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ANALYSIS OF THE DISTRIBUTION OF THE NUMBER OF BIDDERS IN CONSTRUCTION CONTRACT AUCTIONS

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Analysis of the distribution of the number of bidders in construction contract auctions

Abstract

The number of bidders, *N*, involved in a construction procurement auction is known to have an important effect on the value of the lowest bid and the mark up applied by bidders. In practice, for example, it is important for a bidder to have a good estimate of *N* when bidding for a current contract. One approach, instigated by Friedman in 1956, is to make such an estimate by statistical analysis and modelling. Since then, however, finding a suitable model for *N* has been an enduring problem for researchers and, despite intensive research activity in the subsequent thirty years little progress has been made - due principally to the absence of new ideas and perspectives. This paper resumes the debate by checking old assumptions, providing new evidence relating to concomitant variables and proposing a new model. In doing this and in order to assure universality, a novel approach is developed and tested by using a unique set of twelve construction tender databases from four continents. This shows the new model provides a significant advancement on previous versions. Several new research questions are also posed and other approaches identified for future study.

Keywords: *Modelling; Forecasting; Bidding; Tendering; International comparison, number of bidders*.

Introduction

An important consideration for bidders when preparing a serious construction tender proposal is the likely number and identity of the opponents to be faced. Studies of construction companies in the U.S. (Ahmad & Minkarah, 1988) and the UK (Shash, 1993), for example, have found this to be one of the three most important factors that conditions most bidding decisions. Clearly, any relevant information will be useful when making the decision to bid (d2b) and in strategically setting the bid price to increase the probability of winning the contract and making sufficient profit.

There is also strong evidence that some tender results, or at least their probability of occurring, have systematic differences depending on the number of bidders (N) involved. For example, high values of N tend to increase the correlation between the mean bid and the high and low bids in collective bid tender forecasting models (Ballesteros-Pérez et al., 2012; Shrestha & Pradhananga, 2010; Skitmore, 1981b) and the effect of the *winner's curse* (Capen et al., 1971, Skitmore, 2002). N has also recently been shown to be proportional to the amplitude of the bid standard deviation (Ballesteros-Pérez et al., 2015b). Moreover, N plays an important role in combinatorial auctions, with high N increasing computational complexity when trying to find the best combination of winners (Fukuta & Ito, 2007; Sandholm, 2000).

The traditional approach to anticipating N in practice is through personal experience of the past participation rate of bidders mostly in terms of project characteristics (e.g., owner, type and size) and nearby location (Ballesteros-Pérez, et al., 2010; Fu, 2004). Attempts to forecast N more systematically by mathematic models have met with little success. The most popular of these have been to resort to a probabilistic approach by treating N as a statistical variable. Friedman (1956), for example, suggested that N might follow a Poisson distribution. To a lesser extent, similar approaches have been tried in forecasting the identity of bidders, or a group of specific key competitors that might enter a future tender, but with even less success (Skitmore, 1986).

The debates in the early years became very intense at times, with studies proving and refuting different properties seemingly exhibited by N in specific contexts, countries or just according to the nature of the work involved. By the late 1980s, however, the controversies just stopped without a resolution as researchers gradually ran out of ideas and enthusiasm.

The purpose here is to revive this work by revisiting the major achievements made during the period 1956-1986, along with some untested later ideas and propose a new, improved, model to describe the statistical distribution of N. In doing this, a complete and varied set of twelve construction tender databases from around the world is analysed for the purpose of generalisation. The result is a critical view of past research while stimulating again a productive discussion on a subject that, as previously acknowledged, is of considerable importance from both owner and bidders' standpoint and strongly linked to other tender outcomes.

The paper is structured as follows. In the next section, a thorough but summarised literature review is provided. This is followed in *Materials and Methods* by an outline of the main features and countries of origin of the twelve databases and the subsequent research methodology. Next, the *Calculations* section tests the suitability of a variety of general statistical distributions for modelling N, the effect of contract size and the development of a composite two-distribution model for N. The *Results* section then provides a final comparison of the models, confirming the superiority of the new composite model. The *Discussion* and *Conclusions* sections summarise the work, posing several new research questions and identifying other paths for future study. The American term "(procurement or reverse) auction" and European "tender" are treated here as synonymous.

Literature review

There have been several thorough reviews of the effect of *N* on auction results (e.g., Dyer et al., 1989; Levin and Ozdenoren, 2004; Hu, 2011) and these will not be recounted here. Instead, we are concerned with the statistical nature of *N* as a precursor to its potential prediction. It is well known in both practice and theory that *N* generally varies with the project type and size (e.g., Azman, 2014; Drew and Skitmore, 2006), client and specific location (Al-Arjani, 2002; Benjamin, 1969) and even with market conditions (Ngai et al., 2002; Skitmore, 1981a).

Forecasting its value in advance, however, is more problematic, the earliest treatment being Friedman (1956), who suggested a variety of methods for estimating its expected value. One way is to use the often little information available about a company's competitors' intentions in combination with its managers' experience - an approach reiterated by Rubey and Milner (1966) with especial emphasis on the contract type and size involved.

Another suggestion is to exploit the statistical relationship between *N* and contract size (the complete budget to carry out the project) (Friedman, 1956), a reasonable enough assumption at that time of open tendering in the U.S., as larger projects are generally associated with larger (dollar) profits and therefore likely to attract more bidders. Empirical studies attempting this are quite limited and inconclusive, however. Gates (1967) and Wade and Harris (1976) have applied the method to U.S. data, producing generally weak predictive results. Other empirical U.S. research is even less supportive, finding no significant linear relationship between *N* and contract size, nor between contract size and the number of suppliers and subcontractors involved (e.g. Sugrue, 1977). Skitmore's (1986) analysis of UK construction auctions, however, where selective tendering is the norm, surprisingly found a weak to moderate correlation between *N* and contract size. A possible reason for the general lack of correlation in the U.S. suggested by Park (1966) is that the relationship between *N* and contract size may be nonlinear. Although this has yet to be tested with U.S. data, Skitmore's (1986) UK analysis found the correlation to be certainly more apparent when contract size was transformed to a log scale.

The only other empirical approach to forecasting *N* is Skitmore's (1981b) study of several international tender datasets from different time periods, which identified an apparent relationship between *N* and market conditions. However, no mathematical model was developed for this. Today, the general conclusion is that using some measure of contract size will provide the best means of estimating *N* and certainly an advancement on considering it to be purely random (Ballesteros-Pérez & Skitmore, 2014), a view that has been dominant since Rickwood (1972).

For statistical applications involving *N*, besides estimating its expected value, it is important to be able to make some assumptions concerning its probability density function (pdf). In addition to bidding strategies, this has important ramifications in Auction and Game theory (Klemperer, 2004), driven by the different outcomes it produces on several types of auctions formats and under different types of valuations used by bidders of the auctioned items. Nevertheless, there is a long list of proposed candidates. These include the normal (Ballesteros-Pérez et al. 2013a, 2014), uniform (Ballesteros-Pérez et al. 2013b), gamma (Engelbrecht-Wiggans, 1980), Laplace (Ballesteros-Pérez et al. 2015a) and Weibull (Ballesteros-Pérez & Skitmore, 2014).

Of particular interest is the Poisson distribution, considered by Friedman (1956), as likely to “furnish a good fit” for *N* values, reasoning that similar individuals independently deciding whether or not to bid for a particular item is equivalent to *N* following the binomial distribution which, when the average of the number of bids is a small fraction of the total possible, is well approximated by the Poisson. This was later seemingly confirmed by Keller and Bor's (1978) empirical analysis of the bidding patterns of a significant number of similar construction contracts in which their results agreed with the Poisson distribution. In contrast, Skitmore’s (1986) empirical analysis of three sets of UK construction tenders found no significant fit with the Poisson (N=51, $\bar{x}=6.2$, sd=2.1, $\chi^2_{(4)}=20.7$; N=218, $\bar{x}=5.7$, sd=1.1, $\chi^2_{(8)}=16.4$; N=373, $\bar{x}=5.1$, sd=3.8, $\chi^2_{(8)}=31.4$). Meanwhile, others making use of, for example, U.S. Outer Continental Shelf Statistical summary of 1976 to 1978 oil tract auctions, found N might follow not only a distribution different from the Poisson but even bimodal distributions (Engelbrecht-Wiggans, 1980).

On being criticized by other researchers on theoretical grounds, Friedman then modified his original assertion to the distribution of the residuals of a regression between N and contract size (Engelbrecht-Wiggans, Dougherty, & Lohrenz, 1986). Others, however, have suggested the normal distribution to be a better option to reflect the random variability of such residuals – a point supported empirically by Skitmore (1986) for contract size with and without logarithmic transformation.

Since then, a compromise solution has been to consider the number of bidders as a purely stochastic variable in experimental settings (McAfee & McMillan, 1987) or as a fixed value in Game and Auction theory (Harstad et al., 1990), although quite surprisingly the Poisson model has endured since the very first and celebrated compilation of auction and bidding models from Stark and Rothkopf (1979) and Engelbrecht-Wiggans (1980) to modern and current online auctions (Bajari & Hortacsu, 2003).

A completely different approach to estimating N is to try to identify who the actual bidders might be. As with horse racing, where the same horses often race against each other, many contractors tend to prefer construction work of a certain type, size and location and therefore can be expected to bid against each other quite regularly. In the U.S., however, as Morin and Clough (1969) note, it is quite usual for the same bidder to submit proposals for different types of work. A contractor's decision to bid (d2b) is also limited by the number of contracts that can be managed at any one time (Skitmore, 1988). Both of these factors lead to a situation where the same contractors bid less frequently against each other than might be otherwise imagined, making the prediction of their presence on a single auction a very difficult task in the absence of 'inside' information (which in itself is also difficult to obtain as being tantamount to collusion). An alternative is to simply "go and look". Skitmore's (1987) research in the U.S., for example, identified several informal methods used by contractors to assess the state of opponents' order book, including flying over their main compound to see the amount of machinery lying idle!

The use of statistical methods is possible, with Wade and Harris (1976) for example suggesting to treat the identities of several bidders and their groups probabilistically, but there are difficulties in this, particularly involving the identities of those from whom the forecasting company does not have any information. This, has led the tendering theory literature to classify the potential competitors as "key" and "strangers" (Skitmore, 1986).

Since 1986, however, there has been no further work in this area (Ballesteros-Pérez & Skitmore, 2014) and we will leave its consideration for a separate paper on the topic. Similarly, with the exception of additional studies such as by Athias and Nuñez (2009), Skitmore (2008) and Costantino et al. (2011) there has been no further empirical work concerning the statistical nature of *N* and therefore previous assumptions will not be considered further here.

Materials and Methods

Tender datasets

In order to make a thorough analysis of the distribution of *N*, a comprehensive and representative set of construction tender databases is needed. However, such databases are generally difficult to obtain because there are very few published in the regular scientific construction literature mostly due to their length. Therefore, an intensive and detailed search was carried out and access was obtained to documents only available in printed form, mostly in MSc and PhD theses where the original bidding data was complete and unprocessed. This resulted in the collection of twelve databases - some in the original author's scanned form and others requiring a visit to the respective university repository.

The twelve databases contain construction bidding data from four continents: Europe (United Kingdom and Spain), America (United States), Asia (Hong Kong) and Oceania (Australia), all featuring different types of construction work from different time periods. Table 1 summarizes the most important aspects of each database.

For the sake of clarity, the tender databases are referred to by the numerical identifier stated in the column marked "ID".

< Insert Table 1 here >

In general, the sample described in Table 1 is considered sufficiently representative, since the twelve databases analysed encompass different works such as: buildings (housing, aeronautics, schools, hostels, police and fire stations), civil works (waste water treatment plants, railways) and services (specialized and general). All decades from the sixties until now are represented either completely or partially by at least one dataset and their sizes are large enough (from tens to hundreds of contracts) to carry out thorough statistical analyses. Furthermore, concerning the variable number of bidders, the databases range from low (mean N of around 5) to high (around 31) numbers of bidders, whose dispersion values are more or less scattered (see standard deviation column), have different levels of positive skewness (no dataset has negative skewness), as well as different levels of positive and negative kurtosis.

Finally, it is also noted that, among the twelve databases, the six from the United Kingdom and Australia used selective tendering, that is, the owner invites only certain bidders and therefore sets an upper-bound on the value of N. However, the results obtained later seem to be very similar for both open and selective tendering processes.

Outline of Methodology

In the next two sections, several factors that either directly or indirectly affect *N* are identified from the twelve databases. First, the analysis begins with an extensive comparison of the goodness of fit of a range of common statistical distributions and an attempt to deduce why some distributions perform better than others. Next, the relationship between *N* and contract size is analysed in both natural and logarithmic scales, and studied to see how predictably the statistical mean, standard deviation, skewness and kurtosis vary when plotted against contract size, and some general behaviour patterns are provided. Finally, a new model for describing the statistical variation of *N* is presented along with the justification of its main assumptions - that both the frequency of contract sizes and that the population of potentially interested participating bidders are log-normally distributed. As is eventually demonstrated from the large variety of statistical curve shapes that can stem from this model and the thorough statistical distribution fit tests performed, the model represents a significant step forward in this topic. The next section is divided into three subsections describing these analyses in more detail.

Calculations

Comparison of goodness of fit of standard statistical distributions

Of the many statistical distributions proposed to date to model *N* (Poisson, normal, gamma, Weibull, Laplace, etc.) no clear single distribution has yet been found, with different studies making use of databases with different characteristics that are not always identified. To analyse the twelve databases, a χ^2 test is applied to every distribution, which are then ranked according to the number of times the sum of the squared residuals are below the critical χ^2_α values (using three levels of significance α=1%, 5% and 10%) and the p-values. The more times the actual χ^2 values are below the critical χ^2_α values (from 0 to 3 on average), the lower is the p-value (from 0 to 1, on average) and hence the better fit of the distribution.

The range of distributions tested is basically restricted to the location-scale family, as the parameters that define these distributions have true physical meaning, improving the understanding of the underlying distribution involved. In addition, the gamma and Weibull distributions were also tested because of their prevalence in the literature. Of the location-scale distributions tested, seven symmetrical distributions (skewness=0) are of particular interest: the uniform (kurtosis close to -1.2), raised cosine (kurtosis close to -0.6), normal (kurtosis 0.0), logistic (kurtosis 1.2), hyperbolic secant (kurtosis 2.0), Laplace (kurtosis 3.0) and Cauchy (kurtosis undefined). These latter distributions are chosen to map in detail the level of kurtosis that might better fit the *N* distribution in terms of either platykurtic or leptokurtic behaviour. Asymmetrical forms of these seven distributions are also tested by transforming the *N* values into *log N* values (i.e., for testing against the log-uniform, log-raised cosine, log-normal, log-logistic, log-hyperbolic secant, log-Laplace and Log-Cauchy distributions) to map positive skewness with different kurtosis levels and, by using the N^2 values to test for negative asymmetries with different kurtoses (i.e., square-uniform, square-raised cosine, square-normal, square-logistic, square-hyperbolic secant, square-Laplace and square-Cauchy). Furthermore, the Poisson distribution is also tested with the natural, logarithmic and square *N* values. A flexible array of means and variances calculated by the method of moments are therefore tested using location-scale distributions and a representative grid of skewness and kurtosis levels tracked and checked, amounting to 24 combinations in all, plus the Weibull and gamma distributions, for each of the twelve databases. It is also to be noted that, despite most distributions being continuous, a discretization of the X values (N values) is performed by obtaining the pdf $f(x=N)$, from the cumulative distribution function, $F(x=N)$, by the simple calculation: $f(x) = F(x+0.5) - F(x-0.5)$.

Table 2 and Figure 1 give the results for the four best distribution fits (normal, log-normal, logistic and log-logistic) together with the Poisson and the Laplace distributions.

< Insert Table 2 here >

< Insert Figure 1 here >

On average, the log-normal distribution produces the highest number of times the χ^2 values are below the critical three χ^2_α values and the lower p-value, although the normal, logistic and log-logistic are also quite close.

These results are not very useful, however, as the fit is not good for any distribution, even the log-normal. That this may be due to the absence of another influencing factor is an issue taken up in the next section.

Improving accuracy by considering contract size

< Insert Table 3 here >

Table 3 gives the regression equations of N with contract size (in terms of the mean bid, B_m) for the twelve datasets. This indicates the existence of a weak correlation in most cases, irrespective of whether B_m is calculated from the natural or logarithmic bids. This may be due to two causes. First, there may be a large variation in N values obscuring an underlying correlation. Second, the distribution B_m observed in every database may not be uniform, so there is an uneven distribution of the N values on the X-axis.

