“How to prevent the risk of business failure: financial characteristics of medium and large-sized distressed enterprises in Italy”

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Henley Business School

Alessandra Moscone

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Declaration: I confirm that this is my own work and the use of all material from other sources has been properly and fully acknowledged.

Alessandra Moscone

ABSTRACT

This study develops a predictive model to identify financial distress in large and medium-sized firms in Italy, focusing on seven key sectors classified under the Standard Industrial Classification (SIC): construction, manufacturing, transportation-communications-electric-gas-sanitary services, wholesale trade, retail trade, finance-insurance-real estate, and services. The model builds on the widely used Z”-Score model for bankruptcy prediction but modifies it by replacing the working capital to total assets ratio with the cash and cash equivalents to current liabilities ratio. This modification offers a more accurate reflection of liquidity pressures, particularly relevant for early-stage financial distress.

Using Linear Discriminant Analysis (LDA) and logistic regression, the study classifies firms into distressed and non-distressed categories and estimates their likelihood of resorting to Troubled Debt Restructuring (TDR). The dataset spans multiple sectors, allowing for a sector-specific analysis of financial distress indicators. The results highlight that liquidity, leverage, and profitability ratios are critical in predicting early financial distress across industries, with distressed firms exhibiting lower liquidity and higher leverage compared to their non-distressed counterparts.

While the study focuses on Italy, the model could be applied in other EU countries, such as Spain and France, which have similar TDR laws, extending its relevance across different national contexts. The findings provide valuable tools for corporate management, creditors, and investors in assessing financial risks.

However, the research is limited by the small sample size, which may restrict its generalizability. Future studies could expand the application of this predictive model to a larger sample and test its effectiveness across all EU member states, enhancing its accuracy and practical applicability in a wider European context.

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CHAPTER 1

INTRODUCTION

1. *MODERNIZING EU INSOLVENCY FRAMEWORKS*

The *European Council's Preventive Restructuring Directive (EU/2019/1023)***,** adopted on June 20, 2019, marks a pivotal development in the European Union's approach to financial distress and insolvency. This directive aims to harmonize restructuring frameworks across EU member states by offering financially distressed firms the tools to restructure and recover before reaching insolvency. By emphasizing early intervention and facilitating out-of-court settlements, the directive represents a shift in focus from traditional reactive measures, which often come too late to prevent costly bankruptcies, to proactive mechanisms designed to preserve value for both businesses and their stakeholders.

This preventive approach has reignited academic and policy discussions on the development of *financial distress prediction models***,** tools that can identify firms at risk of financial collapse well before formal insolvency is necessary. These models are increasingly seen as critical for enabling businesses to access restructuring options in a timely manner. The ability to predict financial instability accurately helps not only the firms but also their creditors, investors, and broader economic ecosystems by providing early warning signals. In turn, this facilitates the strategic interventions required to stabilize firms and avoid the far-reaching negative consequences of insolvency, such as mass job losses, economic disruption, and social instability.

In response to the directive, scholars have explored ways to refine existing predictive models, incorporating a broader array of *financial* and *non-financial indicators*. Early models, such as *Altman's Z-Score (1968)* and *Ohlson's O-Score (1980)***,** relied primarily on linear relationships between financial ratios like *liquidity***,** *leverage***,** and *profitability* to predict distress. However, the complexity of financial distress has become more apparent in recent years, particularly during periods of global crises like the *2007-2009 Global Financial Crisis* and the *COVID-19 pandemic***.** These events underscored the need for more sophisticated models that account for non-linear interactions**,** external shocks, and industry-specific risks**.**

The directive also underscores the EU's commitment to fostering a more resilient financial system. Member states have different legal and economic contexts, which makes developing universal predictive models challenging, particularly in cross-border insolvency cases. Still, harmonizing restructuring processes across the EU helps create a more stable and predictable environment for businesses operating across multiple jurisdictions, enhancing overall market stability.

