Vines climbing higher: risk management for commodity futures markets using a regular vine copula approach

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Accepted Version

Li, H., Liu, Z. and Wang, S. ORCID: https://orcid.org/0000-0003-2113-5521 (2022) Vines climbing higher: risk management for commodity futures markets using a regular vine copula approach. International Journal of Finance and Economics, 27 (2). pp. 2438-2457. ISSN 1099-1158 doi: 10.1002/ijfe.2280 Available at https://centaur.reading.ac.uk/92905/

To link to this article DOI: http://dx.doi.org/10.1002/ijfe.2280

Publisher: Wiley

Vines Climbing Higher: Risk Management for Commodity Futures Markets using a Regular Vine Copula Approach

ABSTRACT

The volume of trading activity relating to China's commodity futures has grown rapidly over the course of the last decade. To improve risk management in China's commodity futures markets, this paper employs a regular vine (R-vine) copula model to study the dependence structure of commodity futures and to enhance Value-at-Risk (VaR) forecast. In doing so, we find that China's commodity futures market is not centered on one category of commodity futures and the tail dependence between different categories of commodity futures varies significantly. Based on the dependence structure analyzed using the R-vine copula model, we forecast the VaR of individual indices, which are formed of several commodity futures, as well as forecasting the VaR of an equally-weighted portfolio. Our method can outperform the standard GARCH-VaR method in terms of VaR backtesting. The tool developed within this study will enable those involved in commodity futures markets to fundamentally improve their risk management.

Keywords: R-vine copula, Commodity futures, Dependence structure, Tail dependence, Value-at-Risk
JEL Classification: G10; C22; C53

1. INTRODUCTION

The World Bank has suggested that risk management for highly volatile commodity futures is of vital importance to commodity-dependent economies (Dana, 2005). China, as the world's largest developing economy, is a typical example of a commodity-dependent country with fast-developing markets of commodity futures (Shao et al., 2019). This study examines the underlying dependence structure between commodity futures in China and how it can be exploited to improve risk management. Specifically, the first research objective is to investigate the multivariate dependence structure of China's commodity futures from the perspective of financial returns. To this end, a regular vine (R-vine) copula model is employed to study high dimensional dependence structures, due to its flexible tree-like structure. The second research objective is to demonstrate that the dependence structure, identified using the R-vine copula model, can be utilized to improve Value-at-Risk (VaR) forecasting for individual futures, as well as for portfolios. This tool would be of significant value to participants involved in commodity futures markets, as it would enable them to diversify and adjust investments according to dependence structures and risk measurement.

In recent years, China's commodity futures market has been ranked first in terms of the trading volume of commodity futures and options.[1] At the same time, there was a substantial increase in the number of futures contracts in China's commodity futures market. Only 18 commodity futures were listed on China's futures markets on 4th January 2010. By 29th March 2019, the number of listed commodity futures had reached 53.[2] With the number of commodity futures in China increasing sharply, one of the issues faced by investors is the complex dependence structure between these futures. A substantial increase in the degree of financialization of the commodity futures market in China has also been witnessed. As a result of rapid financialization, a further issue is the exacerbated risk caused by highly volatile commodity futures, particularly during the financial crisis (Lien and Zhang, 2008). This high volatility, compounded by a complex dependence structure, causes considerable challenges in risk management. Thus, this study selects 31 out of all 53 commodity futures with sufficient trading liquidity and investigates their dependence structure, in order to improve risk management for China's commodity futures markets. The selected commodity futures can be classified under nine categories according to the WIND database, shown in Table 1.

Studies show that a traditional correlation matrix cannot capture full dependence structure, leading to the omission of important factors in risk management, such as tail dependence (Boero et al., 2011; Wen et al., 2019). There are rank-based dependence coefficients, such as Kendall τ and Spearman ρ coefficients, which reflect the bivariate nonlinear dependence structure to some extent. However, it is difficult for these coefficients to fully reflect the complex multivariate dependence structure in financial markets, such as asymmetric dependence and tail dependence. Copula models overcome these limitations and provide a variety of distributional shapes for measuring aggregating risk types (Junker and May, 2005; Rosenberg and Schuermann, 2006; Christoffersen et al., 2012). The copula model is first proposed by Sklar (1959), and it measures the dependence structure between variables by linking their marginal distributions (Hammoudeh et al., 2014). Copula has been widely used to describe the dependence structure in commodity futures markets, and many of

[1] Data is sourced from the Futures Industry Association (https://www.fia.org/articles/fia-releases-annual-trading-statistics-showing-record-etd-volume-2018 , accessed on 1st September 2020). In 2018, the Soybean Meal futures of Dalian Commodity Exchange ranked first in the agricultural futures and options, with 238.16 million contracts; the Steel Rebar futures of Shanghai Futures Exchange ranked first in the metals futures and options, with 530.98 million contracts.

[2] It is worth mentioning that some of the tradable commodity futures suffer from a lack of liquidity.

them focus on the bivariate dependence structure between futures market and spots market (Zhao et al., 2019), or two different commodity futures (Li and Yang, 2013). However, multivariate dependence structures are more complicated (Patton, 2004). As a result, vine copula began to be employed in high-dimensional dependence structures, due to its diversity in pair-copula selection (Aloui and Aïssa, 2016; Apergis et al., 2020). Further, compared with other copulas, R-vine copula is more flexible, as it is not constrained by any kind of form to begin with.

The contributions of our paper are twofold; first, we identify the dependence structure of China's commodity futures using the R-vine copula model. We find that China's commodity futures market is not centered on one category of commodity futures. *Chemical Product* and *Coke and Steel* are more closely related to other commodity futures. In addition to this, positive tail dependence exists between some categories of commodity futures, e.g. *Chemical Product* and *Oil and Meal*. Second, we show that the dependence structure revealed by using the R-vine copula model can be used to improve risk management in commodity futures markets. Based on the R-vine copula, we are able to forecast the VaR of individual futures indices, as well as that of an equally-weighted portfolio. Our method can outperform the standard GARCH-VaR method in terms of VaR backtesting. The tool developed within this study will enable those involved in commodity futures markets to improve their risk management process.

The rest of the paper is organized as follows: Section 2 discusses the relevant existing literature. Section 3 introduces the vine copula and VaR forecasting application. Using the data described in Section 4, we employ R-vine copula to investigate the dependence structures of China's commodity futures and subsequently forecast VaR in Section 5. We conclude the study in Section 6.

2. LITERATURE REVIEW

Commodity futures markets in different countries have seen rapid development since the early 2000s. It has been observed that the volatility of commodity futures has increased significantly, and the financialization of commodities is the main reason for this increase (Silvennoinen and Thorp, 2013; Hamilton and Wu, 2014). Following the financial crisis of 2008, both the financialization and risk spillover effect of the commodity futures market have notably intensified (Berger and Czudaj, 2020). The risk premia of the commodity futures market consists of two key aspects: cross-sectional spot premia and term premia, which account for most of the expected future returns (Szymanowska et al., 2014). The fast-growing markets of commodity futures have provided investment opportunities and portfolio diversification benefits. Consequently, a number of trading strategies have been developed and studied, including both cross-sectional and time-series momentum strategies (Fuertes et al., 2010; Ham et al., 2019).

Previous studies have confirmed the existence of a risk spillover effect between global commodity futures markets, particularly between the US markets (Du et al., 2011; Trujillo-Barrera et al., 2012). With increasing numbers of futures contracts available for trading, China's commodity futures market has played an important role in global finance in recent years, which has led to considerable levels of interest in the risk spillover effect of China's commodity futures market. Fung et al. (2013) find that the returns of most of China's commodity futures are mainly driven by the domestic market, as opposed to foreign markets. Nguyen and Bhatti (2012) obtain similar results using copula models. Recently, Meng et al. (2020) uses copulas to prove the asymmetrical upside and downside spillover effects between China's crude oil markets and crude oil markets globally.

Since the proposal of the copula by Sklar (1959), bivariate copula has been widely used to measure the correlation of financial assets. This is due to the advantages it provides by characterizing tail dependence, and it is suitable for risk management in financial markets, particularly asymmetric volatility risk (Patton, 2004; Jackwerth and Vilkov, 2019). With the development of copula models, many novel copula models have been proposed, such as the Frank copula, Gumbel copula, and Symmetrized Joe-Clayton (SJC) copula. Additional mixture copulas are also employed in futures markets (Chang, 2012; Mensi et al., 2017; Bedoui et al., 2019; Yahya et al., 2019). For example, Chang (2012) uses a mixture of Gumbel copula and Clayton copula to capture the time-varying and asymmetric dependence structure between crude oil spots and the futures market. Mensi et al. (2017) and Yahya et al. (2019) investigate the dependence structure between several commodities using a wavelet-based copula approach.

