rate is 0.03% and revaluated the trading performance of the developed three trading strategies (without short-selling), which is presented in Table A3 of the Appendix. While the evaluation measures of all three trading strategies have been modestly deteriorated after the consideration of trading fees, the FRAR remains the superior strategy with a Sharpe ratio of 0.74 if trading is done in the entire sample period with $S = 182$. It is also noteworthy that the impact from the trading fee is much smaller if the trading is restricted to only the sample

[10] The first-ever Bitcoin ETF was launched by the purpose investment in Canada on February 18, 2021. This allows investors a channel to sell short bitcoins.

[11] https://www.bitstamp.net/fee-schedule/, accessed on March 14, 2021.

in which the first and second scores are serially correlated. This can be intuitively explained by the fact that the trading frequency is much lower in such setting.

Table 4. Performance evaluation of trading strategies based on three functional forecasting method

	S=182			S=365		
	Fmean	**FPES**	**FPAR**	**Fmean**	**FPES**	**FPAR**
Trading in the entire sample period (trading everyday)						
Annualised Return	60.86	63.60	**64.70**	48.84	39.68	**53.03**
Annualised Volatility	59.04	58.76	**58.15**	65.51	66.14	**63.58**
Sharpe Ratio	1.03	1.08	**1.11**	0.75	0.60	**0.83**
Maximum Drawdown	**-82.48**	-94.62	-99.25	-134.51	-163.24	**-127.95**
Trading only in the sample that the first or second scores are serially correlated						
Annualised Return	16.35	20.64	**20.19**	37.90	39.73	**42.10**
Annualised Volatility	29.71	32.58	**27.92**	43.03	46.33	**40.03**
Sharpe Ratio	0.55	0.63	**0.72**	0.88	0.86	**1.05**
Maximum Drawdown	-42.85	-53.32	**-37.27**	-66.36	-67.45	**-53.95**
Trading only in the sample that the first and second scores are serially correlated						
Annualised Return	0.21	0.26	**0.66**	-8.49	-8.40	**-7.52**
Annualised Volatility	1.31	1.31	**0.78**	14.23	14.26	**13.36**
Sharpe Ratio	0.16	0.20	**0.85**	-0.60	-0.59	**-0.56**
Maximum Drawdown	-2.45	-2.45	**-0.44**	-51.84	-52.87	**-50.44**

Notes: *Fmean* stands for functional mean; *FPES* represents the method based on exponential smoothing to forecast one-day-ahead scores; and *FPAR* is the method based on AR(1) to forecast one-day-ahead scores. The parameter S is length of rolling window. The bold number is the one with the best performance among the three methods.

To show the evolvement of strategy, Figure 6 present the cumulative return of the strategy based on *FPAR* if we trade every day (without short-selling and no trading fee), along with the drawdown and the indicators of dates when the first/second scores are serially correlated. There is a clear patten that this strategy can be consistently profitable before February 2017, as indicated by the up-trending cumulative return curve. However, the strategy became less effective afterwards and reached its maximum drawdown in December 2018. After this turning point, the cumulative return starts to increase. Another interesting observation is that we have the dates when the second scores are serially correlated, mainly in the early 2017 and 2019, while the dates when the first scores are serially correlated are distributed in early 2015, 2016, and around December 2018.

Lastly, it is worthwhile to discuss the impact from the latency on the practical implementation of the developed strategies. Before March 2019, it is challenging to trade at a decent speed in the Bitstamp exchange.[12] Due to the design of our forecasting methods, the latency can be effectively mitigated. This is because the forecasted CIDR on the day $t+1$ can be obtained once all intraday data on day t becomes available,[13] and then the actions of buy and sell based on the forecasted CIDR can be planned at the beginning of day $t+1$. In order to offset the latency, one can place buy/sell orders slightly earlier than the planned timing.[14]

Figure 6. Cumulative return curve of the trading strategy based on $FPAR$ with $S = 182$.

[12] On March 14,2019, the Bitstamp exchange introduced a new WebSocket solution which reduces latency by 250-270 milliseconds and significantly increases the speed of pushing order book messages. See details: https://www.bitstamp.net/article/new-websocket/ (assessed on March 15, 2021).