The role of creditors and financial institutions is crucial in this framework. With access to advanced predictive models, creditors can better assess a firm's likelihood of distress and act swiftly to restructure debt or renegotiate terms, thereby mitigating potential losses. Without reliable predictive tools, the window for taking such actions narrows, increasing the risk of late-stage insolvency and the broader economic fallout that often follows.

The *Global Financial Crisis (2007-2009)* provides a stark reminder of the economic and social costs of delayed interventions. As noted by *De Luca F. and Meschieri E. (2017)***,** the crisis severely impacted firms globally, especially in industries that depended heavily on debt financing. The collapse of major financial institutions and liquidity shortages led to widespread bankruptcies, highlighting the critical importance of *early warning systems* in preventing similar future crises. The *COVID-19 pandemic* has further demonstrated how quickly firms, even those with strong financial foundations, can face financial strain when confronted with unforeseen economic shocks. As *Hassan M.K. et al. (2022)* point out, the pandemic's economic fallout has been widespread, and traditional models based solely on historical financial data have often proven inadequate in predicting the scale of the disruption.

Both the *Global Financial Crisis* and the *COVID-19 pandemic* illustrate the need for governments and regulatory bodies to take an active role in preventing financial distress. The EU has implemented several initiatives aimed at reinforcing financial stability, including the creation of the *European Stability Mechanism (ESM)* in 2012, which provides financial assistance to member states facing severe economic distress. Additionally, regulatory reforms such as *Basel III* and the *Single Supervisory Mechanism (SSM)* have sought to strengthen the resilience of the financial sector by ensuring that banks maintain adequate capital reserves and are subject to centralized oversight.

Further, the EU has prioritized sustainable economic growth as a way to prevent future financial crises. Programs such as the *European Green Deal* are designed to encourage investment in green technologies, fostering a low-carbon economy that can withstand external shocks like those caused by financial or environmental crises. By focusing on long-term sustainability, the EU aims to build an economy that not only recovers from shocks but emerges stronger and more resilient.

The *Preventive Restructuring Directive* supports this broader vision by empowering businesses with the legal tools to restructure their operations and finances before insolvency becomes inevitable. Early detection of financial distress is critical in this context, especially for large multinational firms that operate across borders and face complex financial environments. The directive's focus on early intervention aligns with the EU's commitment to ensuring a stable and competitive internal market, in which firms can manage financial difficulties proactively, reducing the need for costly insolvency proceedings and safeguarding the broader economy.

Since 2016, the European Commission has made significant strides toward reforming insolvency frameworks across the European Union (EU) by issuing various documents and guidelines aimed at facilitating more efficient and streamlined insolvency procedures. This momentum culminated in the adoption of the *"Preventive Restructuring" Directive (EU/2019/1023)* by the European Council on June 20, 2019, a directive that represents a major shift in how financial distress and insolvency are handled within the EU's internal market. The directive is seen as a critical step toward ensuring the proper functioning of the internal market, particularly in terms of offering financially distressed businesses an opportunity to restructure their debts and avoid formal insolvency. As noted by *Arias Varona F. J. et al. (2020),* the directive is designed to harmonize and modernize restructuring frameworks across member states, thus supporting the EU's broader objective of economic stability and growth.

At its core, the directive addresses several key areas of preventive restructuring, aimed at providing struggling firms with tools to manage their financial difficulties before they are forced into formal insolvency proceedings. This preventive approach reflects a shift in the EU's insolvency strategy, moving away from reactive solutions (which typically occur once firms are already in significant financial distress) toward proactive interventions. By encouraging earlier engagement with restructuring options, the directive seeks to minimize the costs and negative economic impacts typically associated with insolvency, including job losses, business closures, and broader market disruptions.

