

Variational data assimilation for morphodynamic model parameter estimation

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Smith, P. J., Dance, S. L. ORCID: <https://orcid.org/0000-0003-1690-3338>, Baines, M. J., Nichols, N. K. ORCID: <https://orcid.org/0000-0003-1133-5220> and Scott, T. R. (2009) Variational data assimilation for morphodynamic model parameter estimation. In: The 8th International Workshop on Adjoint Model Applications in Dynamic Meteorology, May 2009, Tannersville, PA, USA. Available at <https://centaur.reading.ac.uk/1706/>

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Variational data assimilation for morphodynamic model parameter estimation

Department of Mathematics, University of Reading:

Polly Smith*, Sarah Dance, Mike Baines, Nancy Nichols,

Environmental Systems Science Centre, University of Reading:

Tania Scott

*email: p.j.smith@reading.ac.uk

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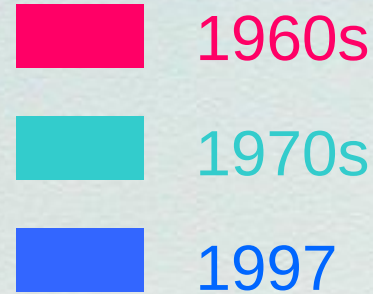
Outline

- Background/ motivation
 - what is morphodynamic modelling?
 - why do we need morphodynamic models?
- A simple 1D morphodynamic model
- Data assimilation and parameter estimation
 - how can we use data assimilation to estimate uncertain model parameters?
 - how do we model the background error covariances?
- Results
- Summary

Terminology

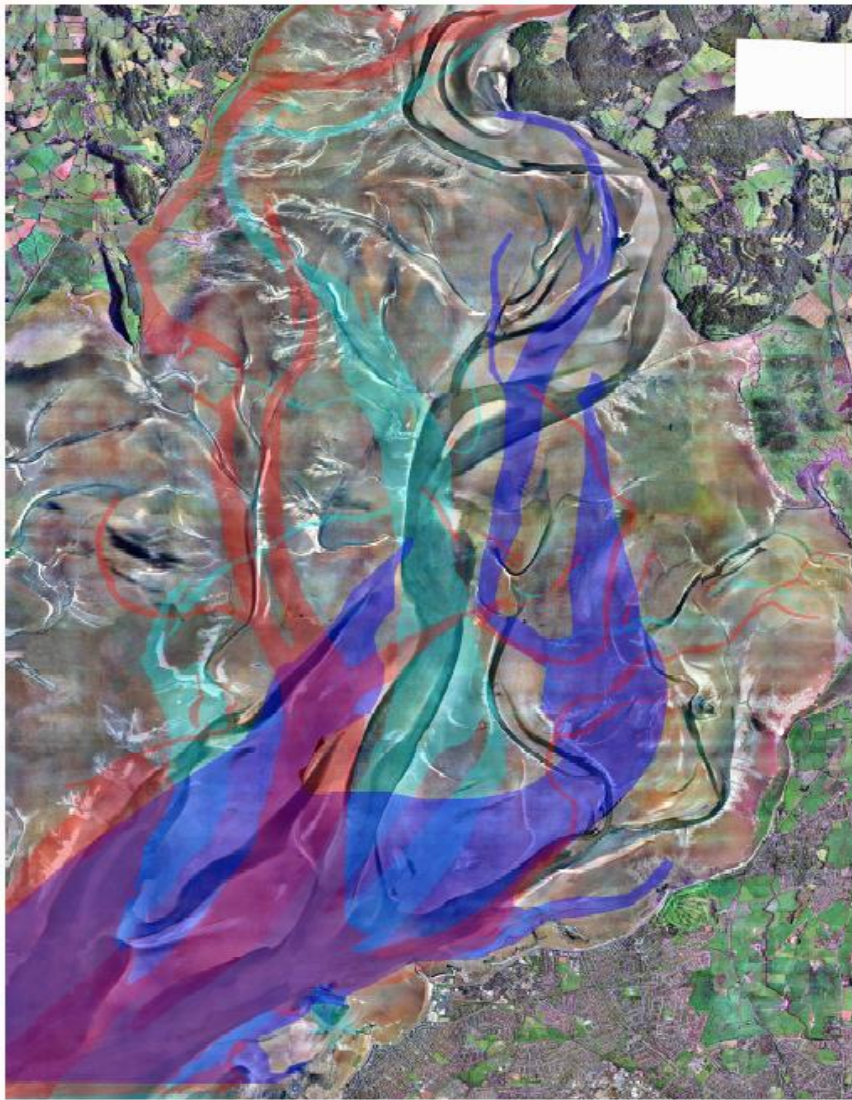
- Bathymetry - the underwater equivalent to topography
 - coastal bathymetry is dynamic and evolves with time
 - water action erodes, transports, and deposits sediment, which changes the bathymetry, which alters the water action, and so on
- Morphodynamics - the study of the evolution of the bathymetry in response to the flow induced sediment transport
- Morphodynamic prediction
 - why?
 - how?