The setting of copula models in high dimensions is more complicated. Joe (1997) proposes pair-copula construction to investigate multivariate distributions and multivariate dependence structures. Bedford and Cooke (2001) introduce a vine model to the multivariate distribution. Aas et al. (2009) use pair-copula to decompose multivariate distributions, in order to exhibit high dimensional dependence structures. Fischer et al. (2009) compare several elliptical copulas, such as Student's t copula, to explore whether they can outperform benchmarks in the high dimensional cases. In recent years, vine copulas and dynamic vine copulas have been developed (Weiß and Supper, 2013; So and Yeung, 2014). Based on vine copulas, Weiß and Supper (2013) successfully forecast VaR by estimating the joint distribution of returns and bid-ask spreads. Using a dynamic vine-copula GARCH model, So and Yeung (2014) verify that the dependence between several blue-chip stocks in Hong Kong is time-varying. Nagler and Czado (2016) prove that the convergence speed is independent of the dimension in the vine copula model. That is, the vine copula model can effectively solve the "Curse of Dimensionality".

Existing literature shows that copula is often used to study the dependency structure of financial markets, and vine copula is suitable for the high-dimensional dependency structure of three or more assets. At present, studies have mainly focused on the dependence between China's commodity futures market and the stock market or the global futures markets. However, few studies have examined the dependence structure between all tradable commodity futures in China's commodity futures market. Compared with other vine copulas, the R-vine copula can allow for the analysis of more flexible high-dimension structures. Therefore, we employ the R-vine copula to investigate the dependence structure of 31 commodity futures in China.

3. METHODOLOGY

3.1. Marginal model

The financial time series is known to have some stylized facts, including weak autocorrelation and conditional heteroskedasticity (Mittnik et al., 2000). Consequently, the ARMA-GARCH model has been widely used in the modeling of univariate financial time series (Engle, 1982; Bollerslev, 1986). For the returns on commodity futures, one of the simple marginal models is the AR(1)-GARCH(1,1) model, as follows[3]:

$$X_t = \phi_0 + \phi_1 X_{t-1} + \varepsilon_t \tag{1}$$

$$\varepsilon_t = \sigma_t \cdot Z_t \tag{2}$$

and

[3] Considering Nelson (1990), Lamoureux and Lastrapes (1990), and Hillebrand (2005), we choose AR(1)-GARCH(1,1) model as the marginal model of R-vine copula.

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3}$$

where ε_t is disturbance term, Z_t is the standardized residual which has independent and identical distribution with mean 0 and variance 1, $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, and $\alpha + \beta < 1$.

In addition to this, the asymmetric effect can be characterized by extended types of GARCH models, including the EGARCH model and the GJR-GARCH model. For example, the GJR-GARCH model adds an indicator function as a dummy variable into the GARCH (1,1) model, where the indicator function depends on the sign of the error term ε_{t-1}. Thus, we will also consider the EGARCH (1,1) and GJR-GARCH (1,1) as other candidates for the marginal model. The distribution of financial time series has features of fat tail and non-zero skewness. Thus, the error term should follow non-Gaussian distributions, and we will consider Student's *t* distribution, skewed Student's *t* distribution, generalized error distribution (GED), and skewed generalized error distribution (SGED). Finally, we will use the Bayesian Information Criterion (BIC) to select the best marginal model from different types of GARCH models with various distributions of error terms.

3.2. Vine Copula

Pearson correlation coefficient is limited by the fact that it cannot measure nonlinear dependence. Sklar (1959) proposes the copula function, which is capable of capturing a nonlinear dependence structure. If we suppose that n-dimensional random vector $(X_1, \cdots, X_n)$ has the joint distribution function $F(X_1, \cdots, X_n)$, the joint density function $f(X_1, \cdots, X_n)$, the marginal distribution function $F_i(X_i)$, and the marginal density function $f_i(X_i)$, $i = 1,2, \cdots n$. Then there is a copula function $C(\cdot)$ such that:

$$F(X_1, \cdots, X_n) = C(F_1(X_1), \cdots, F_n(X_n) \tag{4}$$

and

$$f(X_1, \cdots, X_n) = c(F_1(X_1), \cdots, F_n(X_n); \theta) \cdot f_1(X_1) \cdots f_n(X_n) \tag{5}$$

where θ is the parameter vector and $\mathrm{c}(\cdot)$ is the copula density function.

Conventional pairwise copulas, such as Gaussian copula, Student's *t* copula, and Archimedean copula, can be extended for the multivariate setting. However, there are some shortcomings in such multivariate copula methods (Nelsen, 2007). The major issue is that the dependence structures formed by the multivariate copulas must have the same copula function for each pair, which largely restricts its applicability. For example, some multivariate copulas, such as multivariate Student's *t* copula, can only be used for the symmetric tail dependence between all possible pairs. In contrast, vine copulas allow different pair-copula functions to describe any symmetric or asymmetric tail dependence. To better identify the dependence structure between commodity futures markets, a flexible method, which does not impose any restrictions on the dependence structure, should be employed (Delatte and Lopez, 2013). R-vine copula is driven by real data and does not need to be constructed by any uniform form in advance (Zhang et al., 2014). Therefore, it is superior to other copulas and can better capture financial risks in the commodity markets (Koliai, 2016; Yu et al., 2018).

Bedford and Cooke (2001) originally propose the method of the vine copula, which splits a high dimensional copula function into multiple pair-copula functions in the form of vines. A vine is a nested set of trees with nodes and edges. Each tree has several nodes and the connection between two nodes is called an edge. Each node corresponds to a variable or conditional variable; that is, nodes of the first tree are variables, and nodes of the second tree are edges of the first tree. Each edge corresponds to the dependence structure, which is expressed by a conditional probability distribution function (that is a pair-copula function). In summary, vine copulas aim to express multivariate joint probability density function as the product of multiple pair-copula functions and the marginal probability density functions.

A constraint set defines the dependence structure of a vine copula model and, according to different constraint sets, the vine copula has different copula functions. For example, the C-vine copula limits the dependence structure to a star structure; the D-vine copula limits the dependence structure to a path structure; the R-vine copula can be regarded as a general case, where the C-vine copula and D-vine copula are two special cases. We describe three forms of vine copulas below:

C-vine copula: The C-vine copula model is suitable to use in cases where there is one central variable that strongly influences the other variables. The central variable and the edges associated with it are viewed as the root nodes. The C-vine copula exhibits a star structure starting from every root node. The n-dimensional joint probability density function in the C-vine copula model can be decomposed as the following equation:

$$f(X_1,\cdots,X_n)=\prod_{k=1}^{n} f_k(X_k)\prod_{j=1}^{n}\prod_{i=1}^{n-j} c_{j,j+i|1,\cdots,j-1}\left(\mathrm{F}\left(X_j|X_1,\cdots,X_{j-1}\right)F\left(X_{j+i}|X_1,\cdots,X_{j-1}\right)\right). \quad (6)$$

D-vine copula: The D-vine copula model is suitable for cases where every variable has the same influence on the others. The D-vine copula exhibits a path structure. The n-dimensional joint probability density function in the D-vine copula model can be decomposed as the following equation:

$$f(X_1,\cdots,X_n)=$$
$$\prod_{k=1}^{n} f_k(X_k)\prod_{j=1}^{n}\prod_{i=1}^{n-j} c_{i,j+i|i+1,\cdots,i+j-1}\left(\mathrm{F}\left(X_i|X_{i+1},\cdots,X_{i+j-1}\right)F\left(X_{j+i}|X_{i+1},\cdots,X_{i+j-1}\right)\right). \quad (7)$$

R-vine copula: The R-vine copula model can be applied to more general cases. R-vine copula model does not impose any constraint on edges in advance, therefore it is more flexible and can reflect a more realistic dependence structure between multi-variables (Zhang et al., 2014). The n-dimensional joint probability density function in the R-vine copula model can be decomposed as the following equation:

$$f(X_1,\cdots,X_n)=$$
$$\prod_{k=1}^{n} f_k(X_k)\prod_{j=n-1}^{1}\prod_{i=n}^{j+1} c_{m_{j,j},m_{i,j}|m_{i+1,j},\cdots,m_{n,j}}\left(\mathrm{F}\left(X_{m_{j,j}}|X_{m_{i+1,j}},\cdots,X_{m_{n,j}}\right)F\left(X_{m_{i,j}}|X_{m_{i+1,j}},\cdots,X_{m_{n,j}}\right)\right) \quad (8)$$

where $m_{i,j}$ is the R-vine matrix, and there are 2^{n-1} matrices for the n-dimensional R-vine.