1.1. Key Aspects of the Directive

One of the central elements of the directive is the creation of preventive restructuring frameworks, which provide businesses with the opportunity to restructure their debts and liabilities at an early stage of financial difficulty, often before the firm's situation becomes irrecoverable. These frameworks are designed to give debtors the legal means to negotiate with creditors and other stakeholders, with the aim of reaching an agreement that allows the business to continue operating. This is particularly important because the success of many businesses hinges on their ability to maintain liquidity and avoid costly court-driven insolvency processes. The directive mandates that these restructuring frameworks be accessible without requiring the business to formally declare insolvency, which helps preserve the firm's reputation and avoids the stigma often associated with insolvency proceedings.

Additionally, the directive introduces provisions that facilitate the discharge of debt and disqualifications, particularly for entrepreneurs and small business owners. Under the new framework, entrepreneurs who have experienced financial failure due to no fault of their own are provided with the opportunity to obtain a fresh start after a defined period, typically no longer than three years. This provision is crucial because it aligns with the EU's broader goal of promoting entrepreneurial activity and innovation, recognizing that business failure is often part of the entrepreneurial process. By allowing a quicker discharge of debt, the directive removes one of the main barriers preventing failed entrepreneurs from re-entering the market and pursuing new business ventures, thereby supporting economic recovery and growth.

The directive also emphasizes the efficiency of restructuring and insolvency procedures, aiming to reduce the length and complexity of these processes across member states. In many jurisdictions, insolvency procedures have traditionally been slow, bureaucratic, and costly, often resulting in the liquidation of businesses that might have been saved through timely intervention. By setting minimum standards for member states regarding how restructuring and insolvency processes should be conducted, the directive seeks to enhance the overall efficiency of these procedures. This includes improving the role of courts and insolvency practitioners, who are often central to the success or failure of restructuring efforts. The directive encourages the use of specialized insolvency practitioners and judges who have the expertise needed to handle complex financial restructurings, ensuring that decisions are made quickly and effectively.

Moreover, the directive introduces *early warning tools* that are designed to help businesses identify the first signs of financial trouble. These tools provide debtors with timely information about their financial situation, allowing them to act before it is too late. Early warning systems might include monitoring financial ratios, cash flow data, or external indicators such as changes in market conditions. By identifying potential problems early, businesses can take steps to restructure their operations or negotiate with creditors well before formal insolvency proceedings become necessary. These early warning mechanisms are particularly important in volatile industries where sudden shifts in demand or supply chains can quickly lead to financial difficulties.

1.2. Implications for the Internal Market

The Preventive Restructuring Directive is not only important for individual firms but also for the broader functioning of the *EU internal market.* By harmonizing restructuring and insolvency procedures across member states, the directive aims to reduce the legal and administrative barriers that have historically made cross-border insolvency cases particularly challenging. Before the directive, differences in national insolvency laws meant that businesses operating across multiple EU jurisdictions often faced a complex web of legal systems, making it difficult to coordinate restructuring efforts. This lack of harmonization led to inefficiencies and inconsistencies in how insolvencies were handled, particularly in cross-border cases where conflicting laws could delay or complicate proceedings.

By establishing common standards for preventive restructuring and insolvency procedures, the directive seeks to create a more level playing field for businesses across the EU. This not only facilitates cross-border investment and trade but also enhances market stability by providing a more predictable legal environment. Investors, creditors, and other stakeholders can be more confident that businesses in financial distress will have access to fair and efficient restructuring mechanisms, regardless of the member state in which they are located. This predictability is essential for fostering trust in the internal market, as it reduces the risk of financial contagion spreading from one distressed firm to others within the same supply chain or market.

The directive also aligns with the EU's broader economic policies, particularly its focus on *sustainability* and *long-term growth.* By supporting businesses in financial difficulty and helping them restructure rather than liquidate, the directive contributes to economic resilience, allowing firms to survive economic shocks and continue contributing to the economy. This is especially relevant in the wake of the COVID-19 pandemic, which has placed unprecedented financial strain on businesses across the EU. The directive's emphasis on early intervention and preventive restructuring provides a framework for helping businesses navigate the current economic uncertainty and emerge stronger on the other side.