Kent channel



18km

- Channel movement
 - impacts on habitats in the bay
 - affects access to ports
 - has implications for flooding during storm events



Picture courtesy of Nigel Cross,
Lancaster City Council

Morphodynamic modelling

- Operational coastal flood forecasting is limited near-shore by lack of knowledge of evolving bathymetry
 - but it is impractical to continually monitor large coastal areas
- Modelling is difficult
 - longer term changes are driven by shorter term processes
 - uncertainty in initial conditions and parameters
- An alternative approach is to use data assimilation



Parameter estimation

- Model equations depend on parameters
 - exact values are unknown
 - inaccurate parameter values can lead to growth of model error
 - affects predictive ability of the model
- How do we estimate these values *a priori*?
 - theoretical values
 - calibration

or ...

- data assimilation
 - choose parameters based on observations
 - state augmentation: model parameters are estimated alongside the model state

Simple 1D model

Based on the sediment conservation equation

$$\frac{\partial z}{\partial t} = - \left(\frac{1}{1 - \varepsilon} \right) \frac{\partial q}{\partial x}$$

where $z(x,t)$ is the bathymetry, t is time, q is the sediment transport rate in the x direction and ε is the sediment porosity.

For the sediment transport rate we use the power law

$$q = Au^n$$

where $u(x,t)$ is the depth averaged current and **A and n** are parameters whose values need to be set

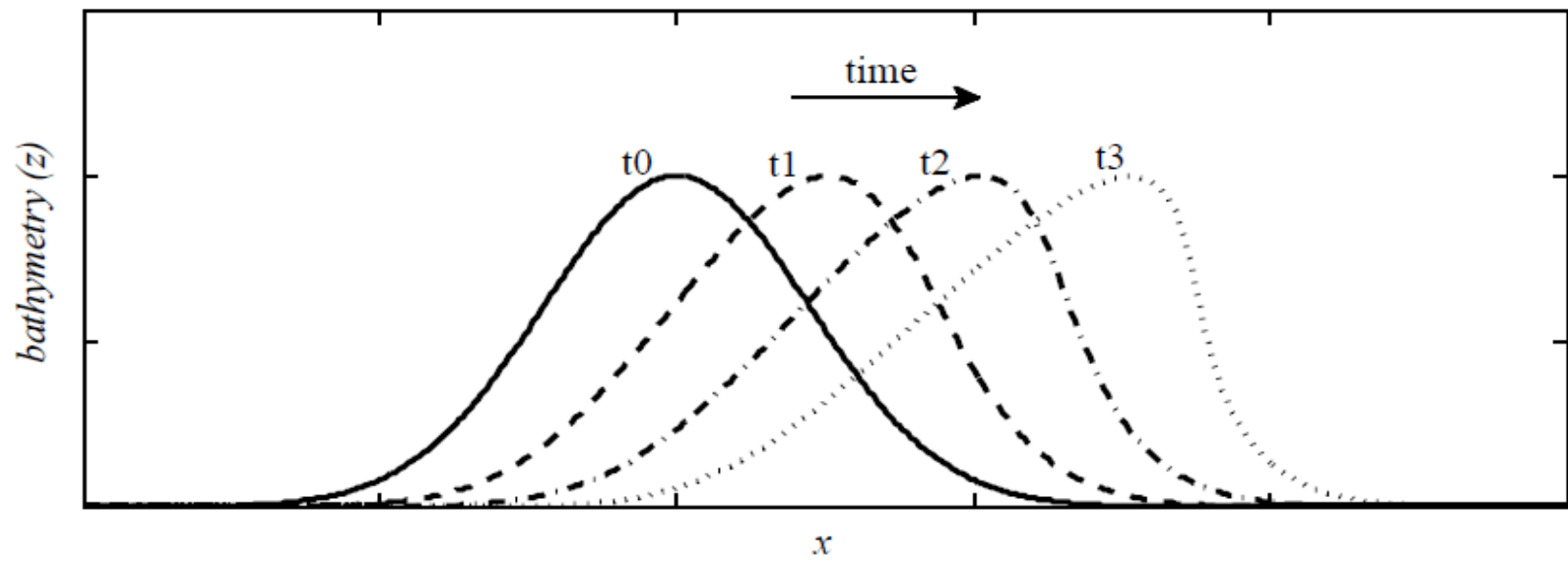
If we assume that water flux (F) and height (H) are constant

$$F = u(H - z)$$

we can rewrite the sediment conservation equation as

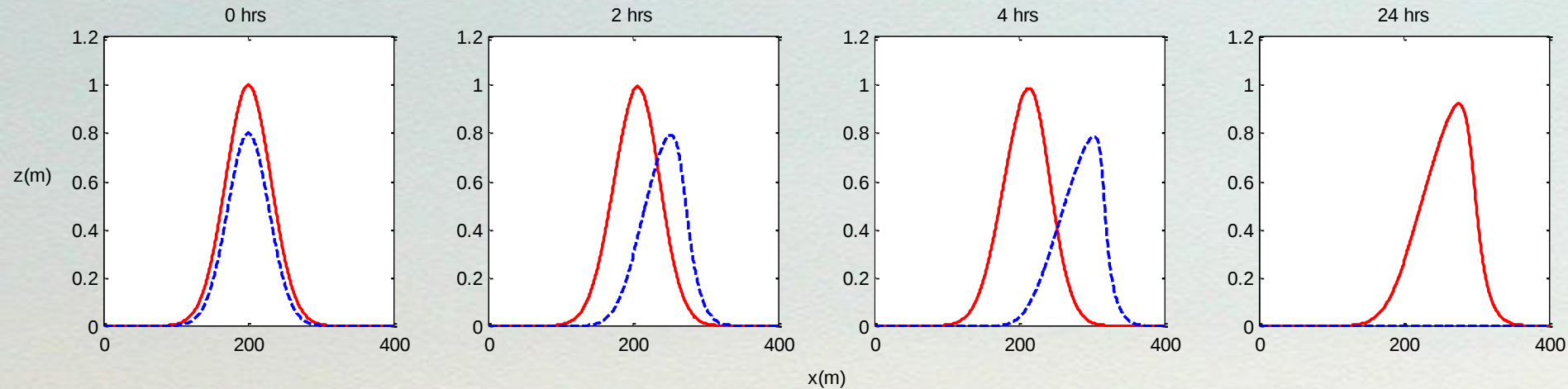
$$\frac{\partial z}{\partial t} + a(z, H, F, \varepsilon, A, n) \frac{\partial z}{\partial x} = 0$$