Compared with the C-vine and D-vine copulas, the R-vine copula provides more flexible dependence structures, which is the incentive for using R-vine copula in this study. The flexibility of R-vine copula comes at the expense of an exponentially increasing algorithm complexity. To simplify the algorithm of selecting reasonable R-vine copula, a simple selection technique is introduced with the simplifying assumption that conditional copula functions in the second or higher trees do not depend on conditional variables (Acar et al., 2012; Dißmann et al., 2013; Kraus and Czado, 2017). Recently, Chang and Joe (2020) proposed a new selection technique without the simplifying assumption, which can give more realistic conditional distributions. However, Chang and Joe (2020) also point out that the previous technique can still depict flexible tail dependence and solve the "Curse of Dimensionality". Therefore, we choose to implement the R package “VineCopula” (Nagler et al., 2019) which is based on the simplifying assumption[4].

[4] Dette et al. (2014) find that misspecification of parametric families will cause a severe error in copula-based regression. This

3.3. GARCH-Vine Copula-VaR method and backtesting

To forecast VaR, we employ the GARCH-Vine Copula-VaR method by the expanding window scheme with re-estimation, and the steps are detailed as follows:

(1) We select a window of data and estimate the marginal model of returns on each asset (X_i) in that window.

(2) We obtain the standard residuals from the marginal model in Step (1) and implement R-vine copula upon them to analyze the dependence structure between all assets.

(3) For each asset, we simulate 10,000 random numbers in the interval [0,1] by the R-vine copula approach in Step (2).

(4) We make a one-step-ahead forecast based on the marginal model in Step (1).

(5) Based on the distribution of returns in Step (4) and simulated values in Step (3), we construct 10,000 simulated returns ($\tilde{X}_i$) for each asset on the next day.

(6) According to simulated returns in Step (5), we can calculate VaR_α ($\alpha = 90\%, 95\%, 99\%$) of each asset and portfolios on the next day, i.e.

$$VaR_\alpha(\tilde{X}_i) = inf\,\{x \in \mathbb{R}: F_{\tilde{X}_i}(x) > \alpha\} \text{ for individual asset}$$

$$VaR_\alpha\left(\sum W_i \tilde{X}_i\right) = inf\,\{x \in \mathbb{R}: F_{\sum W_i \tilde{X}_i}(x) > \alpha\} \text{ for portfolio}$$

where $F_{\tilde{X}_i}$ denotes distribution of simulated returns $\tilde{X}_i$ on asset i, W_i is the weight on asset i where $\sum W_i = 1$, and $F_{\sum W_i \tilde{X}_i}$ represents the joint distribution of simulated returns on the portfolio. The weight vector can be any weight determined by portfolio managers. Since this study does not focus on portfolio optimization, we use equal weight in this study for the purpose of demonstration. It should be highlighted that this framework can be utilized for portfolios of any weight.

(7) We fix the start date of the window and move the end date of the window forward by one day. In the same way, we repeat Step (1) - Step (6) to obtain the VaR forecast for each date in the backtesting period.

Because R-vine copula captures commodity futures' dependence structure in the joint distribution $F_{\sum W_i \tilde{X}_i}$, which refers to assets' co-movements in a portfolio, then the GARCH-Vine Copula-VaR method can give a more conservative aggregated VaR for portfolios than that of traditional GARCH-VaR method (Embrechts et al., 2013; Kakouris and Rustem, 2014). An intuitive simulation is employed to demonstrate this point. We generate three random variables with a correlation of 0.2 as daily returns on three assets, then implement the GARCH-VaR method and GARCH-Vine Copula-VaR method respectively to forecast an equally-weighted portfolio VaR with an out-of-sample period of 200 days. The simulation result is shown in Figure A.1 of the Appendix. We can observe that the GARCH-Vine Copula-VaR method produces a more conservative VaR, as it considers the correlation between multiple assets.

In terms of VaR backtesting, one of the most popular methods is the unconditional coverage (UC) test (Kupiec, 1995), which is based on the VaR violation:

$$I_t = \mathbb{I}(L_t > VaR_t^\alpha) \tag{9}$$

limitation is mainly related to the copula regression. In our paper, the parametric models are selected by BIC, which is applicable according to Dißmann et al. (2013).

where L_t represents the actual loss at time t, VaR_t^α denotes the theoretical loss at the riskiness level α at time t, and $\mathbb{I}$ is the indicator function which equals 1 if $L_t > VaR_t^\alpha$ and equals 0 otherwise. The riskiness level $\alpha = 90\%,\ 95\%,\ 99\%$. Then the number of the observed VaR violation is $S_n = \sum_{t=1}^{n} I_t$, and the observed VaR violation rate is S_n/n. It is necessary to test whether the observed VaR violation rate differs significantly from the theoretical violation rate. Thus, the null hypothesis of the UC test is as follows:

$$H_{UC}: E_{t-1}[I_t] = 1 - \alpha \tag{10}$$

However, the UC test is disadvantaged by the way that it ignores the clustering of violations (Nieto and Ruiz, 2016), and thus we also use the conditional coverage (CC) test developed by Christoffersen (1998). The null hypothesis of the CC test is as follows:

$$H_{CC}: E_{t-1}[I_t | I_{t-1}] = 1 - \alpha \tag{11}$$

Compared with UC test, CC test provides the conditional mean of the failure process. We will employ both UC and CC tests to backtest the VaR forecasting performance.

4. DATA

In China's futures markets, an active contract is a contract with the largest open interest.[5] This study uses the daily returns of active contracts in China's commodity futures market as samples. The data was downloaded from the WIND database. In March 2019, there were 53 listed commodity futures in China's futures markets, but not all commodity futures have enough liquidity to offer daily trading data. Therefore, we choose 31 commodity futures with high liquidity and divide them into nine categories[6] (see Table 1) to generate nine commodity futures indices. On 4th December 2012, *Glass* future was first launched in China's futures markets and there were 19 of our selected 31 commodity futures trading on that day, thus ensuring the construction of most indices. We therefore choose the sample period from 4th December 2012 to 29th March 2019.

For each commodity futures index, the weighted average return is shown as follows:

$$R_i = \sum_j R_{ij} \cdot \frac{Openinterest_{ij}}{\sum_j Openinterest_{ij}} \tag{12}$$

where R_i is the weighted average return of the ith category, $i = 1,2,\cdots 9$. R_{ij} is the return of the jth main future contract in the ith category. Similarly, $Openinterest_{ij}$ is the open interest of the jth main future contract in the ith category.

[Table 1 about here]

The descriptive statistics of returns of nine commodity futures indices are shown in Table 2. We can see that the average return on each index is close to zero, and *Energy* has a much larger average return than the other categories. All indices have non-zero skewness, among which *Chemical Product*, *Nonferrous Metal*, and *Precious Metal* are left-skewed, while *Agricultural and Sideline Product*, *Oil and Meal*, *Coke and Steel*, *Energy*, *Soft Commodity*, and *Non-metallic Material* are right-skewed. In addition to this, the kurtosis of every index is greater than 3, and all Jarque–Bera test (JB test) results reject the normality hypothesis. Therefore, the return of the commodity futures index does not follow a normal distribution.

[Table 2 about here]

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[5] In the international futures markets, an active contract refers to the futures contract with the largest trading volume for the commodity futures.

[6] We follow the classification in the WIND database.

Table 3 shows the Pearson coefficient (upper panel) and Kendall's τ (lower panel) of the nine commodity futures indices. These two dependence coefficients give the different magnitude of the dependence. The Pearson coefficient is a linear measure on the dependence, while Kendall's τ can capture the nonlinear dependence. Taking the coefficients between *Chemical Product* and *Nonferrous Metal* as an example, the Pearson coefficient is 0.5715, which is much larger than Kendall's τ, 0.3899. Additionally, the Pearson coefficient indicates that the correlation between *Agricultural and Sideline Product* and *Energy* is smaller than it is between *Agricultural and Sideline Product* and *Precious Metal*. At the same time, Kendall's τ reflects that the dependence coefficient between *Agricultural and Sideline Product* and *Energy* is larger than it is between *Agricultural and Sideline Product* and *Precious Metal*. To reflect the full picture of dependence structures of the commodity futures indices more accurately, we employ the R-vine copula model, which provides a more effective way of measuring multivariate dependence structures.

[Table 3 about here]

5. EMPIRICAL RESULTS

5.1. Marginal distribution of the commodity futures index

Regarding the choice of marginal model, the AR(1)-GARCH(1,1) model is popular in the field, due to its simplicity (Nelson, 1990; Lamoureux and Lastrapes, 1990; Hillebrand, 2005). We also investigate different forms of GARCH models and identify which is most appropriate. First, we compare the GARCH(1,1), E-GARCH(1,1), and GJR-GARCH(1,1) models based on the normal distribution (see Table 4). According to the BIC, we find that for most categories, the GARCH(1,1) model is superior to other models.[7] We then consider different distributions for fitting the error terms in the GARCH(1,1) model (see Table 5). According to BIC, Table 5 shows that GED is optimal for most categories. Overall, we choose the AR(1)-GARCH(1,1)-GED model as the marginal model of the R-vine copula.[8]

Table 6 shows the AR(1)-GARCH(1,1)-GED model results for the commodity futures indices. Before implementing the R-vine copula, it is necessary to test the autocorrelation and heteroscedasticity of the standardized residuals. Table 6 also shows P-values of the Ljung-Box tests and the LM tests with five order lag and ten order lag: we cannot reject the null hypotheses of "no autocorrelation" and "no ARCH effects" for all indices at the 0.05 level[9], which indicates that the standardized residuals satisfy the prerequisite of vine copulas. We obtain the standardized residuals from the marginal models and use empirical probability integral transform (PIT) to convert the standardized residuals into pseudo-observations, which is uniformly distributed in [0,1]. Finally, we apply the pseudo-observations into vine copula analysis.