The *Preventive Restructuring Directive (EU/2019/1023)* represents a significant step forward in the EU's efforts to promote financial stability and resilience within its internal market. By focusing on preventive restructuring frameworks, the discharge of debt, and improving the efficiency of insolvency procedures, the directive offers businesses a lifeline during times of financial distress. It promotes early intervention, which can prevent costly insolvencies and liquidations, and ensures that businesses have access to the tools they need to restructure effectively. Moreover, by harmonizing these procedures across member states, the directive strengthens the functioning of the internal market, making it easier for businesses to operate across borders and contribute to the EU's long-term economic growth.

2. *STRENGTHENING TDR: EU DIRECTIVE for PROACTIVE RESTRUCTURING of DISTRESSED FIRMS*

The *recent EU directive (EU/2019/1023),* which has been adopted by all EU member states, represents a significant move toward creating *homogeneous Troubled Debt Restructuring (TDR)* laws and procedures across national jurisdictions. This initiative is particularly aimed at providing viable firms and debtors—those facing temporary financial difficulties but with the potential for recovery - with structured mechanisms that enable them to restructure their debts and operations effectively, without resorting to insolvency. By doing so, the directive seeks to prevent unnecessary liquidations that could otherwise lead to the dissolution of economically viable enterprises, a consequence that often has wider ramifications for the economy, employment, and market stability.

At the core of this directive is the principle that early intervention in the restructuring process is essential for the continued viability of debtors in financial distress. The EU's objective is to create a legal framework in which businesses that are facing financial difficulties can initiate restructuring measures early on, before they fall into deeper financial troubles that could lead to formal insolvency. The directive highlights the importance of timely action, suggesting that businesses should not be left to navigate financial distress on their own until their problems become irreversible. Instead, proactive restructuring is seen as the key to preserving businesses, protecting jobs, and ensuring that the broader economic ecosystem remains stable.

A central requirement of the directive is the development of early warning tools**,** which are intended to incentivize debtors to recognize and address their financial difficulties at an early stage. These tools can be developed either by member states or by private entities and are designed to provide firms with an early indication of potential financial distress, allowing them to take preemptive action. By identifying financial issues before they become insurmountable, early warning tools act as a preventive mechanism, offering companies the opportunity to engage with creditors and initiate restructuring negotiations while there is still room for a successful recovery.

The directive encourages the adoption of tailored early warning systems that are responsive to the specific financial environment of each member state, while also setting standards that ensure these tools can be applied consistently across the EU. By making the development of such systems a legal obligation, the directive reinforces the EU's broader aim of creating a more resilient and financially stable business environment. The intention is not only to safeguard businesses and employees but also to support creditors, suppliers, and other stakeholders who could be negatively impacted by business failures.

In addition to creating legal provisions for the restructuring of distressed firms, the directive underscores the importance of cooperation between the public and private sectors in implementing effective early warning systems. Private entities, such as financial institutions and industry bodies, have the expertise and resources necessary to develop sophisticated tools that can provide real-time data and financial analysis, which are crucial for identifying early signs of financial distress. Similarly, member states can play a crucial role by integrating public policies that promote financial literacy, awareness, and access to early restructuring options, especially for small and medium-sized enterprises (SMEs) that may lack the resources to implement such tools independently.

By emphasizing the harmonization of restructuring frameworks, the directive addresses the historical fragmentation of insolvency laws across EU member states. Previously, businesses operating in multiple jurisdictions often faced inconsistent legal frameworks when attempting to restructure, complicating cross-border operations and increasing the risks of financial distress. The *Preventive Restructuring Directive* seeks to resolve these inconsistencies by establishing a standardized legal environment in which businesses can engage in restructuring processes, regardless of where they operate within the EU. This harmonization reduces legal uncertainty and fosters greater confidence among businesses and investors, thereby contributing to the overall stability of the internal market.