where $a(z, H, F, \varepsilon, A, n)$ is the advection velocity or bed celerity.



Can we use data assimilation to estimate the parameters A and n ?

model run with incorrect parameters & without data assimilation



red line = correct parameters

blue line = incorrect parameters (A over estimated, n under estimated)

State augmentation

- Dynamical system model

$$\mathbf{z}_{k+1} = \mathbf{f}(\mathbf{z}_k, \mathbf{p}_k)$$

(discrete, non-linear, time invariant)

- Parameter evolution

$$\mathbf{p}_{k+1} = \mathbf{p}_k.$$

- Augmented system model

$$\mathbf{w}_{k+1} = \begin{pmatrix} \mathbf{z}_{k+1} \\ \mathbf{p}_{k+1} \end{pmatrix} = \begin{pmatrix} \mathbf{f}(\mathbf{z}_k, \mathbf{p}_k) \\ \mathbf{p}_k \end{pmatrix} = \tilde{\mathbf{f}}(\mathbf{w}_k)$$

- Observations

$$\mathbf{y}_k = \mathbf{h}(\mathbf{z}_k)$$

in terms of the augmented system ...

$$\mathbf{y}_k = \tilde{\mathbf{h}}(\mathbf{w}_k)$$

where

$$\tilde{\mathbf{h}}(\mathbf{w}) = \tilde{\mathbf{h}} \begin{pmatrix} \mathbf{z} \\ \mathbf{p} \end{pmatrix} = \mathbf{h}(\mathbf{z}).$$

3D Var

Cost function:

$$\tilde{J}(\mathbf{w}) = (\mathbf{w} - \mathbf{w}^b)^T \tilde{\mathbf{B}}^{-1} (\mathbf{w} - \mathbf{w}^b) + (\mathbf{y} - \tilde{\mathbf{h}}(\mathbf{w}))^T \mathbf{R}^{-1} (\mathbf{y} - \tilde{\mathbf{h}}(\mathbf{w}))$$

$\tilde{\mathbf{B}}$ and $\mathbf{R}$ are the covariance matrices of the background and observation errors.

$$\tilde{\mathbf{B}} = \begin{pmatrix} \mathbf{B}_{zz} & \mathbf{B}_{zp} \\ (\mathbf{B}_{zp})^T & \mathbf{B}_{pp} \end{pmatrix}.$$

$\mathbf{B}_{zz}$ state background error covariance

$\mathbf{B}_{pp}$ parameter background error covariance

$\mathbf{B}_{zp}$ state parameter error cross covariance

augmented gain matrix:

$$\begin{aligned}\tilde{\mathbf{K}} &= \tilde{\mathbf{B}}\tilde{\mathbf{H}}^T \left[\tilde{\mathbf{H}}\tilde{\mathbf{B}}\tilde{\mathbf{H}}^T + \mathbf{R} \right]^{-1} \\ &= \begin{pmatrix} \mathbf{B}_{zz}\mathbf{H}^T \\ \mathbf{B}_{zp}^T\mathbf{H}^T \end{pmatrix} [\mathbf{H}\mathbf{B}_{zz}\mathbf{H}^T + \mathbf{R}]^{-1} \\ &\stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{K}_z \\ \mathbf{K}_p \end{pmatrix}\end{aligned}$$

state & parameter updates:

$$\begin{aligned}\mathbf{z}^a &= \mathbf{z}^b + \mathbf{K}_z(\mathbf{y} - \mathbf{h}(\mathbf{z}^b)) \\ \mathbf{p}^a &= \mathbf{p}^b + \mathbf{K}_p(\mathbf{y} - \mathbf{h}(\mathbf{z}^b))\end{aligned}$$

State-parameter cross covariances

The Extended Kalman filter (EKF)