[Table 4 about here]

[Table 5 about here]

[Table 6 about here]

[7] It is possible to select different marginal models for different categories. For the purposes of simplicity and consistency, we decide to use GARCH(1,1) for all nine future indices. Additionally, the final chosen marginal model, AR(1)-GARCH(1,1)-GED, can pass the Ljung-Box tests and the LM tests.
[8] The ARMA-GARCH models with other orders were also tried and the results are not very different.
[9] The Ljung-Box tests with lag ten on *Coke and Steel* cannot reject at the 0.01 level.

5.2. R-vine copula for the commodity futures index

In this section, we employ the R-vine copula for the nine commodity futures indices, and the dependence structure is shown in Table 7. R-vine copula allows different copula functions for any pair of commodity futures indices and is not limited to a specific structure, such as a star structure (in C-vine) or path structure (in D-vine). For the purpose of comparison, we also apply the C-vine copula to the nine commodity futures indices (see Figure A2 in the Appendix), and the BIC demonstrates that R-vine copula is a more effective model for investigating dependence structures in China's commodity futures market.

In the first layer (Tree 1) of the R-vine copula, there are eight pair-copulas capturing dependence structures between commodity futures indices, and all of them have significant Kendall's τ. Specifically, Kendall's τ between *Agricultural and Sideline Product* and *Oil and Meal* is the strongest, reaching 0.48, followed by *Chemical Product* and *Nonferrous Metal*, reaching 0.38. In addition, *Chemical Product* has four direct dependence edges with other commodity futures indices, including *Coke and Steel*, *Nonferrous Metal*, *Oil and Meal*, and *Soft Commodity*. Moreover, *Coke and Steel* has three direct dependence edges with *Chemical Product*, *Energy*, and *Non-metallic Material* (see Figure 1). We can conclude that China's commodity futures market is not centered on a single category of commodity futures. The second layer (Tree 2) and the third layer (Tree 3) display other conditional Kendall's τ. There are some strong conditional dependence structures, such as the dependence between *Nonferrous Metal* and *Coke and Steel* conditional on *Chemical Product*, which is 0.19 at the 0.01 significant level. The fourth and higher layers (Tree 4 to Tree 8) show there are negligible dependence structures between other commodity futures.

The R-vine copula can reveal the "full" dependence structures of all commodity futures indices, including tail dependence, which is an important factor in VaR forecasting. Some pairwise copulas in the first layer exhibit significant tail dependence. Taking *Chemical Product* and *Oil and Meal* (in Tree 1) as an example, they have both upper-tail dependence (0.03) and lower-tail dependence (0.28), and the lower-tail dependence is much larger than the upper-tail dependence. Besides, *Coke and Steel* and *Energy* only have significant lower-tail dependence (0.17). Among the nine commodity futures indices, *Agricultural and Sideline Product* and *Oil and Meal* have the highest upper-tail dependence (0.46) and the highest lower-tail dependence (0.35).

These results are useful for investors involved in China's commodity futures markets. They confirm the connectedness of commodity futures (Hua and Chen, 2007; Luo and Ji, 2018), which can be explained by the substitutability and complementarity between commodities. Thus, commodity futures prices are no longer affected only by their own supply and demand, but also by the supply and demand of other commodities. It is necessary for investors to consider the linkage effect of other commodities, rather than only focusing on the return of a single commodity. For example, when analyzing the return of *Agricultural and Sideline Product*, investors need to consider the transmission of volatility and tail risk from commodities related to it, such as *Oil and Meal*. Additionally, it is essential to consider, not only the correlation in the traditional mean-variance portfolio framework, but also the dependence structure and tail risk revealed by the R-vine copula. For example, *Chemical Product* and *Coke and Steel* are in the center of the dependence structure and they can lead to a higher level of risk due to their connectedness.

Moreover, our results are insightful for policy makers. We find that *Chemical Product* and *Coke and Steel* are closely related to other commodity futures in China's commodity futures market. The price fluctuation of these two categories of commodity futures will strongly affect the futures prices of other commodities in the market. From the policymakers' perspective, more attention should be paid to the price fluctuations of *Chemical Product* and *Coke and Steel,* as they can have a significant impact on a wide range of commodity futures. By stabilizing the prices of these two categories of commodity futures, price fluctuation in other categories can be mitigated to a certain extent, so as to stabilize the entire commodity futures market.

[Table 7 about here]

[Figure 1 about here]

5.3. Forecasting VaR based on R-vine copula

A critical application of our R-vine copula model is in the forecasting of VaR. Specifically, we employ the R-vine copula model to forecast the one-day-ahead VaR for each commodity futures index, as well as an equally-weighted portfolio. We also implement the standard GARCH-VaR method as the benchmark. To evaluate the performance of VaR from the GARCH-Vine Copula-VaR method and the GARCH-VaR method, we implement two VaR backtesting methods: UC and CC. The backtesting period for VaR is from 5th June 2018 to 29th March 2019 (200 trading days in total).

Figure 2 plots the daily returns, and forecasted 90%VaR, 95%VaR, 99%VaR of each commodity futures index and the equally-weighted portfolio.[10] Table 8 compares the VaR backtesting results of the GARCH-Vine Copula-VaR method and the GARCH-VaR method. Regarding the VaR forecasting for the individual commodity futures index, the GARCH-Vine Copula-VaR method and GARCH-VaR method perform in a similar manner. That is to say, the UC and CC tests indicate the VaR violation rates do not vary significantly from the theoretical violation rates for both methods. In terms of the VaR forecasting for the equally-weighted portfolio, the GARCH-Vine Copula-VaR method is superior to the GARCH-VaR method. Specifically, for the GARCH-Vine Copula-VaR method, the UC and CC tests indicate the VaR violation rates are not significantly different from the theoretical violation rates; for the GARCH -VaR method, the UC and CC tests indicate the VaR violation rates are significantly different from the theoretical violation rates. This can be intuitively explained by the way that the GARCH-Vine Copula-VaR method has the advantage of taking into account the dependence structure and tail dependence identified by the R-vine copula.

The empirical results show that R-vine copula is suitable for analyzing the dependence structure between multiple commodities. Consequently, we can provide more accurate portfolio VaR forecasts. It should be noted that our framework, based on the R-vine copula, can be used to forecast VaR for portfolios with any weights. As a demonstration, we show that the VaR forecasting of the equally-weighted portfolio is improved by using the identified dependence structure. Previous studies found the existence of co-movements between different commodity sectors in New York Mercantile Exchange, the Intercontinental Exchange, and Chicago Board of Trade (Trujillo-Barrera et al., 2012; Yahya et al., 2019). We show that co-movements in China's commodity futures also lead to aggregate risk in the worst cases, due to tail dependence. Our findings contribute to the accurate prediction of portfolio VaR, which is co-dependent on all assets with correlations and tail dependences in a portfolio.

Further to this, VaR can be linked with portfolio optimization. Al Janabi et al. (2017) propose a new portfolio optimization method by minimizing the risk exposure, which is measured by estimated VaR with DCC *t*-copula. Their method considers the multivariate dependence structure and tail risk between assets, as opposed to simple linear correlation. A similar framework based on the R-vine copula can be developed in order to gain diversification benefits, while controlling for

[10] Figure A3 in the Appendix shows the plot of the standard GARCH-VaR method.

the level of risk. Since this study is not focused on portfolio optimization, this is left out for further study.

[Table 8 about here]

[Figure 2 about here]

6. CONCLUSIONS

This study employs the R-vine copula to investigate high dimensional dependence structures in China's commodity futures markets and to improve VaR forecasting of commodity futures. First, according to the BIC, we select the AR(1)-GARCH(1,1)-GED model as the marginal model. Then, we employ the R-vine copula to analyze the dependence structure of commodity futures. Compared with the C-vine copula and D-vine copula, the advantage of the R-vine copula is in the fact that it has a flexible structure, which is not limited to the star structure or path structure. Based on the R-vine copula, we find that China's commodity futures market is not centered on one category of commodity futures. *Chemical Product* and *Coke and Steel* are more closely related to other commodity futures. In addition, *Agricultural and Sideline Product* and *Oil and Meal*, *Chemical Product* and *Coke and Steel* have high lower-tail dependence; *Agricultural and Sideline Product* and *Oil and Meal* have high upper-tail dependence. Finally, we employ an R-vine copula approach to forecast VaR based on the dependence structure and conclude that the GARCH-Vine copula-VaR method outperforms GARCH-VaR method for an equally-weighted portfolio.