In Italy, the introduction of *Article 182-bis restructuring agreements* marks a significant evolution in the country's approach to managing corporate crises. This legal instrument was introduced as part of the broader reforms to the Italian bankruptcy law, initially under Law Decree No. 35 of March 14, 2005, which was subsequently converted into Law No. 80 on May 14, 2005, and further refined by Legislative Decree No. 5 of January 9, 2006 ***(Di Marzio F., 2006).*** These reforms represent a concerted effort by the Italian government to modernize the legal framework governing corporate insolvency and to provide businesses with more flexible and timely tools for navigating financial distress.

The *182-bis restructuring agreements* are designed to offer financially distressed companies an alternative to formal bankruptcy proceedings by allowing them to negotiate a settlement with their creditors under court supervision. Unlike more traditional bankruptcy proceedings, which often result in liquidation, the 182-bis agreements prioritize business continuity and debt restructuring, providing firms with the opportunity to restore financial stability while continuing their operations. This approach is particularly important for viable businesses that are facing temporary liquidity challenges but still have the potential to recover if given sufficient time and support.

One of the most notable features of the 182-bis restructuring agreements is their *pre-insolvency nature***,** which allows companies to seek relief before they reach a point of irrevocable financial collapse**.** The process is initiated voluntarily by the debtor, who submits a restructuring plan to the court, outlining the proposed terms of repayment and the concessions sought from creditors. This plan must be supported by the majority of creditors holding at least 60% of the total debt, making the *creditor's consent* a critical component of the restructuring process. By involving creditors early in the process and securing their approval, the agreements seek to foster cooperative solutions that are less adversarial than traditional insolvency proceedings.

The legal reforms surrounding the 182-bis agreements reflect a broader trend in European insolvency law towards preventive restructuring frameworks**,** which aim to resolve financial distress before insolvency becomes inevitable. This shift aligns with the goals of the *EU's Preventive Restructuring Directive (EU/2019/1023)***,** which encourages member states to adopt early intervention mechanisms that allow companies to engage in debt restructuring without the need for formal insolvency proceedings. In this sense, Italy's introduction of the 182-bis agreements is in line with broader European efforts to harmonize insolvency laws and promote business continuity over liquidation.

From a practical standpoint, the *182-bis restructuring agreements* offer several advantages to both debtors and creditors. For debtors, the agreements provide a stay of enforcement actions**,** meaning that creditors cannot pursue individual legal remedies while the restructuring process is ongoing. This stay offers companies breathing room to negotiate with their creditors and develop a workable plan for returning to financial health. For creditors, the *182-bis agreements* offer a more predictable and orderly process for recovering outstanding debts compared to more fragmented and contentious legal actions. By participating in the restructuring process, creditors are more likely to receive at least partial repayment, as opposed to the uncertain and often diminished returns that come from liquidation.

However, despite its advantages, the 182-bis restructuring framework also faces certain challenges**.** One of the primary concerns is that the process still relies heavily on court supervision**,** which can lead to delays and additional costs**.** Furthermore, the requirement that 60% of creditors must approve the restructuring plan can be a significant hurdle for companies with a large and diverse group of creditors, as reaching consensus can be time-consuming and difficult. Additionally, while the stay of enforcement actions provides protection to the debtor, it can also place additional financial strain on creditors, particularly smaller creditors who may depend on timely payments to maintain their own liquidity.

Another challenge relates to the effectiveness of the restructuring plans**.** For companies to successfully exit the *182-bis agreements*, their restructuring plans must be realistic and viable. If the underlying causes of the financial distress are not adequately addressed - whether they are related to poor management**,** market conditions**,** or operational inefficiencies - the restructuring may only provide temporary relief, leading to recidivism or further financial distress down the line. Therefore, the success of these agreements depends not only on the legal framework but also on the business strategies employed by the distressed companies to return to profitability.