State forecast:

$$\mathbf{w}_{k+1}^f = \tilde{\mathbf{f}}_k(\mathbf{w}_k^a)$$

Error covariance forecast:

$$\mathbf{P}_{k+1}^f = \mathbf{F}_k \mathbf{P}_k^a \mathbf{F}_k^T$$

where

$$\mathbf{F}_k = \left. \frac{\partial \tilde{\mathbf{f}}}{\partial \mathbf{w}} \right|_{\mathbf{w}_k^a} = \left(\begin{array}{cc} \frac{\partial \mathbf{f}(\mathbf{z}, \mathbf{p})}{\partial \mathbf{z}} & \frac{\partial \mathbf{f}(\mathbf{z}, \mathbf{p})}{\partial \mathbf{p}} \\ \mathbf{0} & \mathbf{I} \end{array} \right) \bigg|_{\mathbf{z}_k^a, \mathbf{p}_k^a} = \left(\begin{array}{cc} \mathbf{M}_k & \mathbf{N}_k \\ \mathbf{0} & \mathbf{I} \end{array} \right)$$

Error covariance forecast:

$$\mathbf{P}_{k+1}^f = \begin{pmatrix} \mathbf{M}_k \mathbf{P}_{\mathbf{z}\mathbf{z}_k}^a \mathbf{M}_k^T + \mathbf{N}_k \mathbf{P}_{\mathbf{p}\mathbf{p}_k}^a \mathbf{N}_k^T & \mathbf{N}_k \mathbf{P}_{\mathbf{p}\mathbf{p}_k}^a \\ \mathbf{P}_{\mathbf{p}\mathbf{p}_k}^a \mathbf{N}_k^T & \mathbf{P}_{\mathbf{p}\mathbf{p}_k}^a \end{pmatrix}$$

a new hybrid approach ...

$$\tilde{\mathbf{B}}_k = \begin{pmatrix} \mathbf{B}_{\mathbf{z}\mathbf{z}} & \mathbf{N}_k \mathbf{B}_{\mathbf{p}\mathbf{p}} \\ \mathbf{B}_{\mathbf{p}\mathbf{p}} \mathbf{N}_k^T & \mathbf{B}_{\mathbf{p}\mathbf{p}} \end{pmatrix}$$

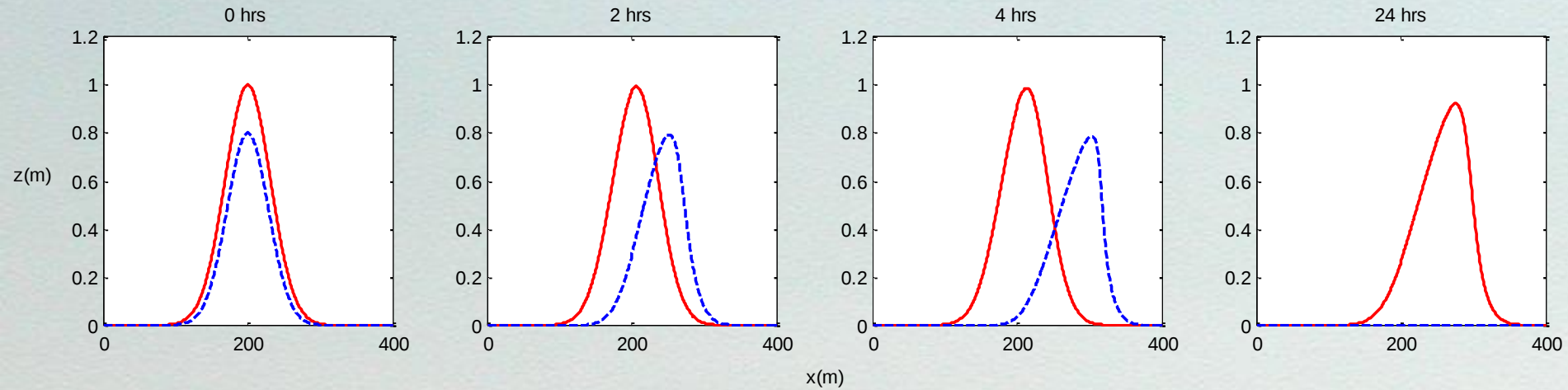
for our simple 2 parameter model

$$\begin{aligned}\mathbf{B}_{\mathbf{z}\mathbf{p}_k} &= \mathbf{N}_k \mathbf{B}_{\mathbf{p}\mathbf{p}} \\ &= \begin{pmatrix} \frac{\partial \mathbf{f}_k}{\partial A} & \frac{\partial \mathbf{f}_k}{\partial n} \end{pmatrix} \begin{pmatrix} \sigma_A^2 & \sigma_{An} \\ \sigma_{An} & \sigma_n^2 \end{pmatrix} \\ &= \begin{pmatrix} \sigma_A^2 \frac{\partial \mathbf{f}_k}{\partial A} + \sigma_{An} \frac{\partial \mathbf{f}_k}{\partial n} & \sigma_n^2 \frac{\partial \mathbf{f}_k}{\partial n} + \sigma_{An} \frac{\partial \mathbf{f}_k}{\partial A} \end{pmatrix}\end{aligned}$$