Our results can benefit investors, policy makers, and other market participants in the futures markets. Using the R-vine copula, the dependence structures of nine indices reveals the connectedness in China's commodity futures market. Thus, investors need to consider the linkage effect of other commodities, rather than focusing only on a single commodity. Additionally, it is essential to consider, not only the correlation within the traditional mean-variance portfolio framework, but also the dependence structure and tail risk revealed by the R-vine copula. From the perspective of policy makers, who aim to stabilize the commodity futures market, more attention should be paid to the central commodity futures, *Chemical Product* and *Coke and Steel*. Their price fluctuations have a significant linkage effect on a wide range of commodity futures. Lastly, the multivariate co-dependencies, depicted by R-vine copulas, indicate co-movements between commodity futures, which, in the worst cases, can lead to aggregate risks for portfolios. Thus, market participants can use our GARCH-Vine copula-VaR method to forecast the VaR of their portfolio. Future research can use dynamic programming, coupled with dependence structure identified by R-vine copula, to study the problem of portfolio construction, such as weight optimization and dynamic adjustment.

Data Availability Statement

The data that support the findings of this study are openly available in [WIND database] at [https://www.wind.com.cn/en/edb.html], reference number [*Agricultural and Sideline Product* (code: APFI.WI); *Chemical Product* (code: CIFI.WI); *Nonferrous Metal* (code: NFFI.WI); *Oil and Meal* (code: OOFI.WI); *Coke and Steel* (code: JJRI.WI); *Energy* (code: ENFI.WI); *Precious Metal* (code: NMFI.WI); *Soft Commodity* (code: SOFI.WI); *Non-metallic Material* (code: NMBM.WI)].

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[Dataset] WIND database: https://www.wind.com.cn/en/edb.html

TABLE 1 Nine categories for commodity futures

No.	Category	Commodity Futures
1	*Agricultural and Sideline Product*	Corn, Corn Starch, Fresh Eggs, Common Wheat
2	*Chemical Product*	Bitumen, Rubber, Polyethylene, Polypropylene, PVC, Methanol, PTA
3	*Nonferrous Metal*	Aluminum, Copper, Nickel, Zinc
4	*Oil and Meal*	No.1 Soybean, Soybean Meal, Palm Olein, Crude soybean oil, Rapeseed Oil
5	*Coke and Steel*	Hot-rolled Coil, Steel Rebar, Coking Coal, Iron, Metallurgical Coke
6	*Energy*	Thermal Coal
7	*Precious Metal*	Silver, Gold
8	*Soft Commodity*	Cotton No.1, White Sugar
9	*Non-metallic Material*	Glass

Source. WIND database.

TABLE 2 Descriptive statistics of the commodity futures index

	observation	mean	maximum	minimum	standard deviation	skewness	kurtosis	JB test
Agricultural and Sideline Product	1536	0.0203	3.7608	-3.4446	0.8244	0.1702	4.9983	263.0046***
Chemical Product	1536	-0.0033	5.3169	-5.8938	1.0783	-0.1338	4.7480	200.1478***
Nonferrous Metal	1536	0.0054	5.0731	-4.6248	1.0009	-0.0843	4.8398	218.4603***
Oil and Meal	1536	-0.0001	3.9581	-4.5143	0.9126	0.0616	4.9076	233.8698***
Coke and Steel	1536	0.0295	6.7845	-7.7050	1.6000	0.0756	5.7591	488.6697***
Energy	944	0.0792	4.5822	-5.5926	1.4181	0.0349	4.3748	74.5303***
Precious Metal	1536	-0.0384	6.2685	-8.5631	1.1353	-0.1996	9.2823	2536.1305***
Soft Commodity	1536	-0.0107	3.7336	-3.6703	0.7833	0.1326	5.2609	331.6453***
Non-metallic Material	1536	0.0355	5.8140	-5.0719	1.3034	0.1598	5.1745	309.1605***

Note. ***，**，* indicate that the coefficient is significant at the level of 1%, 5%, 10%, respectively.

TABLE 3 Dependence coefficient of the commodity futures index

	Agricultural and Sideline Product	*Chemical Product*	*Nonferrous Metal*	*Oil and Meal*	*Coke and Steel*	*Energy*	*Precious Metal*	*Soft Commodity*	*Non-metallic Material*
				Pearson linear correlation coefficient					
Agricultural and Sideline Product	1.0000***								
Chemical Product	0.3360***	1.0000***							
Nonferrous Metal	0.2611***	0.5715***	1.0000***						
Oil and Meal	0.7087***	0.4385***	0.3511***	1.0000***					
Coke and Steel	0.1865***	0.5207***	0.5229***	0.2615***	1.0000***				
Energy	0.1359***	0.3400***	0.3145***	0.1608***	0.4528***	1.0000***			
Precious Metal	0.1531***	0.3112***	0.3469***	0.2271***	0.1764***	0.1083***	1.0000***		
Soft Commodity	0.3416***	0.4024***	0.3363***	0.3671***	0.2317***	0.2032***	0.2022***	1.0000***	
Non-metallic Material	0.2117***	0.4326***	0.3681***	0.2545***	0.5356***	0.3289***	0.1656***	0.2308***	1.0000***
				Kendall's τ					
Agricultural and Sideline Product	1.0000***								
Chemical Product	0.1975***	1.0000***							
Nonferrous Metal	0.1551***	0.3899***	1.0000***						
Oil and Meal	0.4798***	0.2874***	0.2328***	1.0000***					
Coke and Steel	0.1099***	0.3556***	0.3448***	0.1679***	1.0000***				
Energy	0.0725***	0.2431***	0.2132***	0.0983***	0.2760***	1.0000***			
Precious Metal	0.0634***	0.1792***	0.2263***	0.1303***	0.1170***	0.0725***	1.0000***		
Soft Commodity	0.2008***	0.2636***	0.2260***	0.2283***	0.1617***	0.1221***	0.1175***	1.0000***	
Non-metallic Material	0.1191***	0.2936***	0.2455***	0.1581***	0.3642***	0.2265***	0.0959***	0.1470***	1.0000***

Note. ***， **， * indicate that the coefficient is significant at the level of 1%, 5%, 10%, respectively.

TABLE 4 BIC of different GARCH models for the commodity futures index

	Agricultural and Sideline Product	*Chemical Product*	*Non-ferrous Metal*	*Oil and Meal*	*Coke and Steel*	*Energy*	*Precious Metal*	*Soft Commodity*	*Non-metallic Material*
GARCH(1,1)	2.3565	2.9331	2.7815	2.6098	3.5388	3.4807	2.9066	2.3056	3.2970
EGARCH(1,1)	2.3567	2.9383	2.7862	2.6142	3.5447	3.4881	2.9018	2.3053	3.3037
GJR-GARCH(1,1)	2.3587	2.9375	2.7860	2.6085	3.5435	3.4861	2.9064	2.3101	3.3013

TABLE 5 BIC of GARCH (1,1) models with different distributions for the commodity futures index

	Agricultural and Sideline Product	*Chemical Product*	*Non-ferrous Metal*	*Oil and Meal*	*Coke and Steel*	*Energy*	*Precious Metal*	*Soft Commodity*	*Non-metallic Material*
Normal	2.3565	2.9331	2.7815	2.6098	3.5388	3.4807	2.9066	2.3056	3.2970
Skewed normal	2.3609	2.9350	2.7826	2.6146	3.5428	3.4879	2.9077	2.3104	3.3018
Student's *t*	2.3265	2.9073	2.7447	2.5686	3.5170	3.4487	2.7069	2.2527	3.2315
Skewed Student's *t*	2.3312	2.9110	2.7457	2.5733	3.5214	3.4557	2.7116	2.2574	3.2362
GED	2.3217	2.9042	2.7539	2.5695	3.5081	3.4278	2.7116	2.2500	3.2233
SGED	2.3265	2.9080	2.7546	2.5742	3.5126	3.4350	2.7161	2.2546	3.2281