The *182-bis agreements* also interact with other restructuring tools provided by Italian bankruptcy law, such as *concordato preventivo* (preventive arrangement with creditors). While both procedures aim to restructure debt and avoid liquidation, *182-bis agreements* offer more flexibility and can be used even in situations where the debtor is not yet formally insolvent. This makes them particularly useful for companies that are experiencing early signs of financial trouble and are seeking to address their issues proactively**.**

The *182-bis restructuring agreements* represent a progressive step in Italian bankruptcy law, offering financially distressed companies a mechanism to restructure their debts while continuing to operate. By allowing for early intervention, these agreements help prevent unnecessary liquidations and promote business continuity, which is crucial for preserving jobs and maintaining economic stability. As Italy continues to refine its insolvency laws, the *182-bis agreements* are likely to play an increasingly important role in the corporate restructuring landscape**,** both domestically and in the context of broader European harmonization efforts.

The *EU's directive on preventive restructuring* is a critical step toward strengthening the financial resilience of firms across the EU. By mandating the development of early warning tools and promoting early intervention, the directive offers businesses a framework that encourages proactive financial management and timely restructuring efforts. The broader economic implications of this directive are significant, as it helps to preserve viable businesses, protect employment, and maintain the integrity of the EU's internal market. As the directive is implemented and adapted across member states, it is expected to play a central role in mitigating the risks of financial distress and insolvency in the years to come.

3. *ENHANCING FINANCIAL DISTRESS PREDICTION: GLOBAL CHALLENGES and DEVELOPMENTS*

The existing body of literature on financial distress prediction models has consistently emphasized the need for improving their accuracy and reliability. As noted by *Sánchez C.P. et al. (2013)***,** the complexity of financial systems and the variety of factors that contribute to corporate distress require models that can accurately predict financial instability before it becomes critical. In response to this challenge, researchers in recent years have developed more sophisticated models that combine various data sources, including accounting data**,** stock market performance, and macroeconomic indicators**.** These models aim to provide a more comprehensive analysis by incorporating both internal firm-specific factors and broader market and economic trends.

*Accounting data***,** which includes traditional financial metrics such as liquidity ratios, profitability, and leverage, has long been a cornerstone of financial distress prediction models. However, accounting-based models have certain limitations, particularly in rapidly changing market environments, where financial data from previous periods may not fully capture the real-time risks a company faces. To address these gaps, modern models have integrated *stock market data***,** which reflects real-time market sentiment, stock price volatility, and investor behavior. The inclusion of market data allows for a more dynamic approach to predicting financial distress, as stock prices often react to new information about a firm's prospects well before such changes are reflected in the company's financial statements.

Additionally, researchers have recognized the importance of *macroeconomic factors* in shaping a firm's financial health. Variables such as interest rates, inflation, GDP growth, and unemployment rates play a crucial role in determining the broader economic context within which businesses operate. For instance, during periods of economic recession, firms are more likely to experience declining revenues and cash flow issues, which can increase the likelihood of financial distress. By incorporating these macroeconomic variables into prediction models, researchers aim to provide a more holistic analysis of a firm's financial situation, accounting for both internal performance and external economic conditions.

Despite these advancements, there is a recognition within the literature that many of these models have been developed and tested within specific contexts**,** often limited to particular regions, industries, or economic environments. While models that integrate accounting data, stock market performance, and macroeconomic factors have shown promising results in certain settings, their applicability on a global scale remains uncertain. The geographical and economic diversity of firms worldwide, as well as differences in regulatory frameworks and market structures, pose significant challenges to the generalizability of these models.

For example, a model that performs well in predicting financial distress in developed markets like the United States or Western Europe may not yield the same accuracy when applied to emerging markets, where firms may face different financial, regulatory, and operational challenges. In such contexts, market liquidity, transparency, and the availability of reliable financial data can vary significantly, affecting the predictive power of these models. Moreover, macroeconomic conditions can differ vastly between regions, making it difficult to apply a single predictive framework across multiple countries without adjusting for local economic realities.