[13] Although there could be some computational time, the computation cost based on Equation (4) is very small.

[14] The exact amount of ahead time depends on the magnitude of the latency. For example, one can place orders 300 milliseconds in advance.

6. Conclusion

The remarkable evolution of the Bitcoin market has induced heated debate about its finance and economics, especially in regard to price formation and predictability. Recent evidence has been derived from sophisticated methods such as neuro-fuzzy techniques (Atsalakis et al., 2019). However, evidence of Bitcoin price prediction is very limited when intraday price data are used.

In this paper, we use recent developed tools in the field of functional data analysis and uncover evidence of predictivity in the intraday cumulative return of Bitcoin. We also employ the functional times series forecasting methods and explore evidence of intraday trading opportunities of Bitcoin. Our reported evidence showing that Bitcoin CIDR curves are stationary, non-normal, uncorrelated, but exhibit conditional heteroscedastic, represents an important contribution with respect to previous studies dealing not only with conventional assets (e.g., Kearney and Shang, 2019), but also with alternative investment vehicles such as Bitcoin. Such evidence is crucial as it points to the suitability of applying statistical models to Bitcoin price curves that form, among other things, stationary functional time series. This will open the door for future research dealing with Bitcoin price curves within models that involves stationarity of series. Our findings extend previous studies dealing with intraday price dynamics in the Bitcoin market (Eross et al., 2019; Hu et al., 2019; Urquhart and Zhang, 2019; Wang and Ngene, 2020; Petukhina et al., 2020) and also add to the literature dealing with the price prediction and efficiency of Bitcoin (e.g., Urquhart, 2016; Nadarajah and Chu, 2017; Tiwari et al., 2018; Vidal-Tomás and Ibañez, 2018; Al-Yahyaee et al., 2018; Atsalakis et al., 2019; Sensoy, 2019; Naeem et al., 2020). Especially, unlike Sensoy (2019) who use intraday data within a permutation entropy and a rolling window approach, we show the possibility to predict intraday price data and construct a profitable trading strategy within functional data analyses, which contradicts with the efficient market hypothesis in its weak form. In fact, Bitcoin traders can rely on our approach and results to construct an intraday trading strategy based on cumulative intraday return within functional forecasting methods and make abnormal returns, which challenges the efficient market hypothesis. Such findings on the possibility to generate profitable trades using intraday data within high-frequency trading schemes through exploiting intraday points and indications are of crucial importance to day traders in the Bitcoin market. Given evidence of a weak form of market efficiency in other major cryptocurrencies (e.g., Naeem et al., 2020), our findings can matter to traders in those cryptocurrencies who can builds on our models to assess its utility in major cryptocurrencies such as Ethereum, Ripple and Litecoin. Our findings are also important to policymakers and regulators such as central bank officials who are concerned with market efficiency. Notably, they have been putting the fast-growing cryptocurrency – Bitcoin - under surveillance and scrutiny to evaluate its suitability to be incorporated into the global financial system, which ultimately requires a strict regulatory framework.

While our data span is limited to the pre-COVID period to avoid any potential impact from the continuing COVID-19 pandemic on the Bitcoin market, future studies can consider the impact of the pandemic on CIDR curves of Bitcoin. Additional analysis might be needed to optimize the accuracy of the price prediction and increase the risk-adjusted performance of the trading strategy. Therefore, future studies can consider including information on technical indicators such as trading range breakout (Gerritsen, et al., 2019), and machine learning techniques (Nakano et al., 2018) in order to potentially increase the risk-adjusted return of the trading strategy.

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APPENDIX

Table A1. Forecasting errors of the models from the out-of-samples – entire and selected sub-samples of CRIX.

