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Abstract. In wireless communications using massive multiple-input multiple-output (MIMO) channel modalities as would be required for communications with distributed sensing modalities to enable Internet of Things (IoT) connectivity, channel equalization must be performed following channel estimation using system identification tools. This contribution shows the necessity for extending existing subspace multiple output-error state space (MOESP) algorithms with their fractional-order equivalents to perform channel identification.

1. Introduction

Massive multiple-input multiple-output (MIMO) is a technology that uses hundreds of antenna elements at the base station to service tens of terminals in the same time-frequency resource [1]. This technology is being proposed for fifth generation (5G) wireless communications, and is said to achieve the benefits of multiuser MIMO such as increased capacity, increased data rate, enhanced reliability, reduced latency, improved energy efficiency, improved spectrum efficiency and reduced interference but at a greater extend [2] and with simple linear processing. But one of the limiting factors in achieving these benefits in massive MIMO systems is the channel estimation accuracy. In this paper we propose the use of state-space models to estimate the channel, i.e. system identification. Subspace system identification (SSI) algorithms namely the MOESP fractional-order algorithm will be used to identify the system. Works using subspace identification in communications can be found in [3 – 5].

2. System model

We consider a massive MIMO wireless system as shown in figure 1 with a base station equipped with m transmitting antenna elements and a terminal station equipped with p receiving antenna elements, where in massive MIMO systems, $m \gg p$.

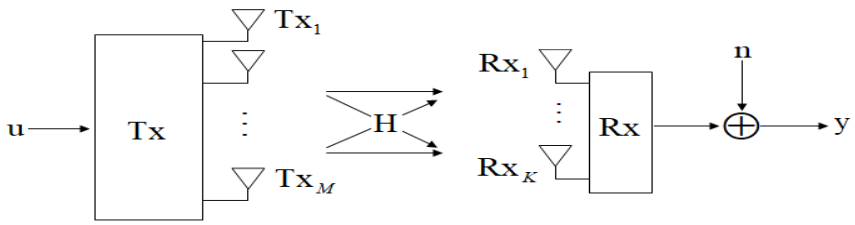

Figure 1. Massive MIMO system.

We assume that the channel is quasi-static. Training symbols known to both the transmitter and receiver are inserted at the start of each frame to assist with channel estimation. The receiver then applies this input-output data to the MOESP fractional-order subspace algorithm to estimate the massive MIMO channel. The following assumptions are necessary for system identification: The system is persistently excited by the training symbols. The system is stable, observable and controllable. The dimension of matrix $\boldsymbol{A}$ as in (2) is known, and $\text{rank}(\boldsymbol{D}) = n$, where n is the order of the system. Lastly, the random noise is irrelevant to the input signal.

The received signal is expressed as:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{u} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{y}$ is the $p \times 1$ received signal vector, $\boldsymbol{u}$ is the $m \times 1$ transmitted signal vector, $\boldsymbol{H}$ is the $p \times m$ Rayleigh fading channel matrix and $\boldsymbol{n} \sim CN(0, N_0 \boldsymbol{I}_p)$ is the $p \times 1$ additive white Gaussian noise vector at the receiver side, with N_0 being the noise power and $\boldsymbol{I}_p$ is the $p \times p$ identity matrix. We consider flat Rayleigh fading in which the fading coefficients are assumed to be independent and identically distributed (i.i.d.) and circularly symmetric Gaussian random variables with zero mean and unit variance, $CN(0,1)$.

3. System identification using the MOESP fractional-order model

The dynamics of the massive multiple-input multiple-output linear-time invariant (MIMO LTI) system can be modelled using fractional-order state-space model and (1) can be expressed as [6]:

$$D^{\alpha}\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t), \;\; \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) \tag{2}$$

where $\boldsymbol{x}(t)$ is the $n \times 1$ state vector, $\boldsymbol{u}(t)$ is the $m \times 1$ input vector, $\boldsymbol{y}(t)$ is the $p \times 1$ output vector, $\boldsymbol{A}$ is the $n \times n$ system matrix, $\boldsymbol{B}$ is the $n \times m$ control matrix, $\boldsymbol{C}$ is the $p \times n$ output matrix, $\boldsymbol{D}$ is the $p \times m$ feed-forward matrix and α is the commensurate fractional-order. It is important to know if a fractional-order system is stable or not, and a fractional-order system is stable if $0 < \alpha < 2$ and $|\arg(\lambda_k)| > \alpha\pi/2$ and $-\pi < \arg(\lambda_k) \le \pi$ where λ_k corresponds to the k-th eigenvalue of $\boldsymbol{A}$ [7].