To observe the variations in the N distribution values, one approach is to place the auctions in ascending order of log contract size (from lower to higher B_m) and plot the first four moments (mean (μ), standard deviation (σ) skewness (γ) and kurtosis (κ)) of groups of *N* values as shown in Figure 2.

< Insert Figure 2 here >

As Newell and Hancock (1984) note, for practical purposes in statistical inference, estimates of γ and κ for sample sizes below 50 can indicate the underlying statistical distribution is normal when it is not. Therefore rolling groups of 50 *N* values are taken. That is, the moments of *N* values from the auctions ranked 1 to 50 are first recorded. Then the moments of *N* values from the auctions ranked 2 to 51 are recorded, and the process continued until reaching the last ordered auction. Four of the datasets do not contain sufficient auctions to do this and are therefore missing from Figure 2.

Even a window width of 50 auctions causes high oscillations in the γ and κ estimates. To clarify the situation, the rule of thumb of usual practice is followed in which only values outside the range of ±1 are taken to be sufficient evidence to conclude the underlying distribution is either skewed or platy/leptokurtic.

As Figure 2 shows, when considered in terms of contract size, with very few exceptions the γ and κ estimates are quite close to zero. Figure 3 provides a first approximation why this might be the case. This contains several interesting features that need to be highlighted since it mirrors some aspects found in Figure 2.

< Insert Figure 3 here >

First, it is quite logical to think that, irrespective of the X axis (contract size) being represented in natural or logarithmic values, contract sizes that are very small or very large will fail to attract bidders, since bidders can make little profit in the former case, and no qualified bidder could submit a proposal in the latter case. This being the case, μ will increase initially from zero until it reaches a zone with relatively stable maximum *N* values, after which it will decrease asymptotically back to zero.

Second, it is expected that the σ curve will behave similarly. However, as Figure 2 shows, in almost all cases there seems to be a gap between the maximum μ and σ (identified as Δ). The reason for this is still to be researched.

Third, the γ and κ curves take on high values when the μ and σ curves are closer to zero. The reason is that when *N* is extremely low, the *N* distribution has to be as shown in the bottom left corner of Figure 3, which is remarkably asymmetrical and leptokurtic since the highest density and cumulative probability will remain between $0 \le N \le 1$. On the other hand, when the *N* curve has γ and κ close to zero, the distribution that models *N* can be assumed nearly normal. However, extremes of high γ and κ are rarely observed, since we can only have a glimpse of those atypical situations with very small and/or large contract sizes and, since they are quite scarce, it is difficult to accurately estimate the γ and κ values involved.

Nevertheless, Databases 4, 7 and 10 in Figure 2 seem to support the assumptions made above about γ and κ. On the other hand, when dissecting the variation in μ, σ, γ and κ along the contract size dimension, the curve modelling *N* (bottom half of Figure 3) for the rest of databases should be close to the normal distribution, as γ and κ are close to zero.

Model proposed

Hossein (1977) found contract size could be modelled by the exponential distributions, while a similar study by Skitmore (1986) however found the log-normal distribution to be more appropriate. Here, two distributions are checked for fit - the log-normal distribution and the Pareto distribution. The former because it has been found to outperform the exponential and the latter because it is closely related to the exponential distribution but has two parameters as has the log-normal. It is noted that both the Pareto and log-normal are alternative distributions for describing the distributions of sizes which abound in natural, physical, economic, and social systems (Malevergne et al., 2011). Fat-tail distributions, such as the log-normal and the Pareto distribution have historically competed for describing with higher accuracy some generating processes and hard-to-distinguish tail properties (Malevergne et al., 2011), and this is the reason why both have been compared here.

The results in comparing both distributions are presented in Table 4, with a representation of the best log-normal distributions found in Figure 4. As noted from Table 4, the Kolmogorov-Smirnov tests indicate the log-normal distribution to generally provide the best fit, even when the datasets seem rather erratic – probably as a consequence of a tender dataset that did not include the complete range of tender sizes.

< Insert Table 4 here >

< Insert Figure 4 here >

On the other hand, a parallel theoretical debate has quite recently emerged concerning the use of a power-law distribution or a log-normal distribution to model firm size (Segarra & Teruel, 2012), as both appear to provide a close fit with real data. If we assume that the number of potentially interested bidders is a fraction or proportion of the population of companies found in a particular area and within a particular market, then the number of bidders should also follow a log-normal distribution with similar location (mean) and scale (variance) parameters, but differing on the Y-axis order of magnitude when representing absolute values, instead of frequency values. This is because the number of potentially interested bidders would be lower when representing the number of companies on the Y-axis compared to the total number of companies by size, but both should probably look quite similar when representing their pdfs, since they would then represent proportions. Therefore, a model is proposed that endeavours to take advantage of the log-normal distributions: (1) the distribution of contract size and (2) the distribution of potentially interested bidders, both considered as log-normal, but with different location and scale parameters (μ_1 and σ^2_1, and μ_2 and σ^2_2, respectively).

What this model tries to represent is that, if there is a different number of bidders who might submit a bid for a future tender as a function of the contract size, and the number of contract size opportunities is known (both being variables well represented by different log-normal distributions), the calculation of the N distribution curve should be according to the representation of Figure 5.

< Insert Figure 5 here >

In particular, Figure 5 represents how, in order to calculate the probabilities associated to every possible value of N (N_i from 0 to $+\infty$), it is only required to add up the two probability bands in the distribution of contract sizes (log-normal whose location and scale parameters are μ_1 and σ_1) that are delimited by the two pairs of X values from the distribution of the number of interested bidders (log-normal whose location and scale parameters are μ_2 and σ_2) whose respective Y values correspond to that specific N_i ±0.5. For instance, in Figure 5 we want to calculate the probabilities of finding N_i=4 in a database. Despite the number of bidders N_i being natural numbers, we need to assume that the number of interested bidders distribution will correspond to a band of Y values between 4.5 and 5.5 (as represented on the right Y axis). Those two Y values each correspond to another two different X values by horizontal intersection first, and then by vertical intersection, in the same log-normal distribution describing the number of interested bidders. But once these four X values are identified, they also define the vertical probability bands within the log-normal distribution of contract sizes that, on being summated, will result in the probability of finding N_i=4 in the database.

Generally speaking, Figure 5 highlights that the final N distribution is affected by the number of bidders that would submit a bid if there was an occasion to do so as well as by the number of times each contract size occurs. In other words if, for each possible N value it is known that there are a fixed number of interested bidders (by means of curve 2, and ±0.5 to discretized the distribution), and if the frequency of that range of contract sizes is calculated (by means of curve 1), then the same frequency will be equivalent to the number of times that that N will be found in the final N distribution.

Concerning the five parameters to be estimated in the model (μ_1, σ_1, μ_2, σ_2 and N_{max}), the mean and standard deviation of the distribution of contract sizes (parameters μ_1 and σ_1) can be directly obtained by calculating the first and second moments from the series of all tender B_m (bid average) values found in a database; N_{max} can be set to the maximum N_i value found in the database (or slightly above); whereas the mean and standard deviation of the distribution of the number of interested bidders (parameters μ_2 and σ_2) cannot be directly estimated (unless extensive and time-consuming field research is carried out for estimating the distribution of nearby firm sizes with potential interest in the type of works contained in the database under study). Therefore, it is recommended that, when looking for the best combination of parameter values, μ_1, σ_1 and N_{max} are calculated as suggested above, while parameters μ_2 and σ_2 are set according to a simple two-variable numerical optimization approach for providing the best overall distribution fit. In this connection, according to the multiple combinations of these five parameter values, the broad range of mathematical shapes that this model distribution can take is represented in Figure 6.

< Insert Figure 6 here >

As can be seen, the model is able to provide a number of statistical curve shapes changing the γ from positive to negative, or reaching higher levels of κ near the N=0 and N_{max} values. For the sake of simplicity however, the most common cases are identified and framed in the thick line on the top rows of Figure 6. This distribution is checked and compared against previous distributions in the next section.

Results

<u>*Comparison of standard statistical distributions considering contract size*</u>

In summary, Table 2 gives the best results for a complete comparison of several statistical distributions irrespective of contract size. However, Figure 3 suggests that, within a certain range of contract sizes, *N* is quite close to a normal distribution. It is also apparent that working with narrower intervals of contract sizes also leads normal-like distributions for *N*, this being the case with the three central images depicted on the top row of Figure 6, where narrow contract size intervals have necessarily quite small variance values from the distribution of contract size opportunities ($\sigma^2_1 \rightarrow 0$) when compared to the variance of the number of potentially interested bidders (σ^2_2), forcing $\sigma^2_1 << \sigma^2_2$.

To examine this further, the Table 2 analysis is repeated but with non-rolling groups of contract sizes as shown in Table 5.

< Insert Table 5 here >

As can be seen, the best pdf for *N* is now the normal distribution with both indicators (the number of times the χ^2 values are below the critical χ^2_α values, and the p-values) significantly improved. However, this improvement also applies to all the other distributions tested, since the two indicators are approximately between 30% and 60% better on average for all of them when compared with the results in Table 2. It is shown, therefore, although the best approximation of *N* is the normal distribution, the log-normal, logistic and log-logistic are not far behind.

<u>*Model validation*</u>

To test the new five parameter (μ_1, σ_1, μ_2, σ_2 and N_{max}) model, it is first effectively reduced to a two-parameter model by forcing μ_1 and σ_1 to take on the values of the actual (log-normal) contract size distributions represented in Figure 4 (that were directly obtained by the method of moments) and having the N_{max} values vary within a nearby range to the actual maximum N values observed in the twelve databases, leaving the only remaining parameters μ_2 and σ_2 to be estimated. This is done by a simple two-dimensional optimization process to find the values that minimize the actual χ^2 values. The results are shown in Table 6 and the model curves illustrated in Figure 7.

< Insert Table 6 here >

< Insert Figure 7 here >

As can be seen, the model outperforms all the distributions tested so far, even when taking narrower intervals of contract size (although the improvements are only around 10% in this latter case). However, both approaches have different aims: the model only provides a better explanation of the distribution of N, while breaking down the series of N values by more compact contract sizes only reduces the amount of randomness when trying to describe the unexplained variation of N.

Summary

Many distributions have been compared in this study by means of multiple chi-square tests performed on twelve databases. Therefore, in order to highlight potential differences between the performance of these statistical distributions, it is convenient to summarise the results in a single ranking table.

This is the aim of Table 7, which presents the average and standard deviation results (the latter not presented earlier to avoid confusion with other variables involved) of the number of times the sum of the squared residuals are below the critical $\chi^2\alpha$ values (from 3 –good fit- to 0 –bad fit–) and the p-values (from 0 –perfect fit– to 1 –worst fit–). Table 7 distributions have been ordered in descending order of the average p-values but, as can be seen, some distributions nearly tie when taking into account both the $\chi^2\alpha$ and p values simultaneously (distributions ranked as 3rd and 8th).

< Insert Table 7 here >

The consequence of a lower p-value is directly indicative of a loss of accuracy when modelling the actual distribution of the *N* values, and this table shows how the new model outperforms (on average) other common distributions. However, it is noted that the standard deviation values obtained, even without the need for carrying out ANOVA tests, denote potential overlaps in the means of the p-values, particularly among the top-ranked distributions. Fortunately, the $\chi^2\alpha$ has zero variance for the new model, which indicates that the model has provided, without exception, what may be considered a reasonable approximation in the twelve databases. This is not the case with the other models.

Further discussion of Table 7 is provided in the next section.

Discussion

From the results obtained in the previous sections, it is clear that the contract size distribution within each database is close to log-normal and strongly conditions *N*. The direct comparison of many statistical distributions (partially shown in Table 1 and Figure 1, as well as in Table 7) is also expected to be biased towards the log-normal distribution. However, as also observed in Tables 5 and 7, the normal distribution naturally presents an acceptable fit (closely followed by the log-normal) when the contract size effect is considered. Therefore, as with many other such goodness-of-fit studies, there is an intermediate situation in which it is difficult to distinguish between the suitability of the normal and log-normal distributions. In addition, the logistic and log-logistic distributions are also good candidates, since they are quite similar in shape to the normal and log-normal distributions respectively, although slightly more leptokurtic. This fact is also frequently observed, since the juxtaposed effect of mixing the normal and log-normal distributions slightly increases the kurtosis coefficient.

In summary therefore, in delimiting the potential values of *N* for a future tender, removing the effect of contract size by using only recent past tenders with a similar contract size or calculating the mean, standard deviation, skewness and kurtosis as in Figure 2, is preferable to directly modelling the whole dataset values of *N* without allowing for contract size. On the other hand, as Tables 6 and 7 show, the new model provides a better fit than the many other statistical distributions examined, although its superiority is not decisive, as indicated by the small differences and high standard deviation between p-values in Tables 5, 6 and 7. In addition, the model assumes the log-normal distribution representing the expected value of the number of interested bidders as fixed, when this curve Y values must necessarily evidence variability since, for instance, it seems counter intuitive to state that the number of potentially interested bidders for a given contract size is constant, but variable as well.

Conclusions

Knowing the statistical distribution of the number of bidders, *N*, for a construction contract is important in real-life bidding because it conditions the decision to bid and how to set the final bid price so as to increase the probability of winning, but also in tendering theory since it affects many related outcomes, such as the correlation between the mean and lowest bids or the dispersion of the bid values, which are key assumptions of many collective bid tender forecasting models. However, little progress has been made despite the many studies from 1956 to 1986 except that there are other variables that seem to condition or have a significant correlation with *N*. Not all of these have been explained in conjunction with measuring their possible interactions.

In this study, a unique set of twelve construction and services tender databases from four continents are used for a thorough comparison of many candidate statistical distributions with the primary aim of determining which are the most accurate and in what conditions.

The univariate results show the log-normal distribution to be the best fit, while the normal distribution provides the best fit when contract size is taken into account. These are basic but important outcomes, since many bidding practitioners and researchers tend to use the normal distribution without distinction when modelling the distribution of bidders, while it is shown here that this distribution is only the most accurate when contracts of similar nature of work and economic size are used. If these conditions are not fulfilled, then the log-normal distribution is the most accurate.

Next, the expected variation of the *N* distribution mean, standard deviation, skewness and kurtosis as a function of contract size is analysed in both natural and logarithmic scales. The four moments are studied to see how predictably they vary when plotted against contract size and some interesting general behaviour patterns are provided. For example, most construction tenders operate within a band of contract sizes that have low levels of skewness and kurtosis, allowing the use normal-like distributions with barely loss of accuracy. However, this situation is no longer valid for extremely high or low contract values, since contractors will usually have less previous experience with such contracts, when the distribution of *N* becomes strongly positive skewed and peaked.

Finally, a new model for describing *N* is presented along with the justification of its main assumptions - that both the frequency of contract sizes and that the population of potentially interested participating bidders are log-normally distributed. As is demonstrated from the large variety of statistical curve shapes that can stem from this model and the thorough statistical distribution fit tests performed, the model results are significantly more accurate in modelling the variations in N than the other alternatives considered for all 12 datasets examined, including the ubiquitous normal distribution which is used in similar studies.

Despite this, however, it is felt that there is still room for further improvement. For instance, research in forecasting the identity of future bidders may, paradoxically, shed further light on the issue. There are also new questions concerning differences in contract size (value) that exist between the maximum expectation and variance of N when represented as a function of contract size. An additional question is how to replace the deterministic number of potentially interested bidders in the model by a distribution with a random component.

The result is a critical view of past research while stimulating again a productive discussion on a subject that, as previously acknowledged, is of considerable importance from both owner and bidders' standpoint and strongly linked to tender outcomes beyond the construction context.