	Entire sample			
	S=182		**S=365**	
	MSD	RPE	MSD	RPE
Fmean	1.8441	0.0000	1.6598	0.0000
FPES	1.8832	-2.0755	1.6802	-1.2188
FPAR	1.8496	-0.2965	1.6630	-0.1929
	The sample period where the first or second scores are serial correlated			
Fmean	1.6412	0.0000	1.4081	0.0000
FPES	1.6784	-2.2173	1.4168	-0.6135
FPAR	1.6466	-0.3331	1.4112	-0.2273
	The sample period where the first and second scores are serial correlated			
Fmean	0.3151	0.0000	0.3235	0.0000
FPES	0.3372	-6.5680	0.3247	-0.3423
FPAR	0.3194	-1.3655	0.3256	-0.6431

Notes: *Fmean* stands for functional mean; *FPES* represents the method based on exponential smoothing to forecast one-day-ahead scores; and *FPAR* is the method based on AR(1) to forecast one-day-ahead scores. The parameter S is length of rolling window. The bold number is the one with the best performance among the three methods. The transaction fee is assumed to be 0.03% in the calculation. No short selling is allowed.

Table A2. Performance evaluation of trading strategies based on three functional forecasting method if short selling is assumed to be allowed

	S=182			S=365		
	Fmean	**FPES**	**FPAR**	**Fmean**	**FPES**	**FPAR**
Trading in the entire sample period						
Annualised Return	61.51	57.93	64.68	40.87	38.52	56.05
Annualised Volatility	63.78	64.66	63.19	67.82	70.47	65.53
Sharpe Ratio	0.96	0.90	1.02	0.60	0.55	0.86
Maximum Drawdown	-71.64	-94.89	-97.98	-164.16	-144.99	-102.08
Trading in the sample that the first or second scores are serially correlated						
Annualised Return	0.21	5.95	3.38	29.97	42.00	45.15
Annualised Volatility	33.33	35.63	32.20	46.46	49.53	43.07
Sharpe Ratio	0.01	0.17	0.10	0.65	0.85	1.05
Maximum Drawdown	-75.73	-64.66	-72.32	-110.63	-59.49	-35.74
Trading in the sample that the first and second scores are serially correlated						
Annualised Return	0.58	0.63	0.66	4.24	9.36	9.35
Annualised Volatility	1.40	1.41	1.07	14.08	13.51	14.23
Sharpe Ratio	0.42	0.45	0.61	0.30	0.69	0.66
Maximum Drawdown	-2.17	-2.17	-1.00	-24.24	-17.70	-19.29

Notes: *Fmean* stands for functional mean; *FPES* represents the method based on exponential smoothing to forecast one-day-ahead scores; and *FPAR* is the method based on AR(1) to forecast one-day-ahead scores. The parameter S is length of rolling window. The bold number is the one with the best performance among the three methods. No transaction cost is considered.

Table A3. Performance evaluation of trading strategies based on three functional forecasting method with transaction fee of 0.03%

	S=182			S=365		
	Fmean	**FPES**	**FPAR**	**Fmean**	**FPES**	**FPAR**
Trading in the entire sample period						
Annualised Return	38.94	41.68	**42.78**	26.92	17.77	**31.12**
Annualised Volatility	59.02	58.74	**58.13**	65.49	66.12	**63.56**
Sharpe Ratio	0.66	0.71	**0.74**	0.41	0.27	**0.49**
Maximum Drawdown	**-112.25**	-116.43	-121.76	-170.41	-195.21	**-149.88**
Trading in the sample that the first or second scores are serially correlated						
Annualised Return	10.20	**14.49**	14.04	25.79	27.63	**29.99**
Annualised Volatility	29.69	32.54	**27.88**	43.00	46.30	**40.00**
Sharpe Ratio	0.34	0.45	**0.50**	0.60	0.60	**0.75**
Maximum Drawdown	-44.10	-55.47	**-40.74**	-89.02	-86.25	**-62.70**
Trading in the sample that the first and second scores are serially correlated						
Annualised Return	0.08	0.13	**0.54**	-9.50	-9.41	**-8.53**
Annualised Volatility	1.30	1.31	**0.73**	14.27	14.29	**13.39**
Sharpe Ratio	0.06	0.10	**0.74**	-0.67	-0.66	**-0.64**
Maximum Drawdown	-2.63	-2.63	**-0.62**	-53.73	-55.08	**-52.64**

Notes: *Fmean* stands for functional mean; *FPES* represents the method based on exponential smoothing to forecast one-day-ahead scores; and *FPAR* is the method based on AR(1) to forecast one-day-ahead scores. The parameter S is length of rolling window. The bold number is the one with the best performance among the three methods. The transaction fee is assumed to be 0.03% in the calculation. No short selling is allowed.