Model setup

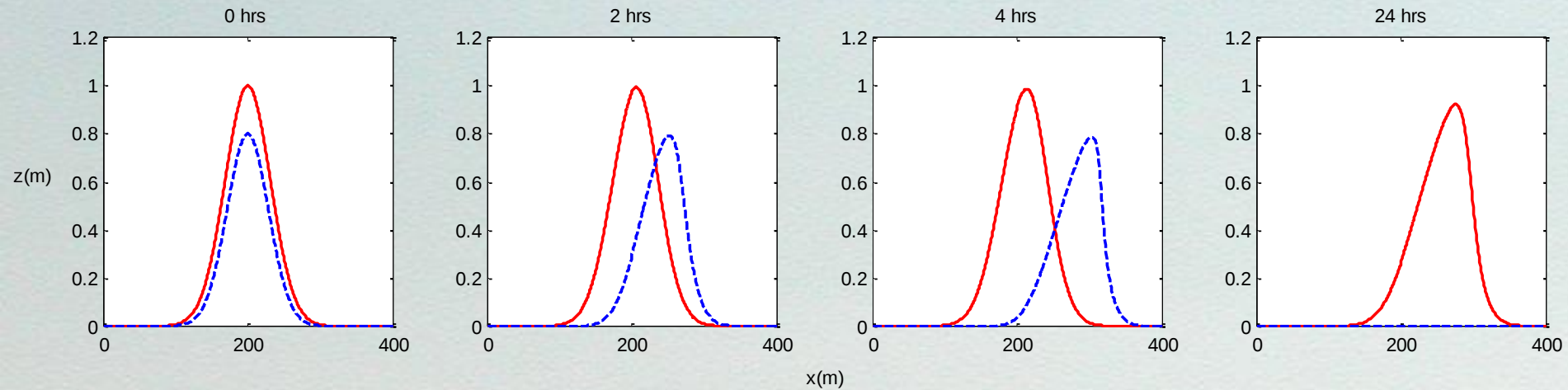
- Assume perfect model and observations
- Identical twin experiments
 - reference solution generated using Gaussian initial data and parameter values $A = 0.002 \text{ ms}^{-1}$ and $n = 3.4$
- Use incorrect model inputs
 - inaccurate initial bathymetry
 - inaccurate parameter estimates
- 3D Var algorithm is applied sequentially
 - observations taken at fixed grid points & assimilated every hour
 - the cost function is minimized iteratively using a quasi-Newton descent algorithm
- Covariances
 - $\mathbf{B}_{zz}$ fixed
 - $\mathbf{B}_{zp}$ time varying