Taking the Laplace transform of (2) the transfer function of the system is written as:

$$\boldsymbol{G}(s) = \frac{\boldsymbol{Y}(s)}{\boldsymbol{U}(s)} = \boldsymbol{C}\left(s^{\alpha}\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{B} + \boldsymbol{D} \tag{3}$$

Since the continuous-time state-space representation of commensurate fractional-order systems is similar to that of integer-order systems, the analysis of the MOESP fractional-order model will follow that of the classical MOESP model as proposed in [6]. For simplicity, we ignore the effects of the additive noise and after several α -order fractional derivatives of (2) we obtain:

$$\underbrace{\begin{bmatrix} \boldsymbol{y}(t) \\ D^{\alpha}\boldsymbol{y}(t) \\ \vdots \\ D^{(i-1)\alpha}\boldsymbol{y}(t) \end{bmatrix}}_{\boldsymbol{y}_i(t)} = \underbrace{\begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{C}\boldsymbol{A} \\ \vdots \\ \boldsymbol{C}\boldsymbol{A}^{i-1} \end{bmatrix}}_{\boldsymbol{\Gamma}_i} \boldsymbol{x}(t) + \underbrace{\begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{C}\boldsymbol{B} & \boldsymbol{D} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{C}\boldsymbol{A}^{i-2}\boldsymbol{B} & \boldsymbol{C}\boldsymbol{A}^{i-3}\boldsymbol{B} & \cdots & \boldsymbol{D} \end{bmatrix}}_{\boldsymbol{\Omega}_i} \underbrace{\begin{bmatrix} \boldsymbol{u}(t) \\ D^{\alpha}\boldsymbol{u}(t) \\ \vdots \\ D^{(i-1)\alpha}\boldsymbol{u}(t) \end{bmatrix}}_{\boldsymbol{u}_i(t)} \tag{4}$$

which can be written as:

$$\boldsymbol{y}_i(t) = \boldsymbol{\Gamma}_i \boldsymbol{x}(t) + \boldsymbol{\Omega}_i \boldsymbol{u}_i(t) \tag{5}$$

where $\boldsymbol{\Gamma}_i \in \mathbb{R}^{ip\times n}$ and $\boldsymbol{\Omega}_i \in \mathbb{R}^{ip\times im}$ are the observability and controllability matrices respectively, and the dimensions of $\boldsymbol{y}_i(t) \in \mathbb{R}^{ip}$ and $\boldsymbol{u}_i(t) \in \mathbb{R}^{im}$. $\boldsymbol{\Gamma}_i$ plays a great role in finding matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ from which the transfer function of the identified system is then derived.

Unlike integer-order systems where subspace methods can be directly used to find $\boldsymbol{\Gamma}_i$ by employing simple mathematical tools such as singular value decomposition (SVD), in fractional-order systems, (5) first has to be transformed after which the mathematical tools can then be applied to the transformed equation to find $\boldsymbol{\Gamma}_i$. If we choose a sampling time of, T_s and setting $\boldsymbol{y}_i(k) \triangleq \boldsymbol{y}_i(kT_s)$ and $\boldsymbol{u}_i(k) \triangleq \boldsymbol{u}_i(kT_s)$, we can write:

$$\boldsymbol{Y}_{i,M} = \begin{bmatrix} \boldsymbol{y}_i(1) & \boldsymbol{y}_i(2) & \cdots & \boldsymbol{y}_i(M) \end{bmatrix}, \boldsymbol{U}_{i,M} = \begin{bmatrix} \boldsymbol{u}_i(1) & \boldsymbol{u}_i(2) & \cdots & \boldsymbol{u}_i(M) \end{bmatrix}, \boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}(1) & \boldsymbol{x}(2) & \cdots & \boldsymbol{x}(M) \end{bmatrix}$$

From the above expressions, (5) can then be transformed into the following equation:

$$\boldsymbol{Y}_{i,M} = \boldsymbol{\Gamma}_i \boldsymbol{X} + \boldsymbol{\Omega}_i \boldsymbol{U}_{i,M} \tag{6}$$

where (6) is the transformed equation of (5), to which the MOESP algorithm and SVD are then applied to determine an estimate of $\boldsymbol{\Gamma}_i$ from which the estimates of $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ can then be obtained according to [6]. In the practical sense, transfer functions such as (3) are not easy to implement, and this has led to the rise in rational transfer functions that can be used to approximate these fractional-order transfer functions. This means that whenever we have a fractional-order transfer function in system identification there is need to replace it with an easier to handle approximate rational transfer function. The following section deals with the approximate rational transfer function.