TABLE 6 The AR(1)-GARCH(1,1)-GED model for the commodity futures index

	Agricultural and Sideline Product	*Chemical Product*	*Nonferrous Metal*	*Oil and Meal*	*Coke and Steel*	*Energy*	*Precious Metal*	*Soft Commodity*	*Non-metallic Material*
ϕ_0	0.0220	-0.0014	0.0181	-0.0093	-0.0361	-0.0106	-0.0229***	-0.0212	0.0048
	(1.2301)	(-0.0657)	(0.9254)	(-0.4725)	(-1.3278)	(-1.5282)	(-4.0642)	(-1.4993)	(0.1719)
ϕ_1	-0.0198	-0.0077	-0.0879***	-0.0229	-0.0063	-0.0209*	-0.0532***	-0.0370	-0.0600*
	(-0.7717)	(-0.4135)	(-3.5855)	(-1.0960)	(-0.3410)	(-1.8724)	(-6.6005)	(-1.3987)	(-1.9543)
ω_0	0.0046*	0.0084*	0.0118*	0.0263**	0.0120*	0.0152	0.0031	0.0120*	0.0072*
	(1.9597)	(1.8643)	(1.8832)	(2.4227)	(1.7069)	(1.4205)	(1.6257)	(1.8741)	(1.7165)
α	0.0392***	0.0431***	0.0482***	0.0493***	0.0572***	0.0447***	0.0276***	0.0491***	0.0259***
	(6.0754)	(4.8822)	(4.2747)	(4.1039)	(4.6723)	(4.4324)	(6.4953)	(3.6532)	(7.8290)
β	0.9546***	0.9507***	0.9409***	0.9182***	0.9391***	0.9496***	0.9689***	0.9320***	0.9696***
	(142.9868)	(100.0429)	(65.3883)	(43.5095)	(74.6707)	(108.2068)	(237.5776)	(46.7927)	(561.7815)
BIC	2.3217	2.9042	2.7539	2.5695	3.5081	3.4278	2.7116	2.2500	3.2233
				autocorrelation and heteroscedasticity test for the standardized residuals					
LB (5)	0.9555	0.8281	0.1581	0.6821	0.1486	0.7469	0.2466	0.4718	0.9642
LB (10)	0.7821	0.1309	0.0762	0.8647	0.0448	0.9431	0.1997	0.4279	0.9923
LM (5)	0.8634	0.0776	0.1061	0.3996	0.9918	0.9475	0.0530	0.9770	0.5464
LB (10)	0.9341	0.1132	0.3308	0.8211	0.8886	0.9142	0.2178	0.9992	0.7793

Note. ***， **， * indicate that the coefficient is significant at the level of 1%, 5%, 10%, respectively, and the number in parenthesis is the t-statistic.

TABLE 7 Dependence structure of the commodity futures indices with R-vine copula

Tree	Edge	Copula	Parameter 1	Parameter 2	Kendall's τ	Upper-tail dependence	Lower-tail dependence
Tree1	5,9	t	0.53	12.26	0.36***	0.07***	0.07***
	5,6	C	0.40	-	0.17***	-	0.17***
	2,5	SBB1	0.24	1.37	0.35***	0.12***	0.34***
	3,7	SBB1	0.18	1.18	0.22***	0.04***	0.20***
	2,3	N	0.56	-	0.38***	-	-
	4,1	BB1	0.42	1.60	0.48***	0.46***	0.35***
	2,4	SBB1	0.15	1.28	0.27***	0.03***	0.28***
	8,2	N	0.39	-	0.25***	-	-
Tree2	2,9;5	t	0.20	12.30	0.13***	0.01***	0.01***
	2,6;5	F	0.73	-	0.08***	-	-
	3,5;2	N	0.29	-	0.19***	-	-
	2,7;3	F	0.60	-	0.07***	-	-
	8,3;2	N	0.14	-	0.09***	-	-
	2,1;4	I	-	-	0.00	-	-
	8,4;2	N	0.22	-	0.14***	-	-
Tree3	6,9;2,5	I	-	-	0.00	-	-
	3,6;2,5	I	-	-	0.00	-	-
	7,5;3,2	I	-	-	0.00	-	-
	8,7;2,3	I	-	-	0.00	-	-
	4,3;8,2	F	0.61	-	0.07***	-	-
	8,1;2,4	N	0.09	-	0.06***	-	-
Tree4	3,9;6,2,5	I	-	-	0.00	-	-
	7,6;3,2,5	I	-	-	0.00	-	-
	8,5;7,3,2	I	-	-	0.00	-	-
	4,7;8,2,3	I	-	-	0.00	-	-
	1,3;4,8,2	I	-	-	0.00	-	-
Tree5	7,9;3,6,2,5	I	-	-	0.00	-	-
	8,6;7,3,2,5	I	-	-	0.00	-	-
	4,5;8,7,3,2	I	-	-	0.00	-	-
	1,7;4,8,2,3	I	-	-	0.00	-	-
Tree6	8,9;7,3,6,2,5	I	-	-	0.00	-	-
	4,6;8,7,3,2,5	I	-	-	0.00	-	-
	1,5;4,8,7,3,2	I	-	-	0.00	-	-
Tree7	4,9;8,7,3,6,2,5	I	-	-	0.00	-	-
	1,6;4,8,7,3,2,5	I	-	-	0.00	-	-
Tree8	1,9;4,8,7,3,6,2,5	I	-	-	0.00	-	-
BIC				-2278.21			

Note. ***, **, * indicate that the coefficient is significant at the level of 1%, 5%, 10%, respectively.

1: *Agricultural and Sideline Product*, 2: *Chemical Product*, 3: *Nonferrous Metal*, 4: *Oil and Meal*, 5: *Coke and Steel*, 6: *Energy*, 7: *Precious Metal*, 8: *Soft Commodity*, 9: *Non-metallic Material*.

t: Student's *t* copula, C: Clayton copula, F: Frank copula, N: Gaussian copula, BB1: BB1 copula, SBB1: Rotated BB1 copula 180 degrees, I: Independence copula.

TABLE 8 VaR backtesting for the commodity futures index

		GARCH-Vine Copula-VaR			GARCH -VaR		
		99% VaR	95% VaR	90% VaR	99% VaR	95% VaR	90% VaR
	violation rate	1.50%	6.00%	9.50%	1.50%	6.00%	10.00%
Agricultural and Sideline Product	UC	**(0.5082)**	**(0.5287)**	**(0.8123)**	**(0.5082)**	**(0.5287)**	**(1.0000)**
	CC	**[0.7673]**	**[0.7768]**	**[0.7507]**	**[0.7673]**	**[0.7768]**	**[1.0000]**
	violation rate	1.00%	4.00%	10.50%	1.00%	6.00%	10.00%
Chemical Product	UC	**(1.0000)**	**(0.3512)**	**(0.8150)**	**(1.0000)**	**(0.5287)**	**(1.0000)**
	CC	**[0.9799]**	**[0.2608]**	**[0.9599]**	**[0.9799]**	**[0.3795]**	**[1.0000]**
	violation rate	1.00%	3.67%	8.50%	1.00%	4.00%	8.50%
Nonferrous Metal	UC	**(1.0000)**	**(0.5020)**	**(0.4691)**	**(1.0000)**	**(0.5020)**	**(0.4691)**
	CC	**[0.9799]**	**[0.4750]**	**[0.6874]**	**[0.9799]**	**[0.4750]**	**[0.6874]**
	violation rate	1.00%	4.50%	8.00%	1.00%	4.50%	8.50%
Oil and Meal	UC	**(1.0000)**	**(0.7416)**	**(0.3303)**	**(1.0000)**	**(0.7416)**	**(0.4691)**
	CC	**[0.9799]**	**[0.6183]**	**[0.5980]**	**[0.9799]**	**[0.6183]**	**[0.7013]**
	violation rate	1.00%	3.50%	10.00%	1.00%	3.50%	10.00%
Coke and Steel	UC	**(1.0000)**	**(0.3047)**	**(1.0000)**	**(1.0000)**	**(0.3047)**	**(1.0000)**
	CC	**[0.9799]**	**[0.2827]**	**[0.6902]**	**[0.9798]**	**[0.2827]**	**[0.6092]**
	violation rate	0.00%	2.00%	8.00%	0.00%	2.00%	8.00%
Energy	UC	**(1.0000)**	(0.0275)	**(0.3303)**	**(1.0000)**	(0.0275)	**(0.3303)**
	CC	**[1.0000]**	**[0.0812]**	**[0.5069]**	**[1.0000]**	**[0.0812]**	**[0.5069]**
	violation rate	2.00%	4.50%	7.00%	2.00%	5.00%	7.50%
Precious Metal	UC	**(0.2109)**	**(0.7416)**	**(0.1370)**	**(0.2109)**	**(1.0000)**	**(0.2196)**
	CC	**[0.4299]**	**[0.6489]**	**[0.1855]**	**[0.4299]**	**[0.6218]**	**[0.3140]**
	violation rate	0.50%	7.00%	12.00%	0.50%	7.00%	11.50%
Soft Commodity	UC	**(0.4315)**	**(0.2197)**	**(0.3591)**	**(0.4315)**	**(0.2197)**	**(0.4887)**
	CC	**[0.7302]**	**[0.1764]**	**[0.4783]**	**[0.7302]**	**[0.1764]**	**[0.4914]**
	violation rate	1.00%	3.50%	11.50%	1.00%	3.50%	11.50%
Non-metallic Material	UC	**(1.0000)**	**(0.3047)**	**(0.4887)**	**(1.0000)**	**(0.3047)**	**(0.4887)**
	CC	**[0.9799]**	**[0.4574]**	**[0.7037]**	**[0.9799]**	**[0.4574]**	**[0.7037]**
	violation rate	0.50%	3.00%	7.50%	6.00%	12.00%	20.00%
Equally-weighted Portfolio	UC	**(0.4315)**	**(0.1622)**	**(0.2196)**	(0.0000)	(0.0001)	(0.0000)
	CC	**[0.7302]**	**[0.1366]**	**[0.3386]**	[0.0000]	[0.0004]	[0.0001]

Note. The first row of each cell is the empirical VaR violation rate. Numbers in parentheses are P-values of the UC test, and numbers in square brackets are P-values of the CC test. P-values which are greater than 0.05 are in bold, meaning that no evidence against the null hypothesis at the 0.05 level.