without data assimilation

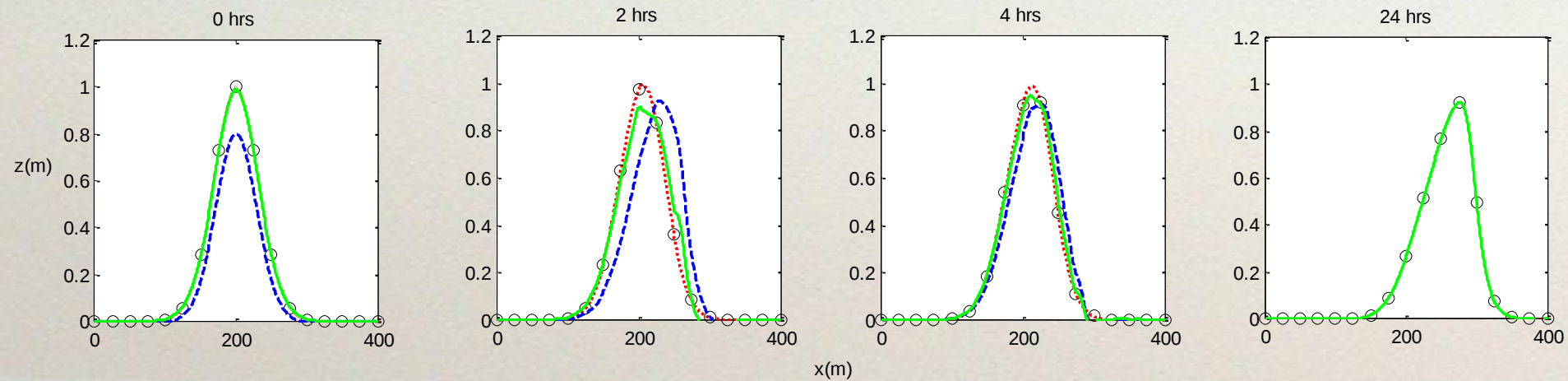


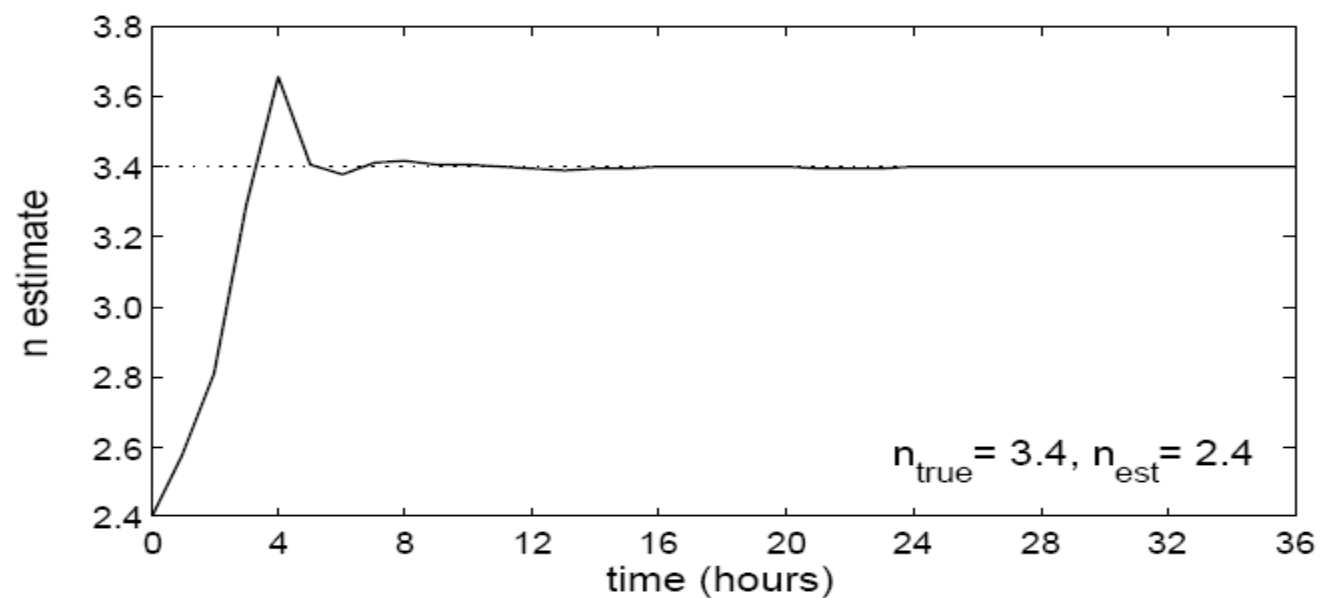
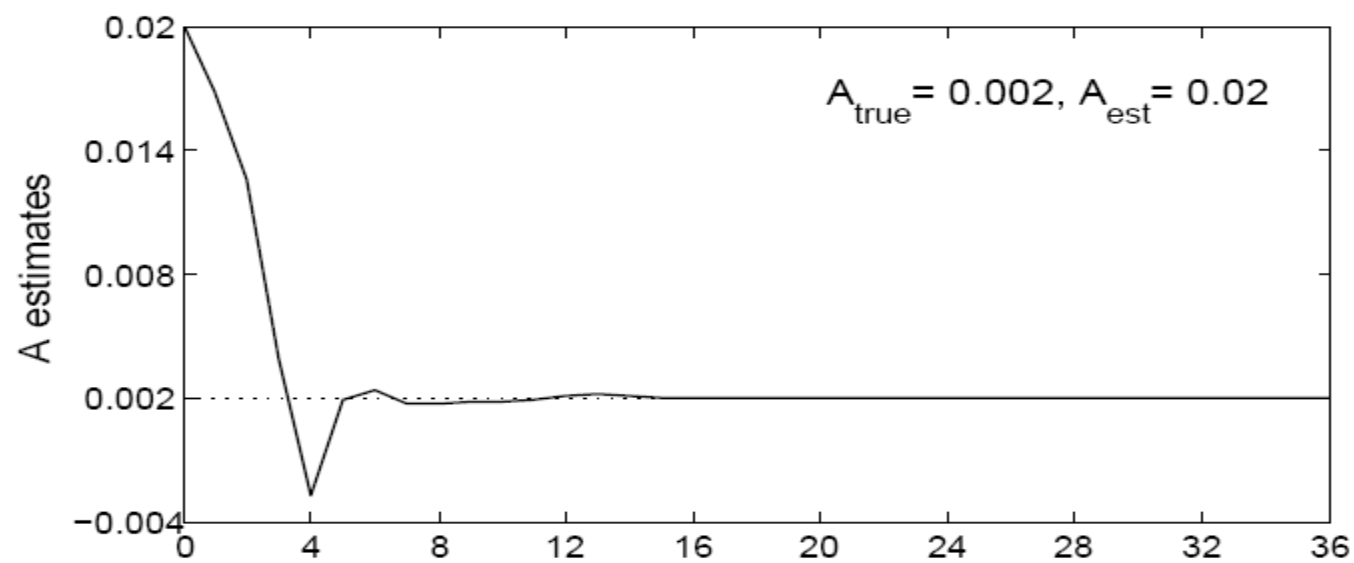
with data assimilation ...