3.1. Fractional-order realisation

In the current work an equivalent continuous-time rational model obtained from approximating a fractional-order differentiation operator by a rational one is used to get the fractional-order model output. The fractional behaviour of systems is usually limited within a specific frequency range, i.e. lower frequency and upper frequency denoted as (ω_L, ω_H) [7]. The lower frequency is limited by the input data spectrum, whilst the upper frequency is limited by the sampling period. Thus fractional-order systems must have the same dynamics as their approximated continuous-time rational counterparts within that specific frequency range. In this paper we consider the Oustaloup's realisation.

4. Results

We present the simulation results to demonstrate the performance of the classical MOESP and the fractional-order MOESP algorithms in the identification of a massive MIMO system. The massive MIMO system was modelled to have the base station equipped with $m = 100$ transmitting antenna elements and the terminal to have $p = 1$ receiving antenna element. A chirp signal with frequency ranging from 0 to $20Hz$, sampling frequency $1kHz$ is used for training. The channel is a Rayleigh fading channel. The classical MOESP algorithm is applied to the input-output data to obtain the integer-order transfer functions for different system orders. Figure 2 compares the actual system and MOESP estimated system outputs for (a) $n = 1$, (b) $n = 2$, (c) $n = 3$ and (d) $n = 4$. It can be seen that their performance improves as the system order increases up to order three, but then the performance reverts to being poor at the fourth order. This is to show that integer-order MOESP algorithms may not be able to sufficiently model the dynamics of the massive MIMO system. We again modelled the same massive MIMO system but this time using the fractional-order MOESP algorithm. Figure 2 (e) compares the actual system and fractional-order MOESP estimated system outputs after applying the

Oustaloup's realisation. It can be seen that there is improvement in the performance of the fractional-order MOESP estimated model compared to the integer-order MOESP estimated model.

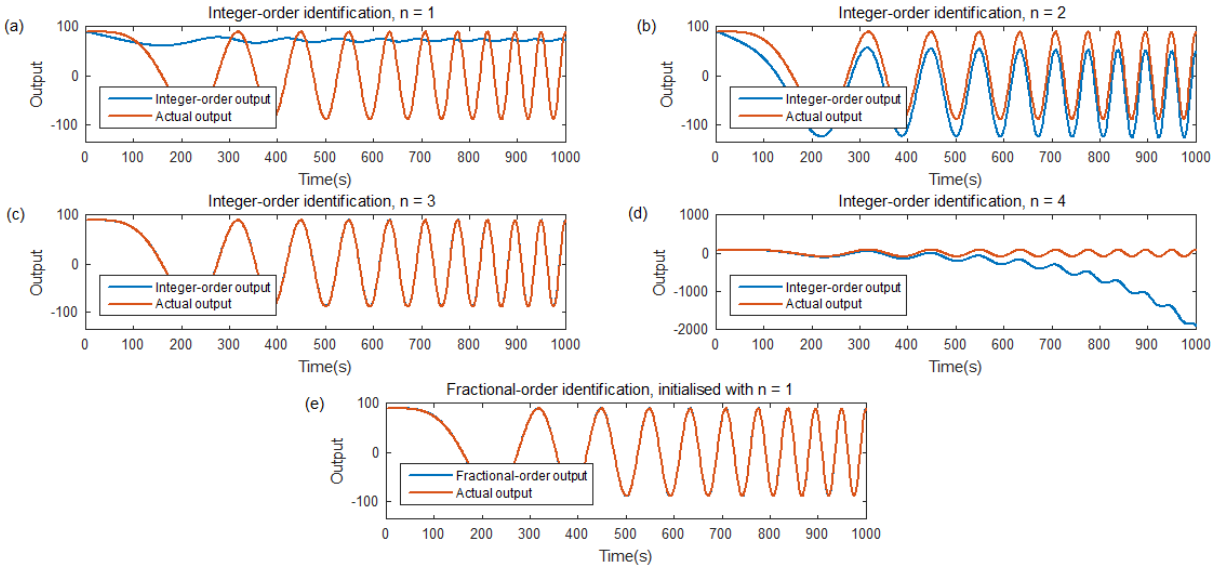

Figure 2. Massive MIMO actual system output and MOESP estimated system output for integer-order (a) $n=1$, (b) $n=2$, (c) $n=3$ and (d) $n=4$. (e) Massive MIMO actual system output and fractional-order MOESP estimated system output for fractional-order $\alpha=0.1$ within the frequency range $(\omega_L,\omega_H)=(20,1000)$ and system initialised with order $n=1$.

5. Conclusion

We were able to perform massive MIMO system identification using the classical MOESP and fractional-order MOESP algorithms. The classical MOESP algorithm showed some improvement with increase in system order but then the performance degraded on the fourth order. We then extended the identification algorithm to the fractional-order MOESP algorithm, where it was shown to outperform the classical MOESP algorithm. These set of results show that the fractional-order MOESP algorithm can be one of the techniques used for channel estimation in massive MIMO systems.

6. References

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