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ID	*Database Alias*	*Description*	*Tender method*
1	UK51	Building-related tenders within the London area with one bidder in common and cover prices	Selective
2	UK272	Construction industry Building Cost Information Service report	Selective
3	UK218	Civil engineering work tenders from the North of England	Selective
4	UK373	Building-related tenders within the London area	Selective
5	US64	Building-related tenders from the US National Aeronautics and Space Administration	Open
6	US50	Building-related tenders from the US	Open
7	HK199	Tenders of buildings for education, police, firemen and hostels in Hong Kong	Open
8	HK261	Tenders from the Hong Kong Administrative Service Department	Open
9	AU152	General contractors' civil engineering works and housing in New South Wales, Australia	Selective
10	AU161	Specialised contractors' civil engineering works and housing in New South Wales, Australia	Selective
11	SP45	Waste Water Treatment Plants and Sewage lines in Catalonia region, Spain	Open
12	SP114	Spanish High-speed Railway Infrastructure Manager (ADIF) tenders	Open

ID	*Number of auctions*	*Period*	*Mean (μ)*	*Std. Dev. (σ)*	*Skewness (γ)*	*Kurtosis (κ)*	*Source*
1	51	1981-1982	6.235	1.464	0.250	0.241	(Skitmore and Pemberton, 1994)
2	272	1969-1979	6.140	1.786	0.265	1.009	(Skitmore, 1981b)
3	218	1979-1982	5.665	2.260	0.497	0.994	(Skitmore, 1986)
4	373	1976-1977	5.134	1.944	0.124	-0.580	(Skitmore, 1986)
5	64	1976-1984	6.734	3.108	1.756	4.763	(Brown, 1986)
6	50	1965-1969	4.680	1.834	0.558	-0.260	(Shaffer and Micheau, 1971)
7	199	1981-1990	12.724	6.262	0.696	0.497	(Drew, 1995)
8	261	1991-1996	13.663	7.279	0.654	-0.498	(Fu, 2004)
9	152	1972-1982	8.651	3.987	0.685	-0.060	(Runeson, 1987)
10	161	1972-1982	6.273	2.877	1.595	3.531	(Runeson, 1987)
11	45	2007-2008	14.133	11.108	1.496	1.706	(Ballesteros-Pérez et al. , 2012)
12	114	2008-2014	31.974	12.082	0.414	-0.345	(Fuentes-Bargues et al. , 2015)

Table 1. Description of the twelve construction tender databases analysed

ID	Database alias	Critical χ² α values			Poisson			Normal		
		$\chi^2\alpha$=0.01	$\chi^2\alpha$=0.05	$\chi^2\alpha$=0.10	χ^2	$\chi^2<\chi^2\alpha$?	p-value	χ^2	$\chi^2<\chi^2\alpha$?	p-value
1	UK51	18.475	14.067	12.017	30.188	0	1.000	8.736	3	0.728
2	UK272	21.666	16.919	14.684	57.384	0	1.000	11.061	3	0.728
3	UK218	21.666	16.919	14.684	16.017	2	0.933	10.136	3	0.660
4	UK373	21.666	16.919	14.684	31.082	0	1.000	17.886	1	0.963
5	US64	23.209	18.307	15.987	14.736	3	0.858	18.793	1	0.957
6	US50	18.475	14.067	12.017	2.077	3	0.045	3.801	3	0.198
7	HK199	48.278	41.337	37.916	10169.1	0	1.000	49.274	0	0.992
8	HK261	49.588	42.557	39.087	4602.7	0	1.000	88.747	0	1.000
9	AU152	30.578	24.996	22.307	78.525	0	1.000	23.064	2	0.917
10	AU161	27.688	22.362	19.812	79.735	0	1.000	109.893	0	1.000
11	SP45	34.805	28.869	25.989	1131.92	0	1.000	26.107	2	0.903
12	SP114	57.342	49.802	46.059	3079.7	0	1.000	54.931	1	0.983
					avg.	0.667	0.903	**avg.**	1.583	0.836

ID	LogNormal			Logistic			LogLogistic			Laplace		
	χ^2	$\chi^2<\chi^2\alpha$?	p-value	χ^2	$\chi^2<\chi^2\alpha$?	p-value	χ^2	$\chi^2<\chi^2\alpha$?	p-value	χ^2	$\chi^2<\chi^2\alpha$?	p-value
1	8.951	3	0.744	7.755	3	0.645	7.650	3	0.636	7.022	3	0.573
2	47.767	0	1.000	10.488	3	0.688	34.870	0	1.000	24.233	0	0.996
3	50.246	0	1.000	10.160	3	0.662	48.580	0	1.000	20.475	1	0.985
4	72.666	0	1.000	30.904	0	1.000	82.635	0	1.000	55.608	0	1.000
5	6.932	3	0.268	15.441	3	0.883	5.708	3	0.161	11.483	3	0.679
6	1.755	3	0.028	4.194	3	0.243	2.399	3	0.066	244.245	0	1.000
7	45.495	1	0.980	47.163	1	0.987	49.564	0	0.993	56.750	0	0.999
8	46.083	1	0.977	109.201	0	1.000	68.904	0	1.000	123.869	0	1.000
9	16.593	3	0.656	25.550	1	0.957	21.134	3	0.867	28.882	1	0.983
10	4.721	3	0.019	39.731	0	1.000	3.101	3	0.002	30.176	0	0.996
11	10.88	3	0.100	26.545	2	0.912	12.467	3	0.178	18.354	3	0.567
12	55.818	1	0.986	63.802	0	0.998	63.864	0	0.998	87.614	0	1.000
	avg.	1.750	0.647	**avg.**	1.583	0.831	**avg.**	1.500	0.658	**avg.**	0.917	0.898

Table 2. Chi-square tests for checking the Poisson, Normal, LogNormal, Logistic, LogLogistic and Laplace distribution fit for *N*

ID	Database alias	Optimal regression curves			
		X=Bm (natural scale) & Y=N	R^2	*X=Bm (log scale) & Y=N*	R^2
1	UK51	Y=-6E-14X²+7E-07X+5.4185	0.109	Y = 0.2649X²-7.0073X+52.236	0.101
2	UK272	Y= 1.7382X^0.1001	0.115	Y= -0.1737X²+4.8162X-26.52	0.146
3	UK218	Y=0.7977LN(X)-3.0126	0.253	Y= 0.7977X-3.0126	0.253
4	UK373	Y=0.372X^0.2034	0.253	Y=0.0063X^2.6271	0.270
5	US64	Y=-0.282LN(X)+10.534	0.018	Y=0.0941X²-2.8759X+28.2	0.024
6	US50	Y=-1E-13X²+8E-07X+4.233	0.038	Y=-0.1128X²+3.2137X-17.948	0.041
7	HK199	Y=-5E-16X²+1E-08X+12.917	0.012	Y=-0.9458X²+30.389X-230.2	0.052
8	HK261	Y=-7E-09X+14.658	0.035	Y=-0.9451X²+34.254X-295.73	0.035
9	AU152	Y=1.6998X^0.1095	0.067	Y=0.1297X^1.5581	0.070
10	AU161	Y=5.5924EXP(1E-07X)	0.009	Y=0.3815X²-9.0763X+59.735	0.070
11	SP45	Y=-2E-13X²+4E-06X+5.5083	0.374	Y=1.0237X²-24.343X+151.48	0.297
12	SP114	Y=-2E-15X²+9E-08X+33.465	0.147	Y=-3.8248X²+125.47X-990.14	0.235
		avg.	0.119	**avg.**	0.133

Table 3. Regression results between variables Contract size (via Mean bid, B_m) and Number of bidders (N)

ID	Database alias	Lognormal						
		D	*Dα=0.01*	*Dα=0.05*	*Dα=0.10*	*D<Dα=0.01?*	*D<Dα=0.05?*	*D<Dα=0.10?*
1	UK51	0.074	0.143	0.123	0.113	Yes	Yes	Yes
2	UK272	0.039	0.063	0.054	0.050	Yes	Yes	Yes
3	UK218	0.051	0.070	0.060	0.055	Yes	Yes	Yes
4	UK373	0.022	0.053	0.046	0.042	Yes	Yes	Yes
5	US64	0.081	0.128	0.111	0.101	Yes	Yes	Yes
6	US50	0.077	0.144	0.125	0.114	Yes	Yes	Yes
7	HK199	0.032	0.073	0.063	0.058	Yes	Yes	Yes
8	HK261	0.060	0.064	0.055	0.051	Yes	No	No
9	AU152	0.070	0.084	0.072	0.066	Yes	Yes	No
10	AU161	0.050	0.081	0.070	0.064	Yes	Yes	Yes
11	SP45	0.060	0.152	0.131	0.120	Yes	Yes	Yes
12	SP114	0.094	0.096	0.083	0.076	Yes	No	No

ID	Database alias	Pareto						
		D	*Dα=0.01*	*Dα=0.05*	*Dα=0.10*	*D<Dα=0.01?*	*D<Dα=0.05?*	*D<Dα=0.10?*
1	UK51	0.088	0.224	0.187	0.168	Yes	Yes	Yes
2	UK272	0.261	0.098	0.082	0.074	No	No	No
3	UK218	0.101	0.109	0.091	0.082	Yes	No	No
4	UK373	0.129	0.084	0.070	0.063	No	No	No
5	US64	0.074	0.200	0.167	0.150	Yes	Yes	Yes
6	US50	0.140	0.226	0.188	0.170	Yes	Yes	Yes
7	HK199	0.200	0.114	0.095	0.086	No	No	No
8	HK261	0.080	0.100	0.083	0.075	Yes	Yes	No
9	AU152	0.118	0.131	0.109	0.098	Yes	No	No
10	AU161	0.122	0.127	0.106	0.095	Yes	No	No
11	SP45	0.133	0.238	0.198	0.179	Yes	Yes	Yes
12	SP114	0.158	0.151	0.126	0.113	No	No	No

Table 4. Kolmogorov-Smirnov tests for checking the LogNormal and Pareto distributions fitting to the distribution of Contract size opportunities

ID	Database alias	Tenders (ordered)	Critical χ^2 α values			Poisson			Normal			LogNormal			Logistic			LogLogistic			Laplace		
			$\chi^2\alpha$=0.01	$\chi^2\alpha$=0.05	$\chi^2\alpha$=0.10	χ^2	$\chi^2<\chi^2\alpha$?	p-value	χ^2	$\chi^2<\chi^2\alpha$?	p-value	χ^2	$\chi^2<\chi^2\alpha$?	p-value	χ^2	$\chi^2<\chi^2\alpha$?	p-value	χ^2	$\chi^2<\chi^2\alpha$?	p-value	χ^2	$\chi^2<\chi^2\alpha$?	p-value
1	UK51	1-26	15,086	11,070	9,236	35,570	0	1,000	10,574	2	0,939	13,568	1	0,981	8,255	3	0,857	10,796	2	0,944	5,340	3	0,624
		27-52	15,086	11,070	9,236	6,604	3	0,748	1,424	3	0,078	3,133	3	0,321	1,748	3	0,117	3,254	3	0,339	2,654	3	0,247
2	UK272	1-30	16,812	12,592	10,645	8,889	3	0,820	2,288	3	0,109	5,736	3	0,547	2,589	3	0,142	5,742	3	0,547	5,058	3	0,464
		31-60	16,812	12,592	10,645	5,459	3	0,514	3,014	3	0,193	5,083	3	0,467	3,569	3	0,265	4,838	3	0,435	5,522	3	0,521
		61-90	18,475	14,067	12,017	6,387	3	0,505	5,983	3	0,458	8,210	3	0,686	5,598	3	0,413	7,224	3	0,594	5,566	3	0,409
		91-120	16,812	12,592	10,645	10,882	2	0,908	3,338	3	0,235	1,396	3	0,034	1,952	3	0,076	0,676	3	0,005	2,178	3	0,097
		121-150	16,812	12,592	10,645	14,941	1	0,979	4,048	3	0,330	10,436	3	0,893	4,369	3	0,373	9,114	3	0,833	5,526	3	0,522
		151-180	16,812	12,592	10,645	12,481	2	0,948	2,775	3	0,164	4,751	3	0,424	3,635	3	0,274	5,087	3	0,467	5,501	3	0,519
		181-210	16,812	12,592	10,645	7,230	3	0,700	3,072	3	0,200	0,602	3	0,004	2,536	3	0,136	0,635	3	0,004	2,843	3	0,172
		211-240	20,090	15,507	13,362	9,531	3	0,701	4,203	3	0,162	4,223	3	0,164	3,347	3	0,089	3,103	3	0,072	2,548	3	0,041
		241-272	18,475	14,067	12,017	13,175	2	0,932	2,539	3	0,076	6,910	3	0,562	1,196	3	0,009	3,048	3	0,119	1,384	3	0,014
3	UK218	1-31	15,086	11,070	9,236	2,425	3	0,212	1,987	3	0,149	4,188	3	0,477	2,698	3	0,254	5,348	3	0,625	3,775	3	0,418
		32-62	16,812	12,592	10,645	5,308	3	0,495	5,591	3	0,529	8,923	3	0,822	7,131	3	0,691	10,742	2	0,903	9,309	3	0,843
		63-93	18,475	14,067	12,017	11,516	3	0,882	8,452	3	0,706	21,285	0	0,997	9,089	3	0,754	18,826	0	0,991	147,947	0	1,000
		94-124	18,475	14,067	12,017	9,124	3	0,756	5,459	3	0,396	12,083	2	0,902	6,138	3	0,476	11,497	3	0,882	8,748	3	0,729
		125-155	18,475	14,067	12,017	1,084	3	0,007	0,861	3	0,003	4,041	3	0,225	1,264	3	0,011	4,092	3	0,231	2,749	3	0,093
		156-186	20,090	15,507	13,362	14,757	2	0,936	14,231	2	0,924	16,403	1	0,963	11,792	3	0,839	12,530	3	0,871	10,406	3	0,762
		187-218	16,812	12,592	10,645	15,144	1	0,981	4,640	3	0,409	4,129	3	0,341	4,260	3	0,358	3,792	3	0,295	7,862	3	0,752
4	UK373	1-37	11,345	7,815	6,251	3,434	3	0,671	5,057	3	0,832	2,094	3	0,447	5,283	3	0,848	2,441	3	0,514	11,192	1	0,989
		38-74	16,812	12,592	10,645	7,211	3	0,698	6,560	3	0,637	6,065	3	0,584	7,038	3	0,683	7,863	3	0,752	9,134	3	0,834
		75-111	16,812	12,592	10,645	1,222	3	0,024	2,468	3	0,128	3,362	3	0,238	4,344	3	0,370	5,224	3	0,485	5,501	3	0,519
		112-148	15,086	11,070	9,236	9,388	2	0,905	3,996	3	0,450	8,642	3	0,876	5,754	3	0,669	10,010	2	0,925	13,000	1	0,977
		149-185	18,475	14,067	12,017	14,135	1	0,951	10,871	3	0,856	17,327	1	0,985	11,424	3	0,879	16,679	1	0,980	14,661	1	0,959
		186-222	18,475	14,067	12,017	9,091	3	0,754	4,476	3	0,276	6,923	3	0,563	3,599	3	0,175	6,959	3	0,567	5,488	3	0,399
		223-259	18,475	14,067	12,017	9,498	3	0,781	3,346	3	0,149	4,847	3	0,321	5,014	3	0,342	6,419	3	0,508	10,900	3	0,857
		260-296	20,090	15,507	13,362	5,558	3	0,303	0,981	3	0,002	5,424	3	0,289	1,203	3	0,003	3,799	3	0,125	2,364	3	0,032
		297-333	13,277	9,488	7,779	7,576	3	0,892	2,117	3	0,286	1,841	3	0,235	3,390	3	0,505	2,923	3	0,429	3,678	3	0,549
		334-373	18,475	14,067	12,017	27,508	0	1,000	9,224	3	0,763	23,669	0	0,999	6,021	3	0,463	13,393	2	0,937	3,667	3	0,183
5	US64	1-32	21,666	16,919	14,684	21,312	1	0,989	19,878	1	0,981	7,653	3	0,431	16,177	2	0,937	6,069	3	0,267	8,722	3	0,537
		33-64	21,666	16,919	14,684	3,152	3	0,042	4,387	3	0,116	2,941	3	0,033	5,830	3	0,243	4,108	3	0,096	7,130	3	0,376
6	US50	1-25	16,812	12,592	10,645	6,086	3	0,586	4,626	3	0,407	7,333	3	0,709	5,220	3	0,484	6,946	3	0,674	76,542	0	1,000
		26-50	18,475	14,067	12,017	2,461	3	0,070	4,329	3	0,259	0,922	3	0,004	4,373	3	0,264	1,347	3	0,013	5,565	3	0,409
7	HK199	1-50	36,191	30,144	27,204	256,220	0	1,000	25,883	3	0,867	13,216	3	0,173	24,880	3	0,835	15,286	3	0,296	21,734	3	0,702
		51-100	37,566	31,410	28,412	215,765	0	1,000	22,422	3	0,682	30,835	2	0,943	25,890	3	0,831	31,177	2	0,947	29,231	2	0,917
		101-150	38,932	32,671	29,615	1199,26	0	1,000	26,811	3	0,823	24,983	3	0,752	31,599	2	0,936	29,246	3	0,892	34,201	1	0,966
		151-199	33,409	27,587	24,769	100,758	0	1,000	8,172	3	0,037	33,241	1	0,989	8,798	3	0,054	23,265	3	0,859	11,412	3	0,166
8	HK261	1-52	40,289	33,924	30,813	301,940	0	1,000	24,471	3	0,677	14,049	3	0,100	24,975	3	0,702	14,481	3	0,116	21,837	3	0,530
		53-104	41,638	35,172	32,007	2194,39	0	1,000	28,378	3	0,798	18,279	3	0,258	30,335	3	0,860	22,008	3	0,480	29,564	3	0,838
		105-156	41,638	35,172	32,007	570,175	0	1,000	22,387	3	0,503	22,657	3	0,519	27,875	3	0,779	28,159	3	0,790	33,969	2	0,934
		157-208	37,566	31,410	28,412	748,019	0	1,000	32,433	1	0,961	30,783	2	0,942	46,466	0	0,999	38,508	0	0,992	54,722	0	1,000
		209-261	37,566	31,410	28,412	760,478	0	1,000	23,974	3	0,756	11,650	3	0,072	27,681	3	0,883	16,349	3	0,305	24,085	3	0,761
9	AU152	1-50	29,141	23,685	21,064	96,359	0	1,000	28,584	1	0,988	19,174	3	0,842	35,346	0	0,999	24,562	1	0,961	38,008	0	0,999
		51-101	30,578	24,996	22,307	102,139	0	1,000	21,111	3	0,867	20,794	3	0,856	19,976	3	0,827	21,327	3	0,873	18,957	3	0,784
		102-152	27,688	22,362	19,812	5,757	3	0,045	3,902	3	0,008	15,191	3	0,704	4,748	3	0,020	11,930	3	0,467	7,866	3	0,148
10	AU161	1-40	23,209	18,307	15,987	71,356	0	1,000	17,711	2	0,940	3,932	3	0,050	13,516	3	0,804	3,714	3	0,041	9,730	3	0,536
		41-80	21,666	16,919	14,684	8,191	3	0,485	12,178	3	0,797	4,320	3	0,111	10,268	3	0,671	3,242	3	0,046	7,681	3	0,433
		81-120	21,666	16,919	14,684	5,776	3	0,238	6,338	3	0,294	2,805	3	0,028	5,474	3	0,209	2,775	3	0,027	6,531	3	0,314
		121-161	23,209	18,307	15,987	3,944	3	0,050	5,238	3	0,125	5,229	3	0,125	4,902	3	0,102	5,123	3	0,117	6,056	3	0,189
11	SP45	1-22	26,217	21,026	18,549	149,734	0	1,000	11,948	3	0,550	8,516	3	0,256	13,760	3	0,684	10,767	3	0,451	13,739	3	0,682
		23-45	30,578	24,996	22,307	310608	0	1,000	22,240	3	0,898	13,279	3	0,419	23,883	2	0,933	15,942	3	0,614	21,298	3	0,872
12	SP114	1-38	38,932	32,671	29,615	2185,9	0	1,000	24,420	3	0,727	26,318	3	0,805	27,857	3	0,856	30,110	2	0,910	36,576	1	0,981
		39-76	32,000	26,296	23,542	83,776	0	1,000	18,223	3	0,689	34,504	0	0,995	17,268	3	0,632	31,804	1	0,989	17,910	3	0,671
		77-114	33,409	27,587	24,769	39,029	0	0,998	23,895	3	0,878	15,926	3	0,471	27,568	2	0,950	20,069	3	0,729	27,077	2	0,943
						avg.	1,679	0,745	**avg.**	2,830	0,492	**avg.**	2,623	0,508	**avg.**	2,811	0,520	**avg.**	2,660	0,535	**avg.**	2,528	0,589