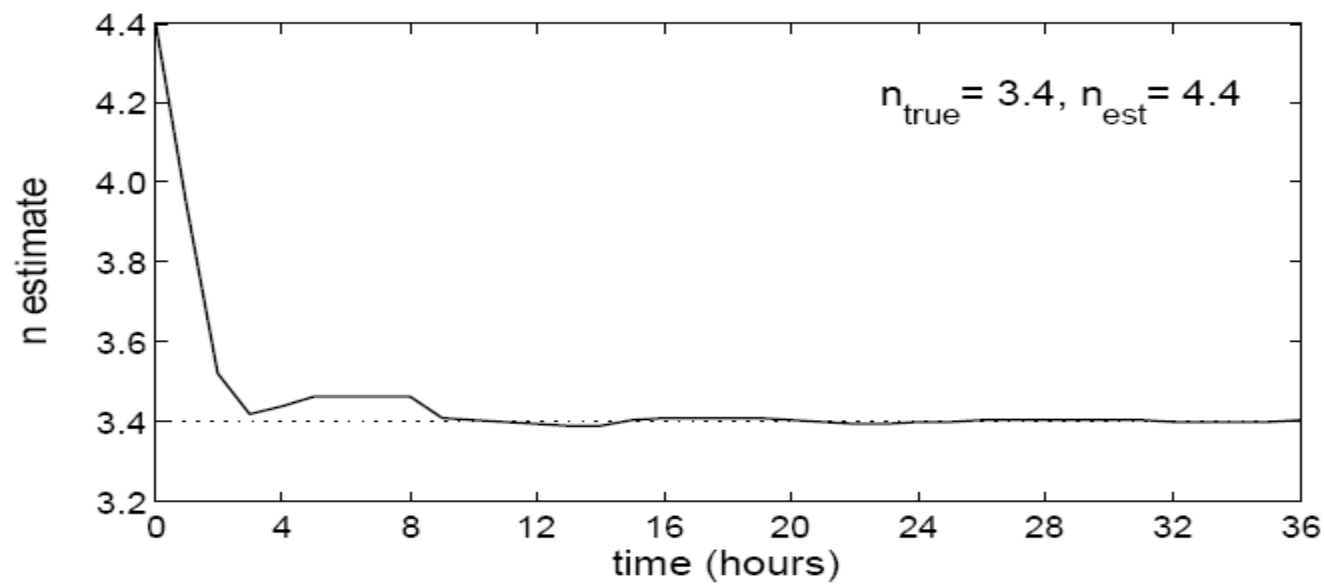
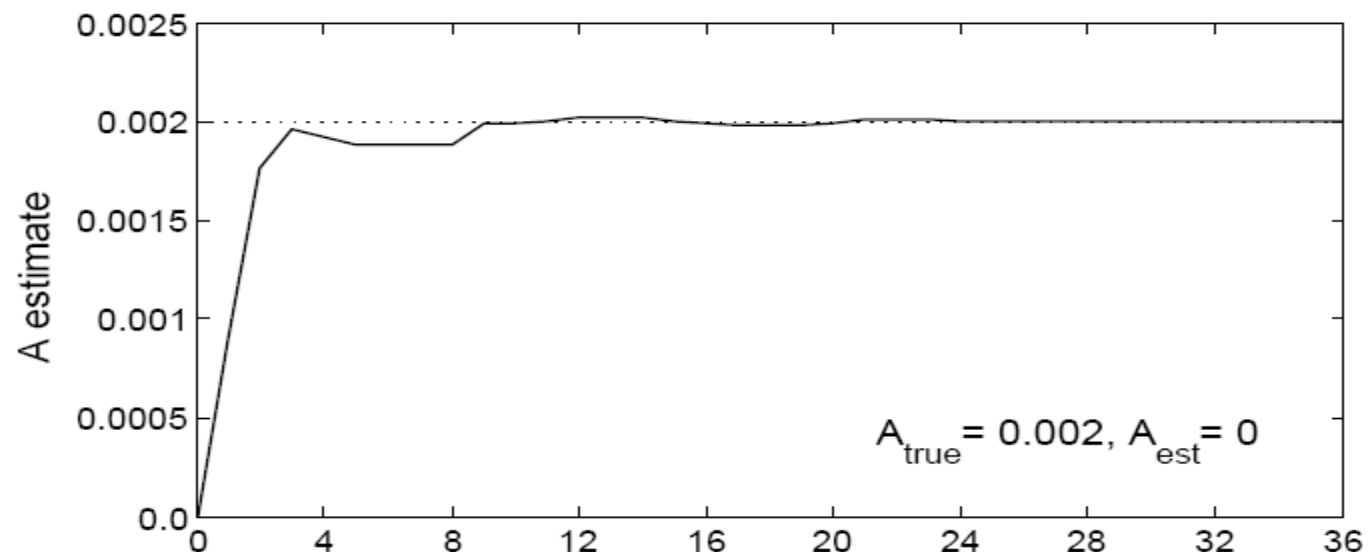
without data assimilation