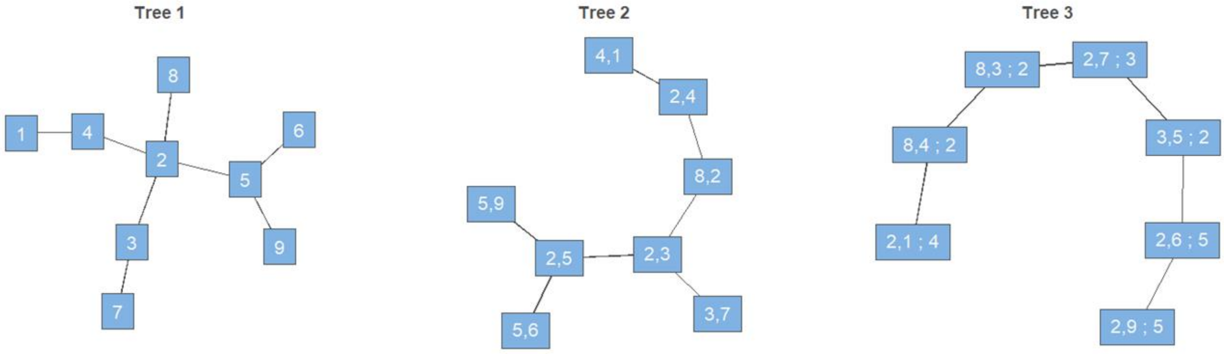

FIGURE 1 The first three layers' structure visualization of R-vine-copula for the commodity futures index (1: *Agricultural and Sideline Product*, 2: *Chemical Product*, 3: *Nonferrous Metal*, 4: *Oil and Meal*, 5: *Coke and Steel*, 6: *Energy*, 7: *Precious Metal*, 8: *Soft Commodity*, 9: *Non-metallic Material*.)

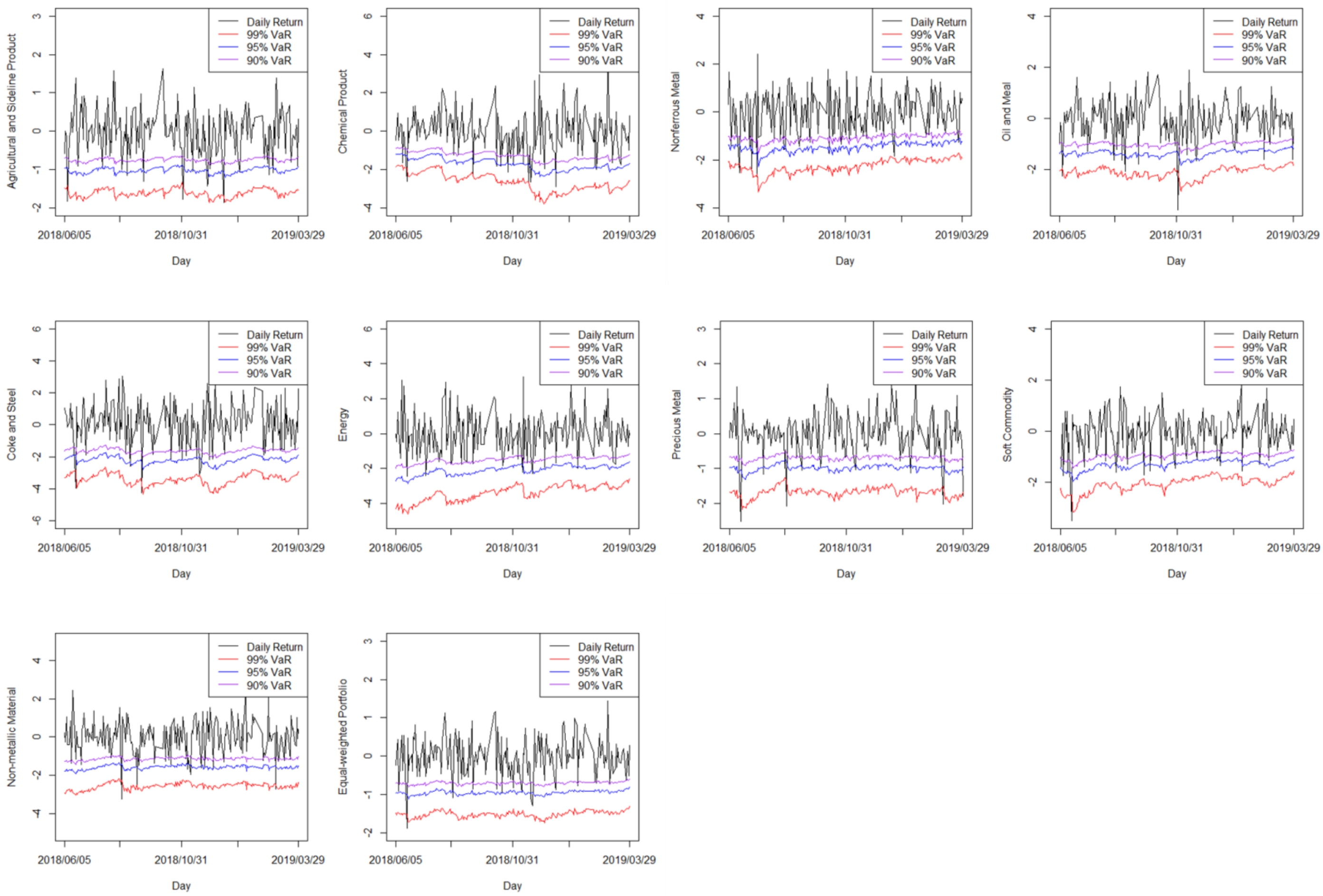

FIGURE 2 GARCH-Vine Copula-VaR method for the commodity futures index

APPENDIX

TABLE A.1 Dependence structure of the commodity futures indices with C-vine copula

Tree	Edge	Copula	Parameter 1	Parameter 2	Kendall's τ	Upper-tail dependence	Lower-tail dependence
Tree1	2,1	t	0.30	11.04	0.19***	0.03***	0.03***
	2,3	N	0.56	-	0.38***	-	-
	2,6	Tawn180	1.41	0.28	0.13***	-	0.17***
	2,8	N	0.39	-	0.25***	-	-
	2,7	BB7	1.11	0.28	0.17***	0.14***	0.08***
	2,5	SBB1	0.24	1.37	0.35***	0.12***	0.34***
	2,4	SBB1	0.15	1.28	0.27***	0.03***	0.28***
	9,2	t	0.43	10.33	0.29***	0.06***	0.06***
Tree2	4,1;2	BB1	0.34	1.54	0.44***	0.43***	0.26***
	4,3;2	F	0.76	-	0.08***	-	-
	4,6;2	I	-	-	0.00	-	-
	4,8;2	N	0.22	-	0.14***	-	-
	4,7;2	SC	0.10	-	0.05***	0.00	-
	4,5;2	I	-	-	0.00	-	-
	9,4;2	I	-	-	0.00	-	-
Tree3	5,1;4,2	I	-	-	0.00	-	-
	5,3;4,2	F	1.80	-	0.19***	-	-
	5,6;4,2	C	0.23	-	0.10***	-	0.05***
	5,8;4,2	I	-	-	0.00	-	-
	5,7;4,2	I	-	-	0.00	-	-
	9,5;4,2	N	0.39	-	0.26***	-	-
Tree4	3,1;5,4,2	I	-	-	0.00	-	-
	3,6;5,4,2	I	-	-	0.00	-	-
	3,7;5,4,2	N	0.23	-	0.15***	-	-
	3,8;5,4,2	N	0.12	-	0.07***	-	-
	9,3;5,4,2	I	-	-	0.00	-	-
Tree5	8,1;3,5,4,2	N	0.10	-	0.06***	-	-
	8,6;3,5,4,2	I	-	-	0.00	-	-
	8,7;3,5,4,2	I	-	-	0.00	-	-
	9,8;3,5,4,2	I	-	-	0.00	-	-
Tree6	9,1;8,3,5,4,2	I	-	-	0.00	-	-
	9,6;8,3,5,4,2	I	-	-	0.00	-	-
	9,7;8,3,5,4,2	I	-	-	0.00	-	-
Tree7	7,1;9,8,3,5,4,2	I	-	-	0.00	-	-
	7,6;9,8,3,5,4,2	I	-	-	0.00	-	-
Tree8	6,1;7,9,8,3,5,4,2	I	-	-	0.00	-	-
BIC				-2262.32			

Note. ***, **, * indicate that the coefficient is significant at the level of 1%, 5%, 10%, respectively.