Table 5. Chi-square tests for checking the Poisson, Normal, LogNormal, Logistic, LogLogistic and Laplace distribution fit with similar contract size subdatasets

ID	Database	LogNormal (Contract size)		LogNormal (Number of interested bidders)			Critical χ^2 α values			Model distribution		
	alias	μ_1 (log)	σ_1 (log)	μ_2 (log)	σ_2 (log)	N_{max}	$\chi^2_{\alpha=0.01}$	$\chi^2_{\alpha=0.05}$	$\chi^2_{\alpha=0.10}$	χ^2	$\chi^2<\chi^2_\alpha$?	p-value
1	UK51	13.937	0.852	17.143	3.305	10	18.475	14.067	12.017	3.803	3	0.198
2	UK272	12.122	0.932	14.731	2.925	9	21.666	16.919	14.684	8.768	3	0.541
3	UK218	10.788	1.357	13.910	3.023	9	21.666	16.919	14.684	10.056	3	0.654
4	UK373	12.457	1.032	15.287	2.565	9	21.666	16.919	14.684	11.819	3	0.776
5	US64	13.336	1.393	17.543	3.452	13	23.209	18.307	15.987	6.808	3	0.257
6	US50	14.125	0.488	15.507	1.167	9	18.475	14.067	12.017	1.403	3	0.015
7	HK199	16.157	1.158	20.490	2.986	35	48.278	41.337	37.916	27.320	3	0.499
8	HK261	18.130	1.023	15.085	2.143	33	49.588	42.557	39.087	31.237	3	0.646
9	AU152	13.813	1.251	18.115	3.078	21	30.578	24.996	22.307	13.386	3	0.428
10	AU161	11.701	1.128	15.219	3.078	11	27.688	22.362	19.812	8.098	3	0.163
11	SP45	14.297	1.274	18.592	2.674	40	34.805	28.869	25.989	12.213	3	0.164
12	SP114	17.054	1.185	20.097	2.991	51	57.342	49.802	46.059	42.942	3	0.832
										avg.	3.000	0.431

Table 6. Chi-square tests for checking the model distribution fitting to the Number of Bidders

Rank	*Distribution*	*Average values*		*Std. Deviation values*	
		$\chi^2<\chi^2_\alpha$?	*p-value*	$\chi^2<\chi^2_\alpha$?	*p-value*
1	**New model** proposed (with two-LogNormals)	3.000	0.431	0.000	0.269
2	**Normal** (discriminating by contract size)	2.830	0.492	0.509	0.327
3	**LogNormal** (discriminating by contract size)	2.623	0.508	0.860	0.335
3	**Logistic** (discriminating by contract size)	2.811	0.520	0.622	0.328
3	**Loglogistic** (discriminating by contract size)	2.660	0.535	0.758	0.336
6	**Laplace** (discriminating by contract size)	2.528	0.589	0.953	0.311
7	**LogNormal** (without discriminating by contract size)	1.750	0.647	1.357	0.420
8	**LogLogistic** (without discriminating by contract size)	1.500	0.658	1.567	0.426
8	**Poisson** (discriminating by contract size)	1.679	0.745	1.384	0.330
10	**Logistic** (without discriminating by contract size)	1.583	0.831	1.379	0.232
11	**Normal** (without discriminating by contract size)	1.583	0.836	1.240	0.234
12	**Laplace** (without discriminating by contract size)	0.917	0.898	1.311	0.178
13	**Poisson** (without discriminating by contract size)	0.667	0.903	1.231	0.274

Table 7. Ranking of distributions analysed for modelling the Number of bidders as a function of the chi-square test results

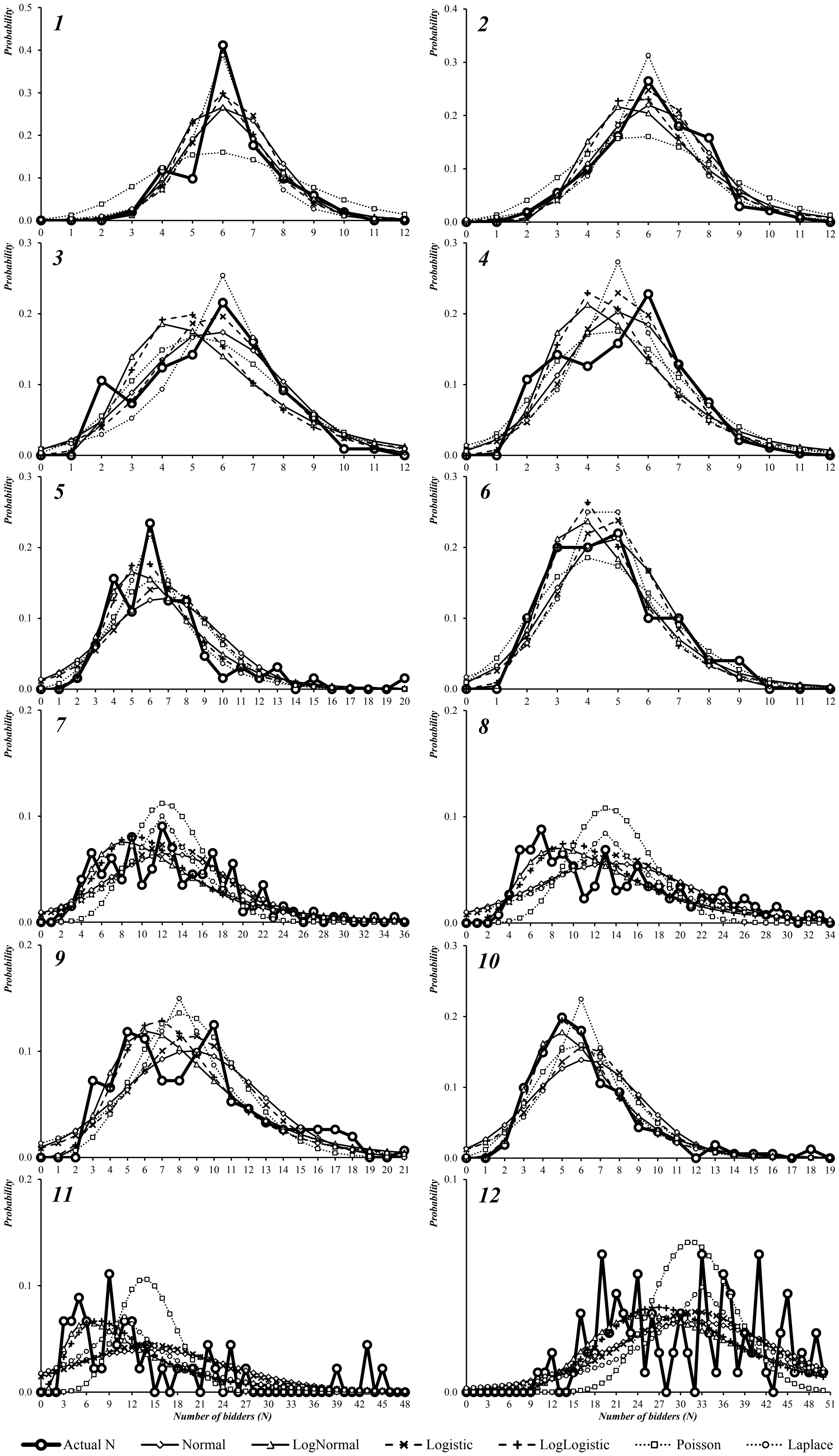

Figure 1. Poisson, Normal, LogNormal, Logistic, LogLogistic and Laplace distribution fitting to the Number of bidders distribution for the twelve datasets

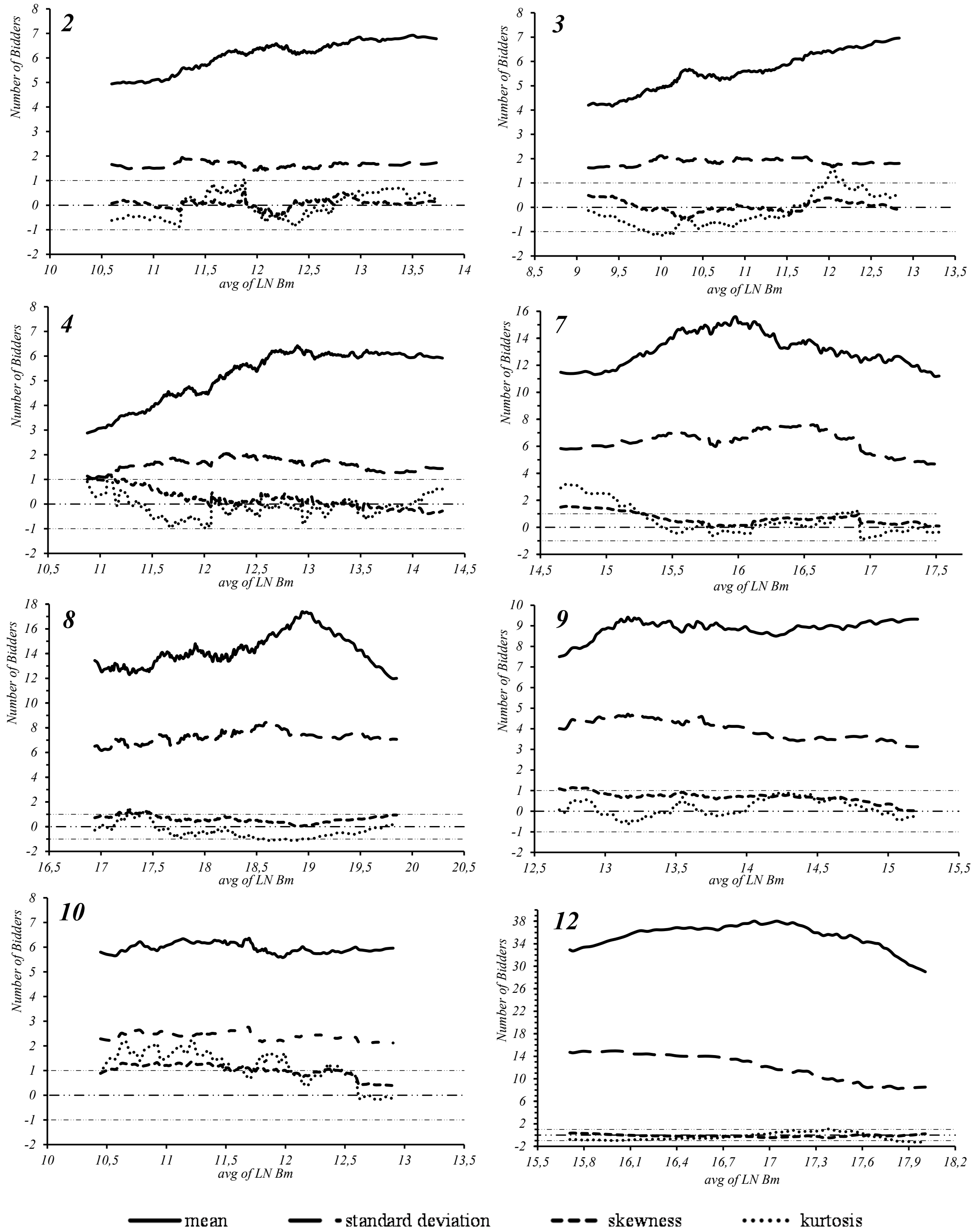

Figure 2. Variation of the mean, standard deviation, skewness and kurtosis of the Number of bidders distribution in sliding windows of 50 tenders for datasets 2-4, 7-10 and 12

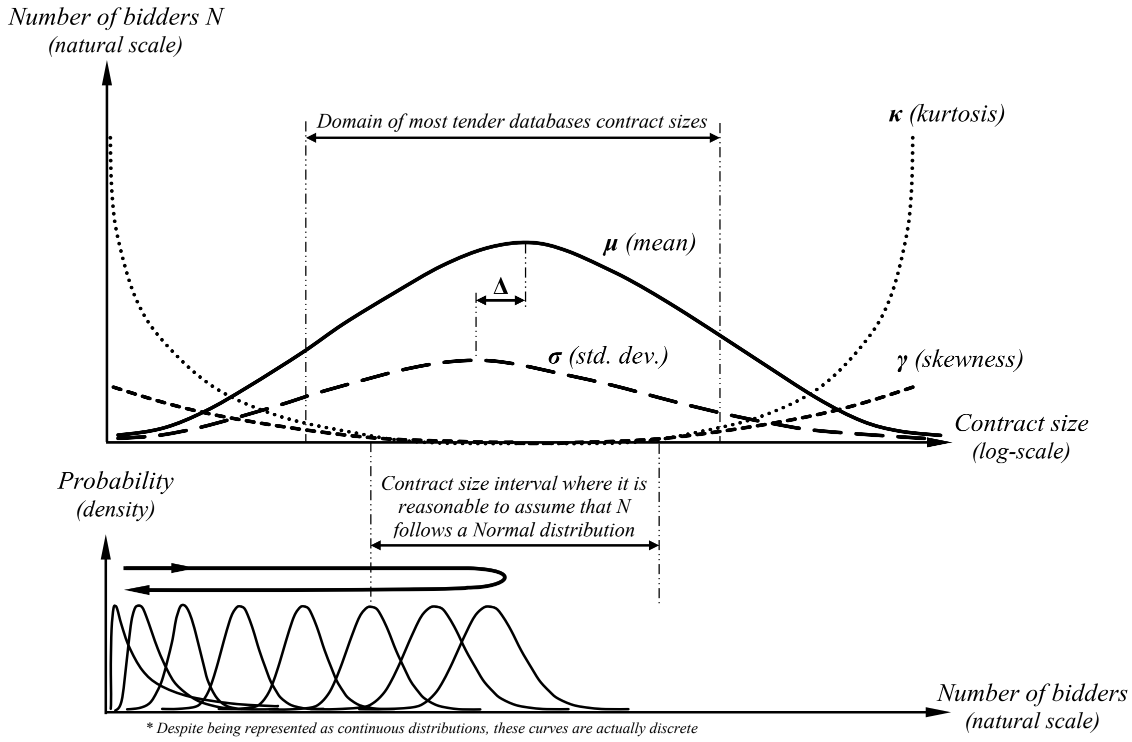

Figure 3. Variation in the mean, standard deviation, skewness and kurtosis of N in terms of contract size