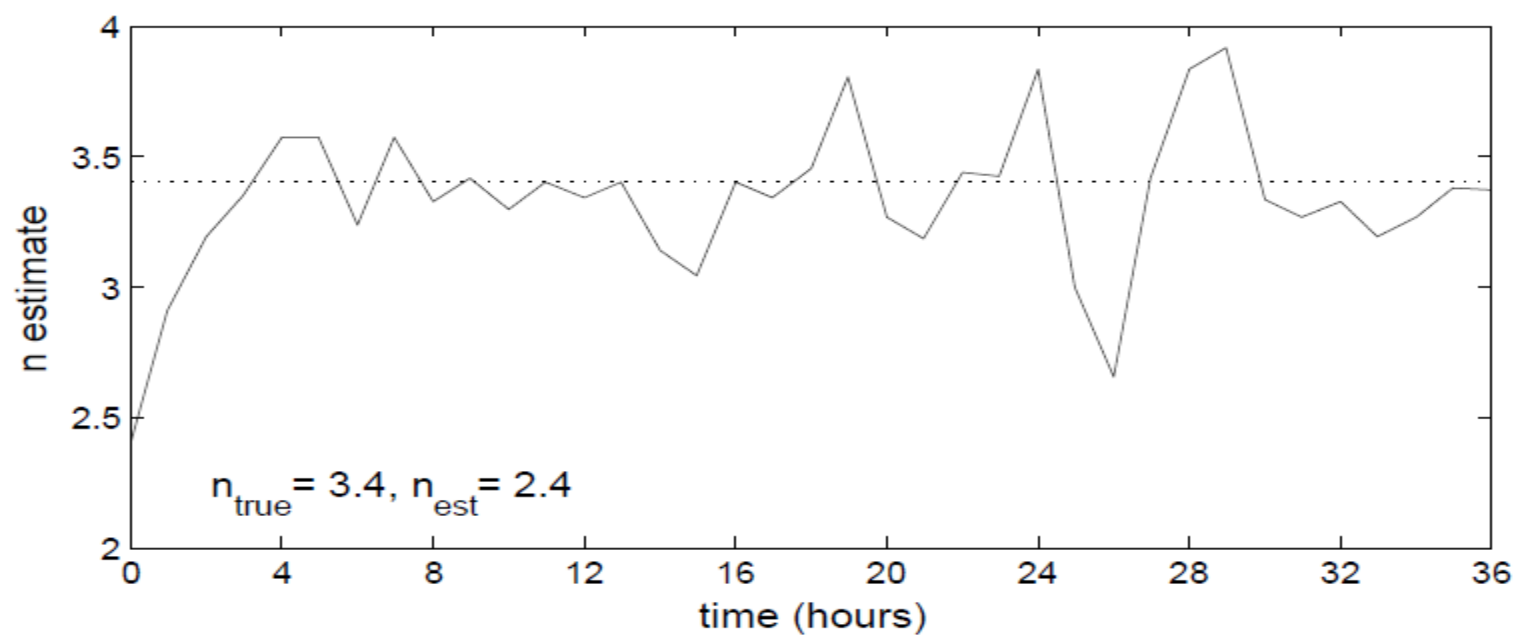
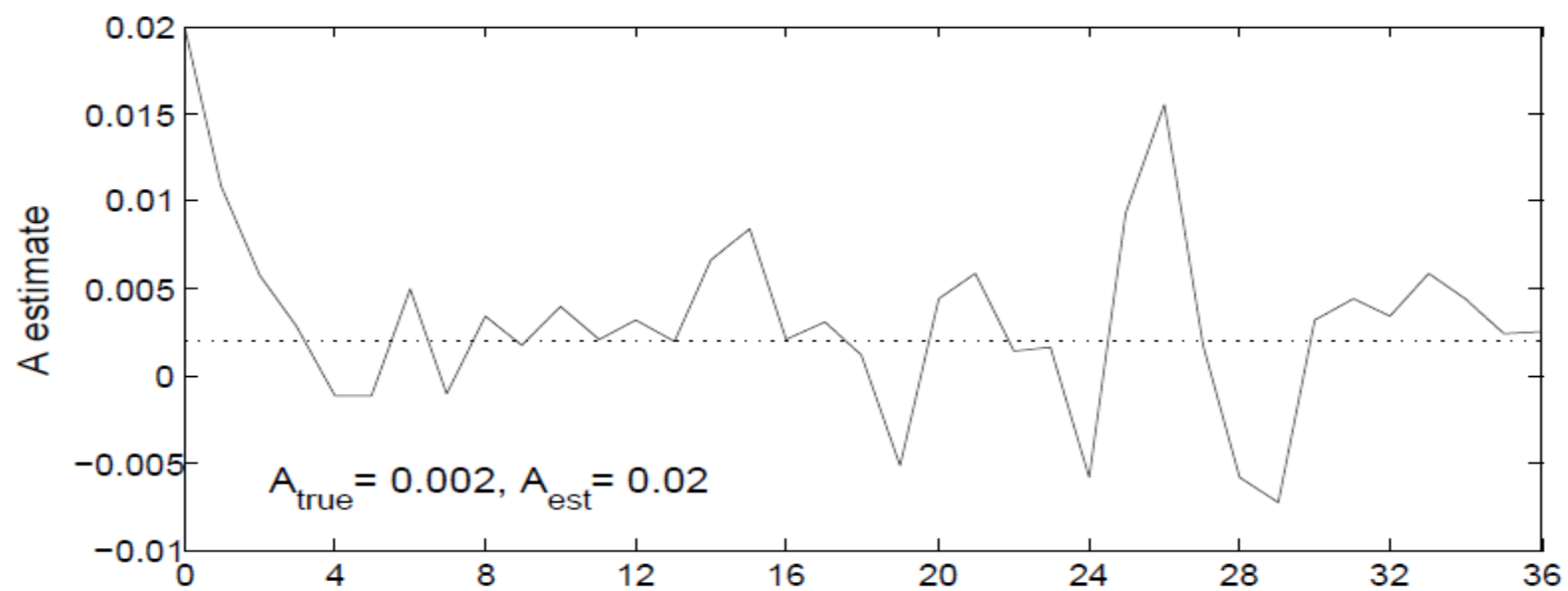


with data assimilation ...









Summary

- Presented a novel approach to model parameter estimation using data assimilation
 - demonstrated the technique using a simple morphodynamic model
- Results are very encouraging
 - scheme is capable of recovering near-perfect parameter values
 - improves model performance
- What next ...?
 - can our scheme be successfully applied to more complex models?
 - can we say anything about the convergence of the system?

Questions?



Simple Models of Changing Bathymetry with Data Assimilation

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Department of Mathematics, University of Reading

Numerical Analysis Report 10/2007*

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Mathematics Report 2/2008*

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