Researchers have therefore called for further validation of these models in a wider array of contexts and regions**.** Expanding the application of these models to include diverse economic environments - such as developing economies, sectors with high financial volatility, or countries with different legal frameworks - would help determine their broader utility and highlight potential areas for improvement. Doing so would allow for a more comprehensive understanding of financial distress across industries and regions and would enhance the predictive power of these models in a global context.

Furthermore, *cross-country comparative studies* could provide valuable insights into the universal applicability of predictive models. By testing these models in different economic settings, researchers could identify the factors that most significantly influence financial distress in various contexts. For instance, while stock market data may be a crucial predictor of distress in countries with well-developed financial markets, it might be less relevant in regions where capital markets are less active or where firms rely more heavily on bank financing. Similarly, macroeconomic factors like inflation or interest rates may have different levels of impact depending on the stability of the local economy.

While significant progress has been made in developing multifactorial financial distress prediction models that incorporate accounting, market, and macroeconomic data, their global applicability remains a key area for future research. As researchers continue to refine these models, it will be critical to test their performance across a range of economic contexts, industries, and regions to ensure that they provide reliable and accurate predictions on a broader scale. This ongoing effort to enhance the accuracy and generalizability of financial distress prediction models will not only improve early detection mechanisms but also contribute to more effective risk management strategies for businesses and financial institutions worldwide.

Recent literature in the field of financial distress prediction has underscored the growing need to improve and refine existing models to better suit the complexities of the modern business environment. *Fernández-Gámez M.A. et al. (2020) and Vo D.H. et al. (2019)***,** among other scholars, have called for the enhancement of these predictive models, noting that while significant progress has been made, further advancements are essential to ensure their ongoing relevance and accuracy. The constantly evolving nature of financial markets, corporate structures, and economic conditions has created an environment in which traditional models—although still useful - require modifications to maintain their predictive power.

One of the most well-known and widely used predictive models in both research and practice is the Altman Z-Score model**.** First introduced by *Altman E.I. et al. (1968)***,** this model has become a global benchmark for predicting financial distress and bankruptcy**.** *Altman E.I. et al. (2017)* emphasize that the Z-Score model continues to be highly relevant across different countries and industries, demonstrating its utility in a variety of international contexts. The model's widespread adoption is largely due to its simplicity and practical applicability, as it uses a combination of financial ratios - such as liquidity**,** profitability, and leverage - to assess a firm's likelihood of experiencing financial distress. Despite its robustness, however, Altman and his colleagues acknowledge that the model, originally developed in the 20th century, may not fully capture the complexities of today's global business environment.

As *Altman E.I. et al. (2017*) point out, the original Z-Score model, while still widely used and effective, was developed in a very different economic context. At the time, the business landscape was relatively stable, with less frequent economic disruptions, slower technological advancements, and fewer international financial linkages. The increasing globalization of markets**,** the rise of digitization**,** and the more interconnected nature of economies today mean that businesses are exposed to a broader range of risks that the original Z-Score model may not have been designed to capture. For instance, factors such as market volatility**,** geopolitical uncertainty**,** and rapid changes in technology play a much more significant role in shaping the financial health of firms in the 21st century than they did in the past.

Given these changes, Altman and his colleagues recommend that their models be modified and extended to better align with the modern financial environment. One area that could benefit from refinement is the incorporation of non-financial factors into the predictive framework. While the Z-Score model relies heavily on traditional financial ratios drawn from a company's balance sheet and income statement, the authors suggest that additional variables - such as market sentiment, industry-specific risks, and macroeconomic trends - could further enhance the model's predictive accuracy. These factors are particularly relevant in today's globalized economy, where external shocks can quickly disrupt even the most financially stable firms.