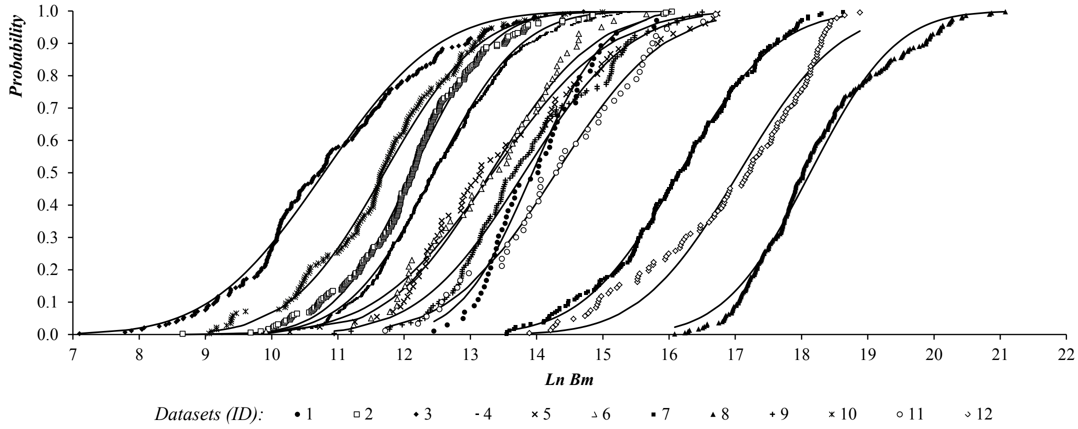

Figure 4. Lognormal distribution fitting to the Contract size opportunities for the twelve datasets

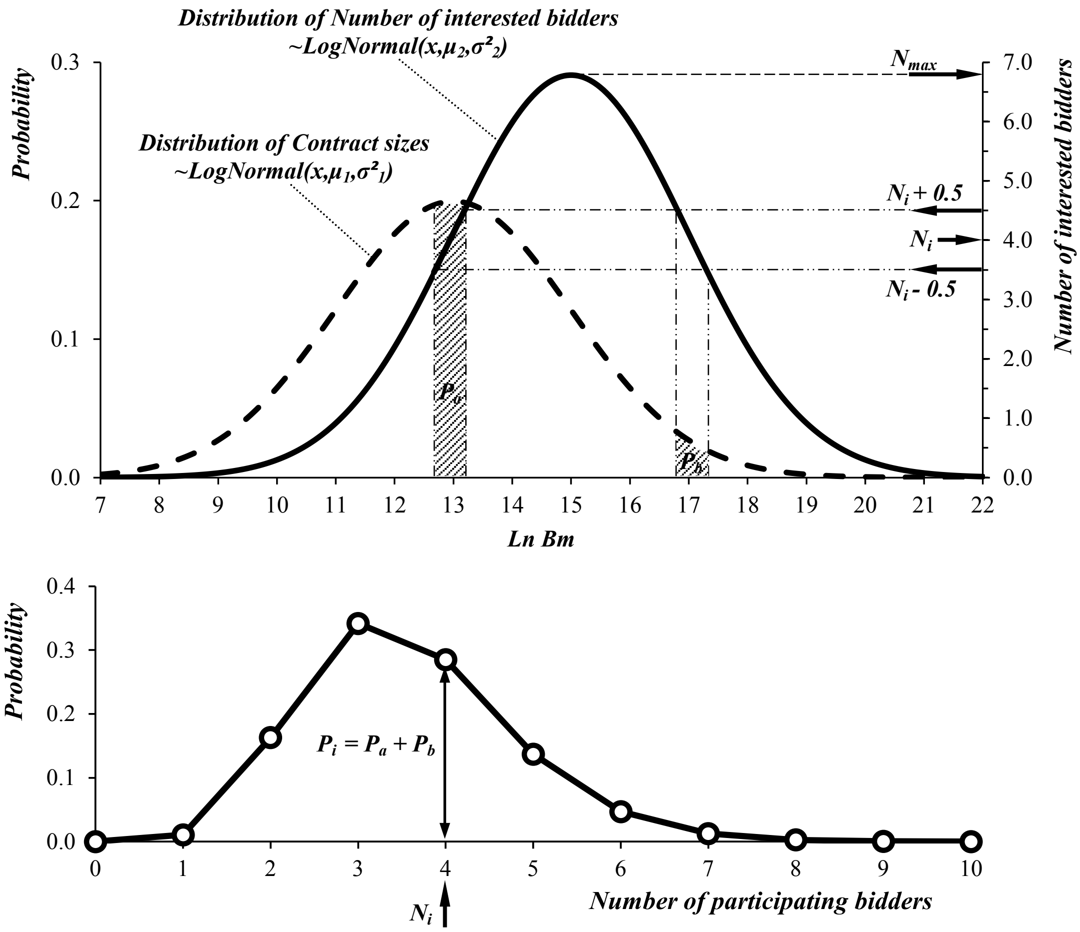

Figure 5. Calculation of the Number of participating bidders distribution as a function of the Contract size opportunities and Number of interested bidders distributions

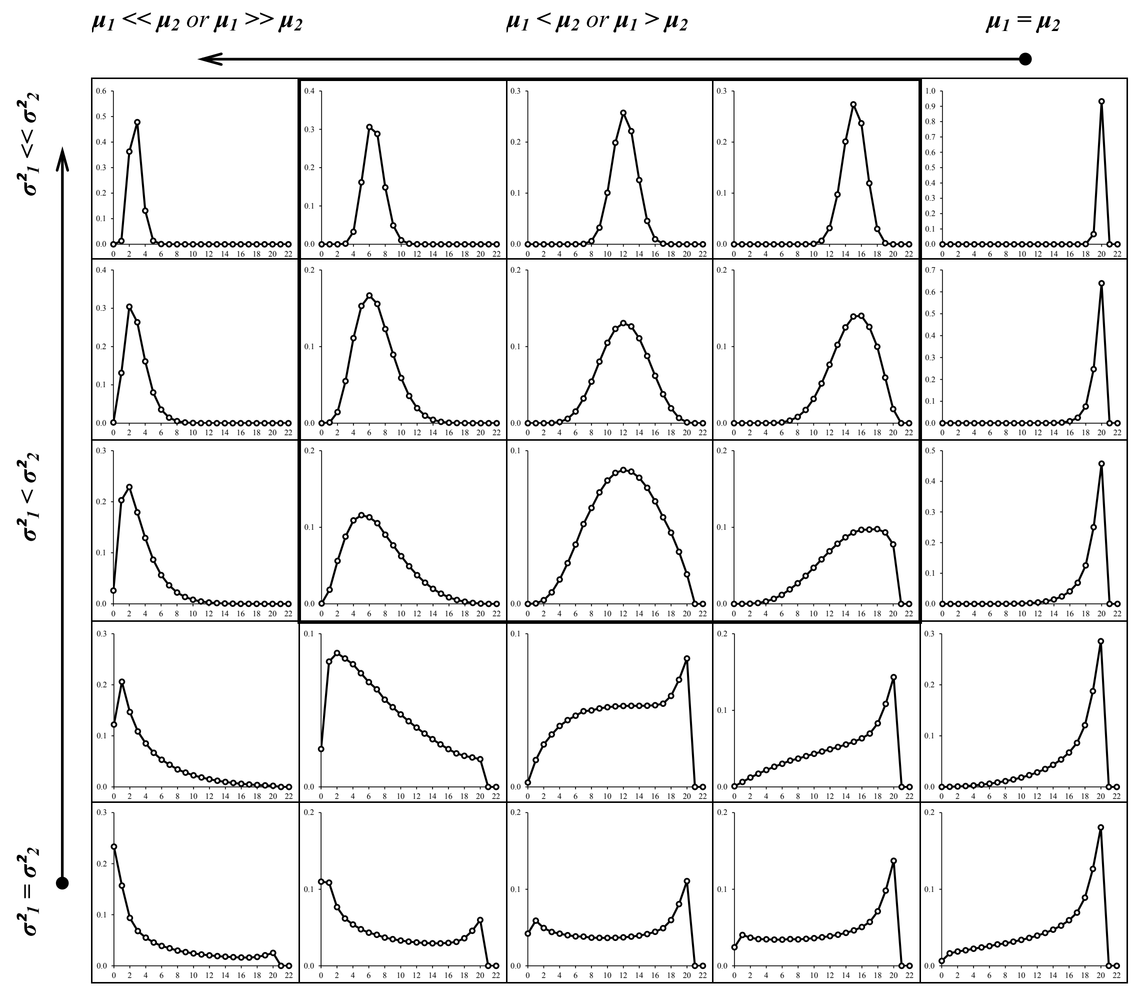

Figure 6. Possible Number of bidders distributions as a function of the relative magnitudes of the Contract size opportunities distribution (1) and the Number of interested bidders distribution (2) assuming both lognormal

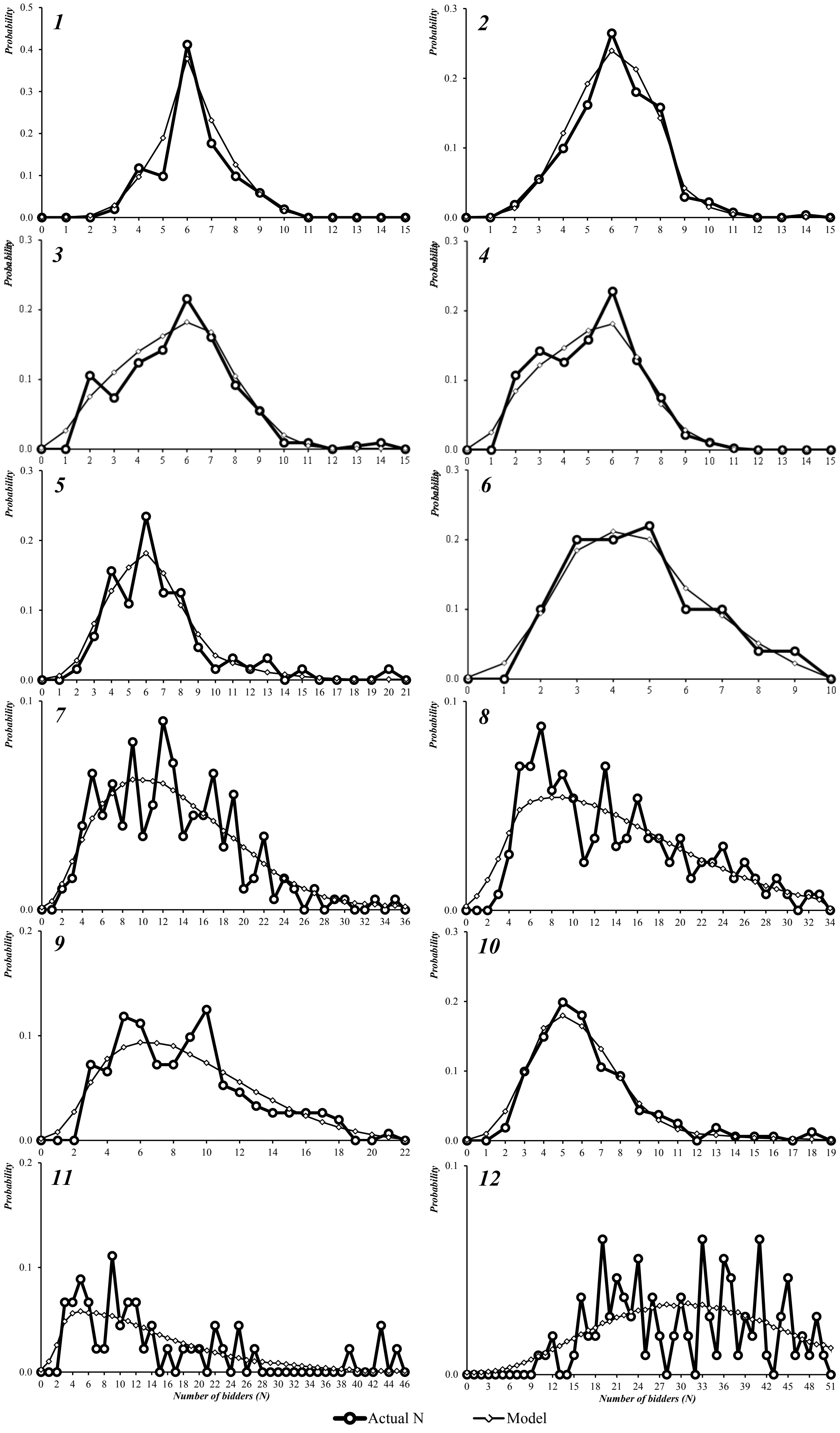

Figure 7. Model distribution fitting to the Number of bidders considering the complete dataset for the twelve datasets

Dataset 1

Order	ID	B_m	N
1	49	255,475	5
2	17	312,916	10
3	50	397,252	5
4	5	406,027	6
5	18	470,578	6
6	26	499,016	7
7	2	520,943	4
8	38	547,634	5
9	32	565,353	6
10	29	580,262	6
11	22	602,953	6
12	51	604,565	6
13	14	638,915	6
14	13	655,085	4
15	31	666,511	6
16	4	679,706	6
17	43	714,806	6
18	39	749,287	7
19	48	764,006	7
20	36	784,334	6
21	33	829,005	6
22	16	865,610	3
23	35	865,760	6
24	9	922,574	6
25	10	1,099,822	4
26	12	1,230,384	6
27	24	1,244,998	8
28	45	1,264,500	4
29	3	1,388,291	7
30	1	1,469,900	6
31	47	1,485,788	8
32	15	1,533,303	6
33	41	1,587,053	6
34	23	1,648,883	5
35	27	1,658,920	4
36	11	1,840,047	6
37	6	2,175,920	9
38	34	2,193,878	7
39	40	2,224,161	8
40	28	2,228,506	7
41	44	2,299,880	5
42	25	2,709,641	6
43	30	2,787,458	6
44	46	2,858,348	7
45	19	3,098,473	9
46	7	3,190,937	7
47	42	3,811,024	8
48	21	4,026,060	7
49	37	7,092,871	9
50	8	7,387,959	4
51	20	8,070,280	8