1: *Agricultural and Sideline Product*, 2: *Chemical Product*, 3: *Nonferrous Metal*, 4: *Oil and Meal*, 5: *Coke and Steel*, 6: *Energy*, 7: *Precious Metal*, 8: *Soft Commodity*, 9: *Non-metallic Material*.

t: Student's *t* copula, F: Frank copula, N: Gaussian copula, C: Clayton copula, SC: Rotated Clayton copula 180 degrees, BB1: BB1 copula, BB7: BB7 copula, SBB1: Rotated BB1 copula 180 degrees, Tawn180: Rotated Tawn type 1 copula 180 degrees, I: Independence copula.

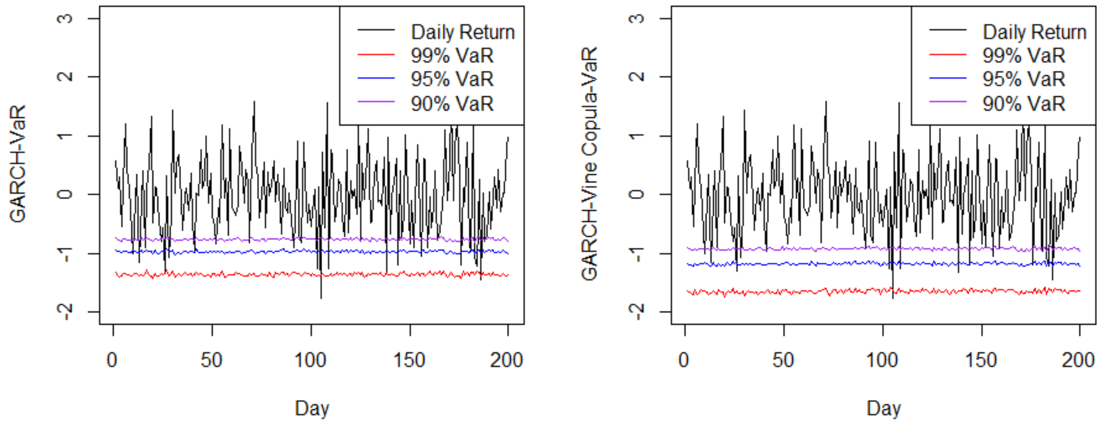

FIGURE A.1 Simulation Results of GARCH-VaR method (left panel) and GARCH-Vine Copula VaR method (right panel)

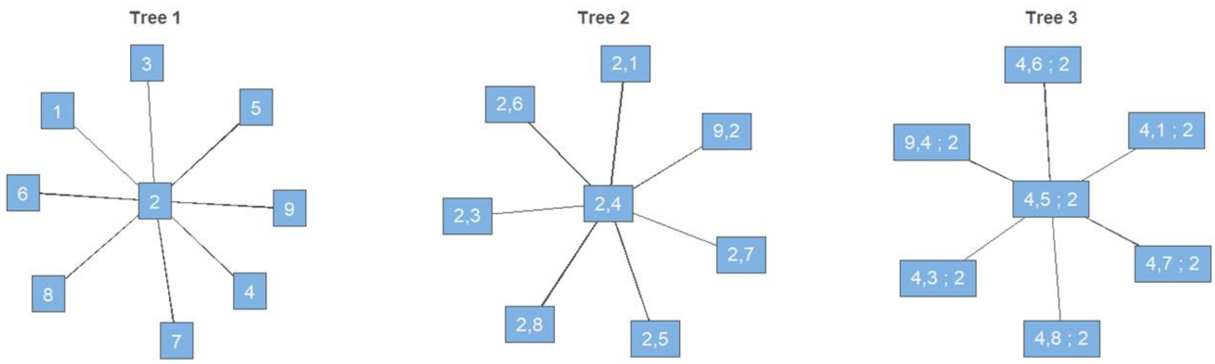

FIGURE A.2 The first three layers' structure visualization of C-vine-copula for the commodity futures index (1: *Agricultural and Sideline Product*, 2: *Chemical Product*, 3: *Nonferrous Metal*, 4: *Oil and Meal*, 5: *Coke and Steel*, 6: *Energy*, 7: *Precious Metal*, 8: *Soft Commodity*, 9: *Non-metallic Material*.)

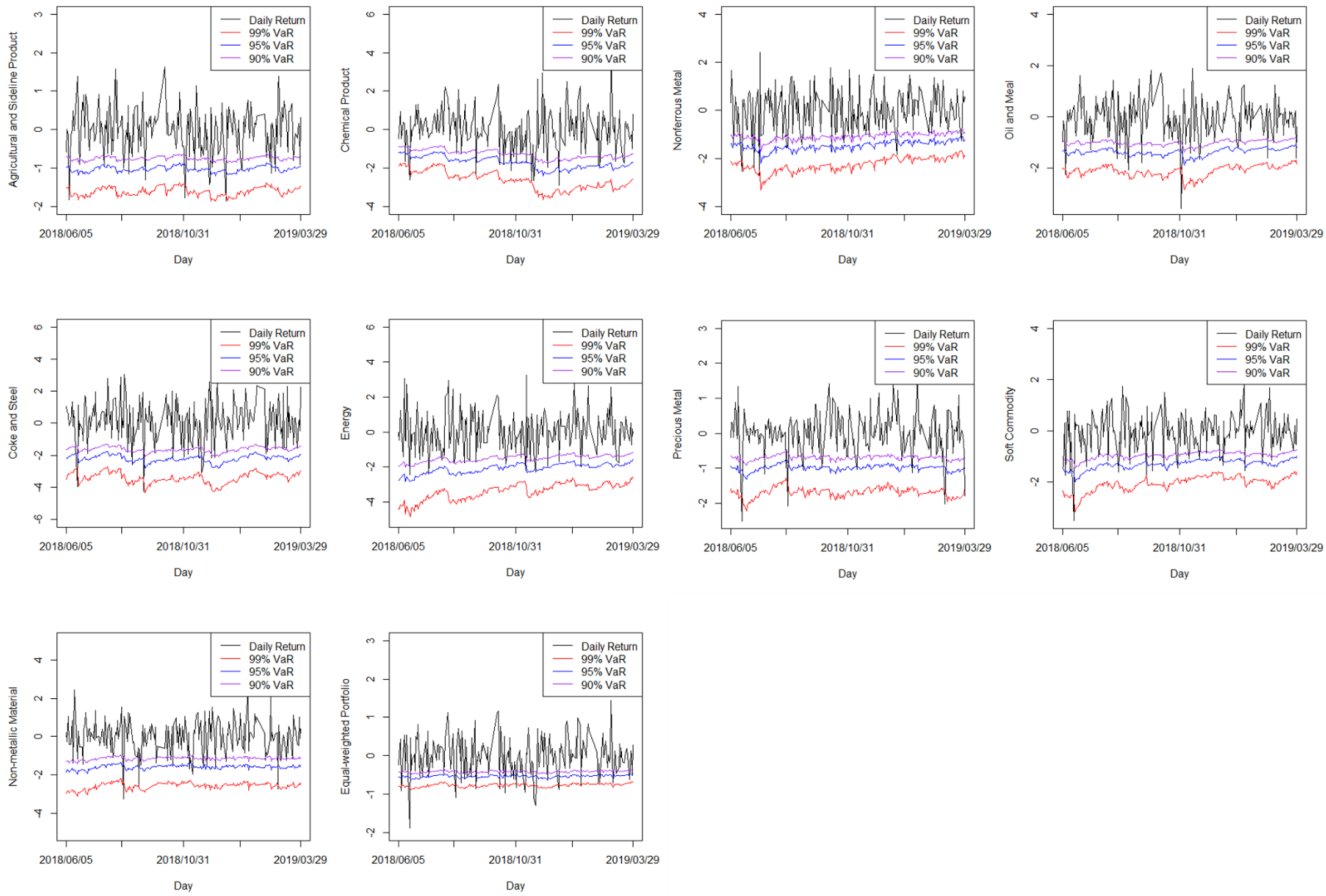

FIGURE A.3 GARCH-VaR method for the commodity futures index