Dataset 2

Order	ID	B_m	N
1	65	5,752	2
2	132	16,055	6
3	36	18,745	2
4	175	20,872	8
5	1	21,043	3
6	271	22,602	4
7	221	22,847	6
8	154	23,606	4
9	148	24,296	4
10	6	26,599	6
11	176	26,985	6
12	38	27,498	5
13	227	29,002	5
14	222	29,309	3
15	155	29,800	4
16	153	30,060	5
17	42	30,846	6
18	105	32,330	7
19	39	38,903	5
20	7	40,225	5
21	162	40,962	6
22	149	41,843	5
23	256	42,201	3
24	129	43,721	4
25	160	44,195	5
26	220	44,690	6
27	163	44,973	5
28	239	45,573	3
29	76	46,808	5
30	156	46,828	3
31	146	48,194	8
32	8	50,173	4
33	9	50,871	5
34	58	51,998	7
35	224	52,655	7
36	232	54,562	4
37	225	55,555	5
38	63	63,381	6
39	130	64,193	8
40	157	64,430	2
41	80	65,612	5
42	61	66,534	7
43	131	67,182	4
44	88	68,601	8
45	62	69,502	3
46	203	71,184	7
47	90	72,605	2
48	50	72,648	4
49	195	76,842	4
50	204	77,745	6
51	255	77,748	4
52	201	78,599	6
53	89	82,153	4
54	86	83,042	6
55	137	83,206	4
56	116	84,398	3
57	67	84,717	6
58	60	85,427	6
59	104	86,547	5
60	181	86,857	6
61	258	87,192	3
62	119	88,932	6
63	98	89,109	6
64	109	91,469	6
65	106	93,184	5
66	95	93,683	6
67	41	95,009	6
68	268	99,293	3
69	68	99,585	6
70	40	100,055	7
71	133	101,205	7
72	147	101,678	5
73	269	101,960	8
74	193	103,719	8
75	71	109,643	2
76	72	112,067	8
77	100	112,958	8
78	174	114,272	8
79	16	114,342	11
80	218	115,621	5
81	244	119,564	6
82	118	119,621	4
83	192	119,960	6
84	191	122,423	5
85	94	123,751	7
86	172	123,752	8
87	110	129,204	8
88	260	129,318	3
89	115	129,918	6
90	14	130,043	7
91	34	134,108	7
92	56	134,131	6
93	185	135,792	5
94	252	136,210	6
95	121	139,907	5
96	145	140,355	8
97	186	141,088	7
98	19	141,172	6
99	77	143,545	5
100	197	145,920	10
101	182	146,810	4
102	241	147,346	8
103	96	147,706	6
104	17	148,493	7
105	161	149,968	3
106	254	150,922	7
107	10	150,978	6
108	97	151,169	5
109	259	151,653	8
110	245	153,161	8
111	230	154,097	5
112	173	154,405	7
113	212	154,486	5
114	81	154,944	5
115	74	155,037	6
116	138	156,122	6
117	179	156,794	6
118	111	157,101	6
119	33	157,391	5
120	59	163,843	4
121	159	164,295	5
122	234	165,137	7
123	253	165,936	6
124	262	166,609	7
125	200	168,631	4
126	23	170,477	6
127	126	174,111	7
128	242	175,115	8
129	152	175,328	8
130	226	176,334	6
131	114	176,778	8
132	70	178,821	4
133	85	180,630	8
134	135	183,550	6
135	11	183,924	7
136	187	184,729	8
137	27	185,639	8
138	171	186,284	8
139	28	187,459	6
140	167	189,190	8
141	12	192,949	7
142	183	193,093	7
143	113	194,181	6
144	238	194,557	7
145	178	195,704	3
146	122	197,390	9
147	206	198,031	9
148	250	199,406	5
149	43	199,638	5
150	188	200,765	8
151	103	202,149	6
152	112	202,168	4
153	202	204,300	6
154	168	204,482	8
155	123	206,321	6
156	180	206,432	9
157	170	207,578	8
158	205	211,129	6
159	210	214,249	6
160	29	217,260	6
161	66	217,446	7
162	189	218,332	8
163	208	219,037	4
164	263	222,316	7
165	158	224,650	7
166	78	225,170	6
167	151	230,210	5
168	266	230,476	7
169	233	230,554	7
170	215	232,022	5
171	30	236,352	4
172	184	237,491	4
173	108	238,521	6
174	31	240,525	5
175	54	243,298	3
176	22	243,855	6
177	199	244,124	8
178	169	245,394	6
179	92	246,461	5
180	165	247,102	8
181	207	251,498	6
182	57	253,158	8
183	24	254,885	8
184	13	257,123	9
185	217	259,894	6
186	117	260,829	4
187	3	262,157	5
188	164	266,082	5
189	190	276,485	6
190	107	276,824	9
191	128	278,246	5
192	150	278,880	4
193	93	279,423	6
194	4	280,954	8
195	177	282,886	5
196	99	286,431	5
197	124	288,278	7
198	166	293,985	7
199	257	298,370	7
200	46	314,221	5
201	246	316,789	7
202	84	332,029	6
203	87	334,262	7
204	194	335,961	7
205	265	336,091	10
206	249	336,868	8
207	102	352,430	6
208	15	356,801	14
209	219	357,349	8
210	83	366,464	6
211	140	371,239	7
212	26	376,399	6
213	53	377,927	6
214	101	378,556	6
215	5	401,972	6
216	261	405,297	8

Order	ID	B_m	N
217	247	406,191	7
218	267	406,191	7
219	51	408,980	10
220	198	409,354	4
221	64	415,820	7
222	55	425,958	6
223	120	431,343	8
224	91	439,911	5
225	127	444,029	5
226	141	455,627	8
227	270	465,382	3
228	44	478,889	7
229	248	486,656	9
230	18	487,318	11
231	251	491,287	5
232	209	494,573	7
233	240	503,235	6
234	25	506,462	10
235	243	511,560	6
236	223	515,963	6
237	134	529,380	6
238	82	538,110	4
239	211	541,719	7
240	2	542,347	8
241	45	560,949	8
242	144	564,537	8
243	69	662,731	7
244	196	664,835	9
245	20	740,024	7
246	21	749,178	7
247	228	760,179	7
248	143	768,544	3
249	216	775,630	6
250	52	776,894	6
251	139	789,119	7
252	214	845,629	7
253	125	871,134	7
254	229	907,487	6
255	48	987,521	9
256	235	987,774	5
257	272	1,004,402	6
258	32	1,006,498	8
259	136	1,030,093	6
260	236	1,057,545	8
261	49	1,059,427	7
262	47	1,238,914	6
263	35	1,767,575	7
264	213	1,780,238	6
265	79	2,279,604	7
266	237	2,683,822	10
267	264	2,683,828	10
268	231	2,776,352	8
269	142	2,815,698	6
270	73	8,234,520	5
271	75	8,489,562	5
272	37	9,296,868	4

Dataset 3

Order	ID	B_m	N
1	189	1,211	2
2	5	2,413	5
3	72	2,654	5
4	31	3,112	2
5	18	3,222	3
6	127	4,036	3
7	136	4,464	6
8	27	4,685	2
9	192	5,217	5
10	201	5,366	2
11	61	5,796	5
12	92	5,861	3
13	8	6,629	8
14	64	6,919	2
15	182	6,927	6
16	195	7,202	6
17	145	8,110	4
18	83	8,282	5
19	7	8,298	3
20	114	8,517	4
21	63	9,159	2
22	90	9,326	4
23	154	9,635	5
24	91	9,670	4
25	167	9,787	4
26	190	9,807	4
27	73	11,545	3
28	69	11,676	6
29	153	11,803	6
30	218	11,889	5
31	177	11,890	5
32	159	12,807	4
33	213	13,666	3
34	168	14,264	6
35	104	14,287	4
36	70	14,314	5
37	116	14,425	2
38	1	15,046	8
39	129	16,996	4
40	60	17,763	5
41	4	17,767	3
42	173	18,010	3
43	14	18,551	6
44	199	18,817	5
45	67	18,973	4
46	55	19,801	2
47	68	20,069	4
48	21	20,116	2
49	118	20,423	3
50	216	20,469	8
51	142	20,545	6
52	37	20,802	2
53	85	20,925	6
54	25	21,219	2
55	19	21,305	2
56	84	21,416	4
57	107	21,787	2
58	175	21,993	6
59	160	22,073	7
60	132	22,210	6
61	186	22,551	6
62	126	22,889	4
63	123	23,524	6
64	86	23,829	7
65	124	23,832	6
66	174	23,858	6
67	121	23,986	6
68	176	24,023	6
69	171	24,310	7
70	22	24,438	2
71	120	24,447	6
72	200	24,624	8
73	170	24,912	8
74	209	24,946	7
75	80	25,279	5
76	23	25,288	2
77	26	25,523	2
78	122	25,623	6
79	198	26,406	8
80	58	26,541	3
81	87	26,960	5
82	193	27,311	9
83	20	27,732	2
84	185	27,757	8
85	82	29,038	4
86	49	29,575	5
87	111	29,713	5
88	156	29,808	5
89	206	29,852	7
90	78	31,271	5
91	24	32,420	2
92	117	32,952	4
93	208	32,962	6
94	215	33,354	7
95	59	33,385	3
96	191	33,509	7
97	113	33,641	3
98	62	34,764	6
99	13	35,358	6
100	71	35,818	6
101	178	36,433	5
102	205	38,145	9
103	135	40,184	6
104	217	40,452	8
105	2	40,862	8
106	34	41,282	6
107	112	42,041	4
108	211	44,620	7
109	36	44,724	8
110	28	45,707	4
111	214	47,248	6
112	44	47,534	7
113	130	48,582	4
114	11	48,682	7
115	115	49,769	5
116	54	50,375	2
117	134	50,520	6
118	179	50,695	4
119	128	50,856	2
120	74	51,410	4
121	133	51,549	6
122	184	52,254	8
123	131	53,644	6
124	148	54,101	4
125	99	55,239	4
126	32	58,195	5
127	163	60,332	5
128	66	65,616	7
129	147	67,072	4
130	197	67,652	7
131	53	69,894	2
132	98	74,280	5
133	45	74,587	2
134	76	76,183	3
135	188	77,321	9
136	79	78,715	5
137	89	79,745	3
138	95	81,210	7
139	149	81,450	6
140	77	84,110	3
141	146	86,481	7
142	12	89,978	13
143	75	91,422	4
144	202	92,063	9
145	6	92,596	10
146	143	93,938	6
147	203	94,027	7
148	183	97,760	5
149	140	99,405	7
150	88	100,567	6
151	93	102,126	6
152	150	104,115	6
153	33	107,038	7
154	48	111,189	6
155	196	112,007	9
156	57	113,796	7
157	137	115,081	6
158	141	115,471	6
159	97	119,885	7
160	50	121,158	3
161	139	122,544	8
162	164	122,895	8
163	94	129,329	6
164	180	130,153	6
165	41	130,359	5
166	17	134,443	14
167	9	146,246	7
168	101	148,013	2
169	187	152,111	9
170	81	155,835	5
171	125	157,494	4
172	169	170,593	7
173	15	174,266	7
174	39	175,981	7
175	155	186,359	7
176	157	195,637	7
177	194	198,405	11
178	165	199,653	7
179	119	204,132	5
180	52	213,544	7
181	40	226,280	4
182	152	227,459	5
183	10	242,757	11
184	100	255,876	5
185	51	257,050	5
186	47	259,844	7
187	43	261,374	6
188	103	264,877	6
189	212	268,807	4
190	106	272,708	7
191	42	284,132	5
192	110	287,760	7
193	35	289,899	6
194	109	336,040	6
195	105	370,847	8
196	144	410,514	7
197	151	426,977	9
198	172	428,115	10
199	138	431,616	7
200	108	445,232	6
201	3	509,741	9
202	102	530,878	8
203	166	537,637	6
204	56	547,061	6
205	181	562,081	8
206	46	580,737	7
207	38	607,239	6
208	29	720,865	8
209	158	779,079	9
210	210	851,480	9
211	204	885,689	9

Order	ID	B_m	N
212	65	1,025,652	8
213	161	1,154,586	7
214	96	1,156,500	7
215	30	1,169,669	14
216	207	1,383,514	9
217	162	1,996,684	8
218	16	2,443,593	8

Dataset 4

Order	ID	B_m	N
1	270	10,266	2
2	201	18,756	2
3	162	20,607	2
4	192	22,609	2
5	10	25,800	2
6	189	32,296	4
7	71	35,050	2
8	131	39,459	4
9	3	42,309	3
10	76	42,335	2
11	59	46,230	2
12	218	46,965	3
13	93	47,257	2
14	127	48,179	3
15	173	48,643	2
16	57	49,786	3
17	166	49,788	2
18	4	50,011	3
19	31	50,709	3
20	135	51,024	3
21	98	51,737	3
22	302	51,762	3
23	46	52,509	4
24	369	52,517	5
25	227	57,866	2
26	88	59,237	2
27	114	59,666	4
28	42	60,154	3
29	170	61,560	2
30	86	62,747	2
31	183	64,407	5
32	168	64,768	3
33	8	64,870	2
34	154	65,584	2
35	259	67,107	5
36	306	69,055	3
37	289	70,535	2
38	25	72,051	3
39	269	73,042	6
40	191	75,073	3
41	23	75,675	2
42	303	77,774	4
43	172	78,675	4
44	279	81,868	3
45	6	83,477	3
46	96	83,722	2
47	185	84,382	2
48	169	84,652	2
49	274	86,141	3
50	116	86,209	4
51	101	88,478	5
52	196	89,054	4
53	359	89,544	6
54	213	90,931	3
55	142	92,008	3
56	159	92,742	4
57	1	93,246	7
58	181	97,951	4
59	15	98,687	2
60	115	99,086	6
61	230	99,339	6
62	297	99,470	6
63	187	100,309	4
64	262	101,130	8
65	37	104,858	4
66	97	105,919	3
67	313	106,138	2
68	48	106,193	5
69	371	106,300	3
70	81	107,887	6
71	206	110,050	2
72	197	110,194	4
73	291	110,884	4
74	246	111,308	3
75	62	112,400	2
76	199	112,882	3
77	271	113,112	6
78	278	114,144	3
79	301	114,593	4
80	163	114,748	2
81	345	116,618	2
82	33	116,931	5
83	91	117,739	7
84	14	118,503	2
85	171	119,032	4
86	272	119,270	6
87	26	119,631	6
88	83	123,004	4
89	219	123,418	6
90	348	124,383	8
91	47	124,537	3
92	182	125,622	3
93	298	125,771	4
94	326	126,402	5
95	212	126,660	5
96	107	126,800	4
97	79	129,460	5
98	84	130,632	5
99	287	131,416	5
100	74	131,565	8
101	312	133,233	3
102	128	133,540	7
103	24	133,971	3
104	38	134,815	7
105	317	135,358	8
106	125	135,369	2
107	307	135,660	2
108	87	144,477	5
109	157	146,759	2
110	343	148,003	7
111	341	148,421	3
112	69	150,081	5
113	70	151,904	5
114	193	152,157	5
115	52	152,238	5
116	237	153,443	4
117	174	153,789	7
118	165	158,196	3
119	7	158,521	3
120	328	158,794	4
121	368	159,037	4
122	19	159,861	6
123	362	160,155	5
124	276	160,607	5
125	325	161,488	6
126	113	163,868	5
127	344	165,774	5
128	207	166,162	6
129	300	168,570	4
130	286	168,656	3
131	92	168,704	4
132	244	171,307	2
133	356	172,202	6
134	21	174,101	2
135	232	175,982	5
136	99	177,015	2
137	51	177,396	3
138	373	177,447	6
139	258	178,802	3
140	332	182,529	2
141	148	183,077	5
142	158	183,573	3
143	215	183,956	3
144	145	185,581	6
145	216	185,766	6
146	315	185,777	3
147	152	186,199	7
148	239	188,155	6
149	273	189,592	5
150	68	190,001	8
151	151	192,221	3
152	220	193,145	5
153	252	195,882	6
154	12	198,895	4
155	119	199,253	7
156	223	199,539	6
157	322	201,565	6
158	240	203,383	6
159	164	203,517	6
160	43	204,325	10
161	285	205,432	7
162	140	207,530	7
163	150	209,994	6
164	108	213,588	5
165	146	217,099	7
166	204	219,550	6
167	295	220,081	6
168	20	221,482	6
169	153	221,736	2
170	241	222,186	6
171	208	222,858	10
172	352	225,887	8
173	40	226,345	3
174	222	228,306	3
175	324	228,548	6
176	28	228,611	2
177	89	228,686	7
178	54	229,444	4
179	370	231,175	6
180	34	234,544	8
181	251	235,048	7
182	226	240,781	3
183	292	240,785	3
184	314	242,726	7
185	264	245,385	3
186	342	247,403	4
187	117	249,180	6
188	355	249,957	4
189	235	250,595	6
190	45	250,649	3
191	147	251,652	6
192	335	252,577	6
193	319	253,958	7
194	134	257,506	3
195	149	259,653	5
196	347	260,232	6
197	138	261,590	7
198	39	263,795	4
199	364	268,421	4
200	120	273,857	6
201	281	275,082	10
202	200	275,553	3
203	339	277,997	7
204	305	292,008	7
205	194	292,696	7
206	121	293,169	5
207	363	293,720	6
208	75	295,163	6
209	311	295,839	9
210	105	296,576	5
211	327	301,196	6
212	85	306,749	6
213	102	307,320	8
214	11	308,365	6
215	309	308,864	5
216	104	315,881	3
217	329	316,722	4
218	367	320,945	7
219	211	325,739	3
220	234	326,936	5
221	132	329,463	5
222	90	330,003	7
223	354	331,138	6
224	243	336,799	8
225	265	339,929	8
226	320	341,272	5
227	144	346,369	11
228	266	346,650	8
229	95	350,441	8
230	53	351,739	3
231	190	354,912	4
232	340	357,137	9
233	109	357,794	8
234	167	358,237	6
235	156	362,705	5
236	56	364,978	7
237	228	367,800	9
238	231	368,037	7
239	349	385,951	6
240	233	390,881	8
241	129	391,214	5
242	372	393,297	7
243	293	393,967	5
244	253	398,477	6
245	257	398,555	4
246	139	398,680	5
247	112	400,523	7
248	136	401,065	5
249	296	401,373	6
250	217	407,210	6
251	82	411,514	6
252	275	412,354	4
253	290	415,477	7
254	358	416,033	5
255	133	416,141	4
256	277	419,637	8
257	66	421,229	7
258	203	423,123	4
259	122	425,352	8
260	180	428,382	6

Order	ID	B_m	N
261	263	430,774	6
262	9	443,592	8
263	13	453,216	8
264	316	457,125	5
265	238	457,601	6
266	73	462,459	4
267	100	471,945	6
268	304	474,179	9
269	111	477,075	7
270	63	495,134	6
271	337	495,743	8
272	357	502,889	3
273	35	505,227	7
274	32	509,133	5
275	186	516,209	6
276	260	516,448	4
277	143	533,213	6
278	310	533,285	5
279	214	534,462	7
280	72	542,917	9
281	308	546,775	7
282	209	547,133	6
283	58	555,493	10
284	282	558,570	5
285	346	562,844	4
286	360	567,769	7
287	124	567,999	5
288	247	578,837	2
289	323	595,331	5
290	64	602,094	3
291	225	603,995	7
292	44	605,672	9
293	236	605,815	5
294	250	609,716	6
295	254	621,330	7
296	141	621,751	4
297	155	623,702	8
298	67	625,582	8
299	248	639,203	6
300	224	647,075	4
301	284	656,363	8
302	294	660,556	6
303	268	667,073	5
304	365	673,507	7
305	29	678,044	6
306	210	684,172	5
307	249	687,499	5
308	55	695,530	6
309	242	711,072	5
310	178	718,094	7
311	353	730,250	6
312	221	731,243	7
313	77	731,953	8
314	118	733,926	6
315	176	734,159	5
316	333	736,153	4
317	80	741,370	6
318	351	760,367	7
319	50	767,523	6
320	49	779,007	8
321	283	798,192	6
322	110	799,855	4
323	195	820,054	7
324	330	841,216	5
325	288	856,166	7
326	366	863,234	8
327	60	890,692	8

Order	ID	B_m	N
328	256	904,120	6
329	18	926,226	6
330	299	927,109	4
331	17	927,566	5
332	334	929,771	4
333	161	959,114	8
334	103	964,725	6
335	229	1,004,630	6
336	5	1,026,480	7
337	41	1,054,996	9
338	188	1,120,178	6
339	130	1,124,186	6
340	30	1,166,292	7
341	255	1,187,921	3
342	338	1,191,547	7
343	184	1,219,109	6
344	94	1,231,401	5
345	137	1,253,170	7
346	202	1,320,747	6
347	336	1,344,893	7
348	261	1,360,551	4
349	205	1,381,556	7
350	2	1,472,162	6
351	177	1,562,410	7
352	22	1,568,224	7
353	179	1,581,625	6
354	321	1,583,369	6
355	36	1,633,361	4
356	106	1,659,637	5
357	78	1,999,840	6
358	350	2,005,694	6
359	61	2,071,215	6
360	198	2,189,824	5
361	160	2,217,761	5
362	267	2,446,742	3
363	318	2,473,213	6
364	65	2,662,722	6
365	175	2,730,352	6
366	27	2,824,961	9
367	123	3,229,167	8
368	126	3,387,660	2
369	280	3,596,202	6
370	245	3,985,016	6
371	331	4,187,295	5
372	16	4,726,542	5
373	361	8,771,818	5

Dataset 5

Order	ID	B_m	N
1	9	29,044	7
2	30	45,653	7
3	13	77,294	7
4	12	107,840	6
5	39	137,589	20
6	26	154,196	4
7	53	162,505	13
8	37	171,720	8
9	8	176,835	3
10	15	186,726	6
11	36	188,702	6
12	32	203,945	6
13	16	204,247	8
14	14	224,953	6
15	57	228,182	8
16	19	229,316	4
17	7	230,766	6
18	45	243,824	4
19	35	263,014	8
20	55	269,513	7
21	64	285,590	13
22	43	286,220	6
23	49	287,666	5
24	62	319,228	15
25	29	382,367	10
26	42	387,837	3
27	52	393,076	5
28	46	402,341	5
29	51	424,831	6
30	27	446,274	6
31	63	478,377	6
32	21	506,707	4
33	61	520,026	4
34	6	524,399	6
35	50	598,986	11
36	47	670,525	9
37	10	747,552	2
38	24	757,616	5
39	11	868,992	4
40	40	949,038	12
41	34	1,316,515	6
42	33	1,376,083	9
43	25	1,481,536	7
44	44	1,484,631	6
45	48	1,515,446	8
46	56	1,584,305	8
47	38	1,601,454	4
48	60	1,883,560	5
49	5	2,003,500	7
50	28	2,110,914	3
51	17	2,303,748	4
52	41	3,013,188	8
53	18	3,197,054	4
54	58	3,214,767	5
55	3	3,492,159	6
56	4	3,676,250	4
57	1	4,150,514	7
58	23	7,532,342	6
59	31	7,865,411	7
60	2	9,463,718	11
61	54	12,201,531	8
62	59	15,379,320	9
63	20	17,429,631	5
64	22	18,465,333	3

Dataset 6

Order	ID	B_m	N
1	16	20,247	3
2	49	76,211	3
3	31	98,962	5
4	37	131,278	3
5	28	145,951	5
6	44	147,051	5
7	26	147,200	5
8	18	165,117	5
9	23	174,157	2
10	47	174,666	3
11	3	174,981	5
12	35	183,188	8
13	9	233,860	7
14	14	249,480	2
15	36	255,602	4
16	40	278,273	3
17	30	305,750	4
18	5	326,991	5
19	45	399,197	6
20	12	438,765	4
21	50	448,149	5
22	2	453,341	7
7	20	522,621	7
24	15	537,500	2
25	6	598,250	4
26	7	686,000	3
27	38	767,358	3
28	46	787,940	4
29	29	804,355	3
30	25	838,808	2
31	4	861,231	7
32	8	904,356	6
33	11	965,780	6
34	27	1,035,251	3
35	19	1,197,103	5
36	13	1,220,968	5
37	24	1,315,074	7
38	21	1,375,299	9
39	41	1,409,432	4
40	43	1,527,585	6
41	17	1,635,392	5
42	34	1,635,426	9
43	10	1,725,065	6
44	32	1,906,357	4
45	22	2,205,996	4
46	48	2,269,116	4
47	39	2,277,616	2
48	1	3,203,337	8
49	42	3,869,281	3
50	33	6,321,544	4

Dataset 7

Order	ID	B_m	N
7	43	763,896	17
2	128	765,264	4
3	77	800,696	9
4	173	852,246	13
5	32	872,496	18
6	34	983,245	9
7	74	1,099,338	14
8	44	1,329,758	11
9	140	1,345,589	9
10	22	1,417,487	5
11	36	1,455,436	7
12	80	1,457,550	12
13	1	1,521,264	5
14	123	1,579,990	11
15	16	1,639,043	7
16	126	1,692,042	5
17	176	1,695,106	5
18	91	1,861,218	8
19	40	1,932,775	12
20	130	1,933,606	11
21	79	1,945,299	16
22	21	2,125,224	6
23	76	2,197,257	10
24	60	2,237,624	12
25	65	2,315,028	9
26	38	2,531,162	11
27	124	2,604,669	9
28	45	2,869,188	16
29	134	2,923,644	6
30	106	2,978,741	12
31	117	3,077,471	10
32	53	3,097,133	16
33	72	3,103,435	7
34	94	3,162,445	6
35	109	3,374,166	18
36	37	3,488,874	8
37	158	3,793,233	21
38	48	3,879,931	25

Order	ID	B_m	N
39	26	3,955,454	9
40	42	4,074,833	7
41	86	4,165,783	15
42	64	4,263,397	11
43	85	4,317,992	24
44	19	4,411,610	5
45	105	4,734,579	9
46	104	4,895,453	19
47	28	4,910,363	9
48	113	4,953,931	10
49	9	5,081,488	13
50	61	5,158,058	33
51	51	5,213,495	13
52	144	5,239,616	3
53	30	5,308,333	10
54	55	5,311,018	16
55	75	5,417,879	22
56	177	5,430,074	5
57	115	5,455,079	7
58	132	5,546,321	12
59	98	5,568,012	12
60	63	5,606,824	13
61	139	5,762,240	5
62	116	5,776,201	17
63	101	5,840,471	5
64	68	5,849,842	27
65	39	5,971,309	12
66	69	6,431,575	12
67	160	6,520,410	13
68	50	6,577,772	19
69	88	6,711,020	17
70	112	6,926,366	21
71	175	6,941,442	7
72	82	6,993,996	22
73	110	7,020,557	4
74	96	7,052,495	14
75	102	7,077,820	18
76	78	7,138,917	24
77	100	7,159,077	17
78	2	7,195,958	19
79	29	7,298,614	8
80	81	7,316,638	25
81	6	7,382,782	18
82	70	7,753,293	13
83	118	7,803,872	27
84	87	8,316,816	19
85	136	8,326,467	17
86	137	8,503,542	15
87	138	8,513,146	9
88	92	8,651,503	17
89	27	8,725,680	15
90	67	8,829,614	6
91	95	9,130,136	19
92	35	9,157,789	13
93	163	9,270,871	13
94	120	9,376,634	18
95	119	9,378,027	19
96	168	9,888,951	14
97	83	9,967,104	21
98	129	10,116,512	7
99	10	10,195,475	5
100	90	10,238,717	12
101	125	10,562,633	16
102	58	11,002,658	22
103	161	11,264,567	13
104	59	11,352,444	30
105	196	11,701,127	16

Order	ID	B_m	N
106	7	11,798,922	4
107	159	11,877,969	15
108	162	11,879,965	12
109	15	12,016,014	6
110	114	12,219,982	11
111	62	12,247,327	23
112	57	12,310,844	29
113	13	12,689,001	5
114	143	12,743,941	6
115	181	12,775,105	14
116	24	12,813,902	6
117	133	12,997,923	13
118	131	13,082,183	19
119	122	13,468,271	17
120	12	14,089,689	4
121	107	14,326,042	18
122	84	14,740,774	35
123	3	14,942,473	4
124	157	14,987,988	4
125	103	15,237,206	17
126	155	15,376,482	9
127	11	15,468,071	5
128	4	15,492,985	9
129	18	15,884,604	8
130	66	16,001,038	19
131	47	16,142,432	22
132	46	17,270,516	19
133	14	17,600,051	4
134	20	18,052,061	5
135	17	18,261,285	3
136	56	18,510,902	13
137	33	19,017,916	11
138	97	19,580,746	15
139	145	19,682,885	15
140	180	19,820,731	19
141	195	20,329,081	22
142	111	20,482,778	13
143	73	2,056,7519	13
144	52	20,763,266	22
145	93	20,987,071	17
146	89	21,721,427	24
147	5	21,897,525	8
148	71	22,005,460	19
149	8	22,005,595	9
150	149	22,273,687	7
151	25	22,523,145	10
152	49	22,792,215	16
153	142	23,374,497	9
154	121	24,397,705	12
155	172	24,736,322	6
156	166	25,553,436	22
157	31	25,691,639	7
158	169	26,488,553	14
159	41	26,866,865	12
160	150	29,074,170	9
161	194	29,158,901	14
162	186	31,219,182	11
163	188	32,162,193	12
164	189	32,320,613	17
165	23	32,466,204	8
166	182	32,504,745	9
167	151	32,836,424	5
168	146	32,898,259	8
169	192	33,442,044	20
170	174	34,477,446	7
171	185	34,966,263	11
172	184	35,050,955	17

Order	ID	B_m	N
173	127	35,470,407	13
174	99	36,150,477	12
175	147	37,571,271	10
176	108	39,589,743	8
177	193	39,629,959	12
178	179	40,797,771	11
179	167	43,470,348	17
180	198	45,444,698	15
181	153	45,638,843	2
182	165	45,827,899	15
183	54	50,844,928	14
184	164	51,272,832	3
185	197	52,622,753	15
186	170	54,801,234	16
187	171	55,467,234	9
188	135	58,550,982	7
189	148	60,559,950	6
190	190	61,415,292	12
191	178	61,650,136	12
192	187	62,332,752	12
193	183	64,564,247	16
194	152	66,006,552	2
195	199	72,353,464	17
196	191	72,488,422	20
197	154	85,998,385	7
198	156	89,072,086	4
199	141	122,432,544	10

Dataset 8

Order	ID	B_m	N
1	235	9,686,030	16
2	237	11,293,063	27
3	240	11,841,023	24
4	259	12,117,616	11
5	261	13,887,971	14
6	250	14,783,092	7
7	245	15,132,109	12
8	234	15,741,697	18
9	133	15,837,168	8
10	244	18,823,189	22
11	142	19,870,200	6
12	241	20,302,245	19
13	215	20,412,549	17
14	209	20,579,837	24
15	202	20,750,492	17
16	208	21,005,604	16
17	189	21,872,827	8
18	220	22,830,562	6
19	1	22,905,464	4
20	251	23,202,525	29
21	151	23,288,908	9
22	200	23,399,980	18
23	219	23,644,428	27
24	197	24,511,056	10
25	132	24,630,300	10
26	258	25,298,293	8
27	183	25,428,526	11
28	191	25,709,445	8
29	184	25,905,435	17
30	162	26,007,830	15
31	139	26,314,349	7
32	84	26,417,821	3
33	174	26,600,432	5
34	76	26,950,453	10
35	238	27,447,056	25
36	236	28,108,553	20
37	92	28,415,299	9
38	169	28,525,712	10
39	172	28,533,663	9

Order	ID	B_m	N
40	60	28,986,427	10
41	167	29,264,893	14
42	176	29,270,416	8
43	199	29,557,430	13
44	130	29,737,681	22
45	54	30,218,068	14
46	56	30,289,323	10
47	127	30,634,246	12
48	131	31,576,756	14
49	185	32,309,757	9
50	158	32,405,302	10
51	5	32,648,679	8
52	20	32,798,912	8
53	77	33,010,313	5
54	230	33,671,263	20
55	123	33,672,702	17
56	120	33,797,575	9
57	246	33,797,664	16
58	126	34,458,393	4
59	217	35,026,179	27
60	121	35,564,261	6
61	224	35,893,943	33
62	134	35,948,491	16
63	47	36,231,371	15
64	22	36,419,133	7
65	50	36,646,493	8
66	252	37,391,261	24
67	201	37,397,035	11
68	64	37,899,102	6
69	211	39,268,954	20
70	216	39,983,159	10
71	87	40,034,426	15
72	193	40,219,489	11
73	148	40,267,410	11
74	119	41,142,768	5
75	179	41,703,108	18
76	88	42,240,240	9
77	228	42,500,698	25
78	63	44,257,298	6
79	71	44,609,273	6
80	94	45,711,952	12
81	248	46,018,107	16
82	147	47,518,324	6
83	90	48,189,273	7
84	186	48,423,414	12
85	247	48,617,410	29
86	118	49,059,223	13
87	16	49,274,964	7
88	42	49,532,531	9
89	72	49,620,702	7
90	11	49,906,898	16
91	70	50,643,989	5
92	154	50,698,846	21
93	157	50,821,541	12
94	239	50,991,436	26
95	253	51,057,579	28
96	214	52,415,737	22
97	45	52,632,198	13
98	89	53,244,674	16
99	256	53,312,678	22
100	99	53,501,390	9
101	52	54,063,116	12
102	124	54,071,508	20
103	86	54,307,385	10
104	38	54,644,554	16
105	68	54,947,923	7
106	204	55,416,740	13

Order	ID	B_m	N
107	105	55,716,395	20
108	156	55,825,133	12
109	225	56,276,136	23
110	58	56,972,579	5
111	24	57,185,850	17
112	229	57,530,816	23
113	9	58,034,490	7
114	75	58,361,917	4
115	80	58,799,099	5
116	107	59,274,762	15
117	178	59,701,239	23
118	187	59,781,748	15
119	106	60,221,650	16
120	12	60,389,249	8
121	180	60,572,413	8
122	213	60,608,254	25
123	59	61,792,701	3
124	13	62,439,050	7
125	255	63,922,601	12
126	152	64,261,762	11
127	218	66,605,605	32
128	61	66,743,092	18
129	57	66,803,823	5
130	78	66,871,052	5
131	155	67,194,397	9
132	164	67,282,323	10
133	188	67,412,187	13
134	165	68,163,986	30
135	260	68,287,273	18
136	15	68,563,040	9
137	91	69,701,410	7
138	83	70,627,860	19
139	254	70,641,167	12
140	181	71,010,773	24
141	227	72,986,042	23
142	79	73,933,651	7
143	30	74,013,304	15
144	41	74,912,322	13
145	210	74,976,914	22
146	243	75,146,431	24
147	36	75,449,647	13
148	21	77,530,258	16
149	32	78,650,299	13
150	35	79,089,305	17
151	8	79,826,297	8
152	31	79,827,787	6
153	49	81,034,245	6
154	128	81,153,956	20
155	26	82,222,571	10
156	222	83,688,694	21
157	182	84,855,011	18
158	33	86,729,872	16
159	18	86,952,841	5
160	19	89,469,514	6
161	34	90,922,565	5
162	137	91,374,772	14
163	166	91,485,441	24
164	37	92,341,007	14
165	29	92,950,923	9
166	85	97,406,505	5
167	98	97,406,505	5
168	194	97,823,404	24
169	14	98,579,660	7
170	168	98,937,736	7
171	136	99,541,286	33
172	10	100,187,412	8
173	103	103,429,075	10
174	17	103,983,088	6
175	146	104,620,712	26
176	69	107,124,675	7
177	108	107,674,517	8
178	173	108,191,498	18
179	23	108,478,803	9
180	104	108,611,921	19
181	62	108,676,000	6
182	198	110,295,802	29
183	161	111,843,652	26
184	40	113,779,940	9
185	125	118,831,902	26
186	28	119,642,127	20
187	48	121,431,073	17
188	43	121,746,843	16
189	203	121,986,939	24
190	171	123,091,301	23
191	140	123,995,149	6
192	145	124,745,749	17
193	65	128,059,822	19
194	27	135,274,596	13
195	25	137,137,793	6
196	102	142,048,216	15
197	257	143,181,832	15
198	232	144,232,980	28
199	177	148,663,307	7
200	44	155,298,815	7
201	160	156,082,500	30
202	109	163,036,576	7
203	149	163,494,651	26
204	82	165,624,964	17
205	144	170,840,390	18
206	7	173,415,798	9
207	242	179,628,219	29
208	195	182,766,423	20
209	112	184,288,849	19
210	221	186,059,008	15
211	3	202,157,572	6
212	192	210,812,717	21
213	110	216,120,285	13
214	95	219,223,842	9
215	39	227,133,200	19
216	141	234,240,839	14
217	143	235,532,709	14
218	159	239,737,552	27
219	2	241,470,617	10
220	153	244,991,457	20
221	206	248,566,380	16
222	226	254,121,267	26
223	175	267,003,262	21
224	212	294,573,317	25
225	196	299,376,262	23
226	207	302,313,596	8
227	223	303,930,143	32
228	97	307,544,521	13
229	113	326,782,705	13
230	129	328,810,633	13
231	163	337,645,719	6
232	150	355,975,318	6
233	115	388,346,117	13
234	116	408,575,647	13
235	46	429,568,372	7
236	66	430,091,931	13
237	117	432,859,750	13
238	205	433,573,963	18
239	170	452,587,145	9
240	81	454,563,758	7
241	114	459,617,877	13
242	67	461,704,853	16
243	51	476,863,622	4
244	233	515,945,074	5
245	249	519,046,161	5
246	93	533,161,986	8
247	135	533,736,354	7
248	96	535,932,449	4
249	55	551,801,748	7
250	101	555,152,362	5
251	6	607,350,915	7
252	73	617,039,057	5
253	231	627,736,136	10
254	100	631,070,960	5
255	111	641,501,137	6
256	138	736,914,373	9
257	190	744,383,266	22
258	122	788,351,471	4
259	4	1,124,639,280	4
260	53	1,158,986,282	5
261	74	1,426,001,288	7

Dataset 9

Order	ID	B_m	N
1	103	56,732	5
2	120	73,266	6
3	122	130,801	5
4	43	132,303	5
5	102	158,165	3
6	17	180,258	4
7	121	206,217	4
8	19	223,148	5
9	32	225,299	4
10	10	227,450	9
11	9	232,775	3
12	13	242,106	5
13	33	244,931	7
14	44	252,145	6
15	51	273,722	5
16	125	276,239	4
17	150	300,408	14
18	69	310,137	5
19	53	315,973	12
20	30	339,635	4
21	6	342,653	8
22	54	350,364	16
23	18	358,884	5
24	4	372,150	3
25	29	377,694	6
26	1	381,391	4
27	76	386,393	10
28	82	388,131	16
29	12	390,904	8
30	56	391,853	15
31	77	392,145	9
32	3	402,472	7
33	81	411,205	15
34	111	428,339	6
35	14	430,338	3
36	7	436,690	7
37	57	447,253	10
38	22	455,859	6
39	66	460,510	6
40	148	465,701	17
41	151	474,516	13
42	65	482,462	8
43	55	499,372	16
44	79	507,584	5
45	106	512,374	6
46	37	512,818	5
47	80	517,141	8
48	68	522,296	11
49	118	531,660	4
50	137	532,393	7
51	70	547,753	10
52	52	550,140	21
53	41	567,554	7
54	31	582,819	3
55	24	594,703	7
56	25	599,392	7
57	149	642,660	11
58	45	649,360	10
59	61	651,707	18
60	87	657,234	15
61	62	660,565	10
62	58	675,993	6
63	88	701,171	10
64	75	701,751	5
65	109	707,188	5
66	73	708,362	17
67	49	711,578	16
68	63	725,123	12
69	26	727,673	6
70	78	734,618	6
71	60	741,877	17
72	23	751,431	9
73	20	751,973	3
74	48	805,337	10
75	95	837,791	3
76	39	840,343	6
77	96	871,212	4
78	89	884,565	7
79	126	890,228	10
80	84	924,607	14
81	110	944,857	11
82	112	947,843	9
83	74	983,520	9
84	15	1,023,110	3
85	114	1,058,313	8
86	64	1,075,612	11
87	152	1,119,235	10
88	71	1,124,010	9
89	101	1,129,887	5
90	40	1,132,020	5
91	72	1,168,567	10
92	46	1,176,415	4
93	8	1,240,503	10
94	127	1,254,329	7
95	146	1,284,611	18
96	83	1,314,678	9
97	21	1,327,789	3
98	91	1,359,160	9
99	104	1,362,291	18
100	34	1,379,128	5
101	100	1,404,513	6
102	67	1,416,992	10
103	113	1,526,470	7
104	129	1,602,781	11
105	133	1,605,577	8
106	97	1,624,700	12
107	135	1,750,689	5
108	132	1,771,181	6
109	124	1,962,038	12
110	92	2,141,937	17
111	5	2,153,301	9
112	141	2,202,905	6

Order	ID	B_m	N
113	27	2,363,846	8
114	38	2,508,048	9
115	2	2,561,172	9
116	107	3,001,627	7
117	59	3,006,421	13
118	16	3,445,830	9
119	140	3,457,929	5
120	99	3,589,080	9
121	134	3,606,215	11
122	85	3,634,841	6
123	138	3,710,882	6
124	136	3,731,703	12
125	50	3,746,593	10
126	130	3,754,805	11
127	123	3,758,103	12
128	28	3,806,591	5
129	115	3,847,477	9
130	86	3,947,517	13
131	119	4,069,052	14
132	36	4,327,794	3
133	131	4,407,398	10
134	98	4,499,075	8
135	117	4,548,821	10
136	128	4,612,678	9
137	105	4,637,181	8
138	35	4,778,909	3
139	147	5,008,153	14
140	42	5,281,779	4
141	139	5,313,098	8
142	94	6,372,181	13
143	47	6,478,101	13
144	108	7,248,956	10
145	93	8,573,757	10
146	90	8,846,539	15
147	145	10,041,043	6
148	143	10,041,371	11
149	142	10,902,881	12
150	144	12,104,566	10
151	116	13,685,053	8
152	11	14,674,315	10

Dataset 10

Order	ID	B_m	N
1	154	8,587	9
2	161	8,919	8
3	122	9,582	5
4	158	9,586	18
5	156	11,159	4
6	130	11,903	11
7	47	11,981	4
8	160	11,989	15
9	68	12,108	3
10	157	12,257	18
11	79	12,917	3
12	159	14,873	13
13	127	24,736	7
14	126	27,070	8
15	46	27,290	6
16	124	27,357	7
17	52	28,229	4
18	31	29,644	4
19	93	31,418	3
20	4	31,602	4
21	34	32,298	3
22	38	33,132	3
23	39	33,132	3
24	53	34,075	4
25	128	34,621	6
26	32	34,785	5
27	33	35,838	8
28	153	36,136	9
29	1	36,406	5
30	75	37,130	7
31	119	38,501	5
32	155	39,087	6
33	112	39,092	5
34	129	39,623	9
35	25	42,790	5
36	91	44,749	7
37	26	48,035	4
38	28	48,104	7
39	37	51,107	5
40	23	51,411	7
41	3	59,141	4
42	49	60,871	6
43	80	62,909	10
44	70	63,753	5
45	35	67,006	5
46	97	67,851	5
47	77	69,098	5
48	125	69,972	7
49	50	74,176	6
50	78	76,067	8
51	41	79,709	4
52	62	80,054	7
53	43	80,104	2
54	76	80,332	5
55	139	81,413	6
56	29	82,727	4
57	82	84,454	14
58	51	87,037	10
59	27	87,929	13
60	131	89,418	6
61	30	94,961	10
62	94	96,562	4
63	134	98,450	6
64	150	102,114	3
65	132	102,160	6
66	152	102,360	4
67	108	102,996	11
68	14	103,907	4
69	12	104,142	6
70	7	105,308	6
71	137	106,113	5
72	36	107,088	7
73	2	108,485	5
74	110	108,806	6
75	123	111,326	8
76	81	112,051	3
77	84	112,104	5
78	135	112,547	6
79	48	112,868	4
80	24	113,926	10
81	141	114,969	5
82	144	116,525	4
83	87	119,033	5
84	66	120,156	8
85	17	122,317	5
86	111	123,475	9
87	69	126,872	8
88	136	127,880	6
89	90	128,207	5
90	42	131,595	5
91	45	131,750	2
92	54	134,590	10
93	121	136,802	3
94	44	137,428	5
95	145	138,328	9
96	18	140,514	5
97	73	144,143	5
98	148	144,165	6
99	116	151,571	4
100	22	155,400	4
101	15	162,267	4
102	13	162,467	6
103	40	164,516	13
104	21	166,160	8
105	151	167,233	5
106	64	169,798	7
107	72	170,011	6
108	106	173,237	6
109	10	175,457	3
110	16	180,415	7
111	149	184,394	4
112	20	187,599	6
113	101	195,421	6
114	138	198,183	6
115	117	202,766	3
116	133	206,866	8
117	95	208,540	3
118	104	211,821	4
119	5	220,093	5
120	114	229,196	4
121	8	236,497	6
122	83	244,161	3
123	113	245,606	2
124	86	272,561	7
125	140	274,551	6
126	120	285,500	5
127	65	296,019	11
128	74	313,076	6
129	56	314,483	9
130	118	343,941	11
131	92	346,422	5
132	11	355,518	7
133	146	359,106	10
134	59	363,022	3
135	103	374,287	3
136	60	377,712	7
137	67	380,539	4
138	85	381,766	5
139	96	382,789	5
140	99	389,007	5
141	19	421,600	4
142	115	449,603	8
143	88	455,400	5
144	63	459,292	9
145	142	474,916	7
146	61	475,413	5
147	100	508,394	3
148	57	515,450	8
149	89	525,365	6
150	58	531,609	6
151	147	538,566	7
152	71	547,322	8
153	109	611,131	8
154	107	732,683	7
155	98	734,439	4
156	55	924,217	16
157	6	1,005,086	6
158	143	1,237,836	8
159	105	1,775,726	6
160	102	1,857,215	6
161	9	3,260,798	8

Dataset 11

Order	ID	B_m	N
1	4	122,657	3
2	39	201,402	5
3	11	229,253	11
4	35	277,774	5
5	3	287,515	14
6	44	308,259	4
7	17	364,765	18
8	38	380,479	12
9	26	433,576	9
10	40	713,573	12
11	6	713,584	5
12	34	771,586	9
13	2	775,167	4
14	7	883,144	6
15	31	987,694	3
16	37	1,002,202	3
17	10	1,077,819	25
18	5	1,082,862	11
19	42	1,243,280	20
20	23	1,260,234	4
21	22	1,275,801	19
22	21	1,286,376	7
23	32	1,616,923	6
24	41	1,708,144	8
25	43	1,764,662	11
26	18	1,856,797	14
27	25	2,179,868	25
28	33	2,536,349	9
29	45	2,612,412	5
30	36	3,071,591	9
31	9	3,080,428	10
32	19	3,363,910	6
33	8	3,885,370	22
34	24	4,289,341	22
35	29	4,749,403	9
36	1	5,450,186	13
37	27	5,719,436	16
38	12	5,959,673	43
39	20	6,062,617	45
40	13	6,664,934	43
41	28	6,927,068	10
42	30	7,312,009	27
43	16	8,625,632	39
44	14	14,894,721	12
45	15	18,027,863	23

Dataset 12

Order	ID	B_m	N
1	15	1,079,144	24
2	28	1,488,330	21
3	79	1,537,471	22
4	31	1,612,499	24
5	107	1,653,353	26
6	37	2,103,391	19
7	4	2,499,595	12
8	41	2,626,856	24
9	17	2,716,395	36
10	26	2,960,181	22
11	97	3,017,126	36
12	108	3,063,002	33
13	47	3,086,445	20
14	114	3,195,185	48
15	109	3,559,762	34
16	16	3,682,856	45
17	24	4,048,593	34
18	94	4,198,634	45
19	93	5,517,449	49

Order	ID	B_m	N
20	5	5,519,531	16
21	112	5,582,020	22
22	22	5,705,802	41
23	100	6,534,653	16
24	113	6,552,800	37
25	105	7,001,062	16
26	72	7,054,504	65
27	95	8,389,273	40
28	6	8,516,973	21
29	101	8,852,925	15
30	85	9,802,996	47
31	38	10,495,291	50
32	106	10,615,254	16
33	46	10,816,891	39
34	49	11,177,786	47
35	40	11,488,513	49
36	104	12,315,545	63
37	84	14,132,357	44
38	103	14,227,093	60
39	64	14,341,530	23
40	110	16,251,125	33
41	99	16,860,295	10
42	50	17,588,667	49
43	111	17,746,139	25
44	96	18,619,007	12
45	34	19,595,731	42
46	27	20,153,397	41
47	59	20,561,970	59
48	98	20,563,467	11
49	3	21,353,568	30
50	10	21,708,000	33
51	63	22,143,465	23
52	102	22,318,154	55
53	51	26,479,141	31
54	1	27,040,451	45
55	55	27,910,450	52
56	39	28,331,375	45
57	61	29,000,444	41
58	25	30,249,575	41
59	2	30,551,542	45
60	30	30,770,871	30
61	12	31,590,763	36
62	14	31,590,763	36
63	69	33,210,700	39
64	7	34,764,003	34
65	45	35,455,363	41
66	29	35,987,767	41
67	71	39,283,078	30
68	32	41,228,951	44
69	65	41,597,001	40
70	8	42,025,452	38
71	33	42,092,509	46
72	70	43,561,070	31
73	54	45,077,325	37
74	11	45,260,730	37
75	43	45,925,278	41
76	35	49,140,339	36
77	60	49,876,103	37
78	56	51,567,774	44
79	9	51,730,833	33
80	13	53,330,879	27
81	73	56,420,628	33
82	66	59,361,774	37
83	75	59,997,000	29
84	20	60,296,637	19
85	42	62,189,663	35
86	21	64,914,393	27
87	76	66,094,206	29
88	48	69,184,189	39
89	44	70,540,511	36
90	57	71,468,346	33
91	62	71,756,192	26
92	19	74,858,840	19
93	23	74,858,840	19
94	77	75,413,556	24
95	52	75,417,028	33
96	90	78,509,325	24
97	91	79,529,913	23
98	67	82,812,206	30
99	86	83,070,638	17
100	74	83,210,900	26
101	78	84,037,235	24
102	87	87,245,462	17
103	89	88,574,213	21
104	92	88,614,685	21
105	83	89,355,119	20
106	80	91,994,522	19
107	53	93,425,682	26
108	81	95,744,520	19
109	88	99,982,162	22
110	36	101,166,388	18
111	18	102,559,740	19
112	82	112,041,174	20
113	68	126,157,555	21
114	58	158,509,777	18