In particular, *Altman E.I. et al. (2017)* introduce the Z”-Score model, an international adaptation that has been shown to perform well across different countries and regions. This model builds on the original Z-Score by incorporating variables that are more suited to the dynamics of global business. The Z”-Score is better equipped to handle the cross-border nature of firms and the different accounting standards, regulatory environments, and financial systems that exist across countries. Despite this progress, the authors still urge further improvements, particularly in adapting the model to address regional economic conditions and sector-specific challenges, which have become more pronounced in recent years.

Furthermore, Altman and his colleagues stress that the business environment of the 21st century is marked by much higher volatility than in previous decades. Events such as the 2007-2009 Global Financial Crisis and the COVID-19 pandemic have demonstrated how quickly firms can go from financial stability to distress, sometimes with very little warning. These crises have exposed the limitations of older models, which may not be able to predict such sudden disruptions. As a result, *Altman E.I. et al.* recommend that predictive models incorporate early warning systems that can detect potential financial distress before it becomes too severe. By identifying signs of trouble earlier, firms can take corrective actions - such as restructuring their debt or cutting costs - before they reach the point of bankruptcy or insolvency.

Another important consideration in updating financial distress models is the need for sector-specific modifications. Different industries face distinct risks and challenges, and a one-size-fits-all model may not be effective in all contexts. For example, the technology sector is highly dynamic, with firms often experiencing rapid growth followed by sharp declines, whereas more traditional industries such as manufacturing may face different risk factors, such as supply chain disruptions or labor shortages. By adapting predictive models to account for the specific risks associated with different sectors, the accuracy and usefulness of these models can be significantly enhanced.

The Altman Z-Score model remains one of the most important tools in financial distress prediction, but as *Altman E.I. et al. (2017)* argue, it must continue to evolve to reflect the changing global business environment. The recommendation to modify and extend existing models is critical for maintaining their relevance in an era of rapid technological advancement, globalization, and economic volatility. As businesses and financial markets become more complex, so too must the models used to predict financial distress, ensuring that they remain effective in providing early warnings and helping to mitigate risks. Future research and model development will likely focus on integrating a wider array of variables, incorporating real-time data, and adapting models to the specific needs of different industries and regions.

4. *FINANCIAL DISTRESS PREDICTION in ITALY: a MODEL for EARLY INTERVENTION*

In this research, my primary objective is to develop a predictive model that can effectively identify financial distress at an early stage, with a particular focus on improving prediction accuracy**.** The need for early detection is crucial, as it allows businesses to implement timely corrective measures to avoid insolvency or unnecessary liquidation. By focusing on the early identification of financial distress, my model seeks to provide firms and stakeholders - such as creditors and investors - with the tools necessary to make informed decisions about the future of the business.

For this study, I concentrate on private large and medium-sized firms operating within Italy**,** which serves as a significant economy within the European Union (EU)**.** Italy is subject to EU-wide regulations, particularly in the areas of *preventive restructuring* and *troubled debt restructuring (TDR)****,*** but it also has specific national legal frameworks for managing financial distress. The choice of Italy is not only important for understanding the country-specific dynamics of financial distress but also provides a valuable case study for developing a model that could be adapted for use in other countries with similar legal frameworks, such as Spain and France**.** These nations share common regulatory approaches to corporate restructuring and debt management**,** making it feasible to apply and extend the findings of this research beyond Italy.

To ensure that my model accounts for sector-specific risks and financial characteristics, I have classified firms according to the Standard Industrial Classification (SIC) system. This allows me to analyze firms in seven key industries: *construction****,*** *manufacturing****,*** *transportation-communications-electric-gas-sanitary services****,*** *wholesale trade****,*** *retail trade****,*** *finance-insurance-real estate*, and *services****.*** These sectors represent a broad cross-section of Italy's economic activity, each with unique financial dynamics and exposure to different risk factors.

By focusing on these seven SIC - classified sectors within Italy, my study aims to provide a comprehensive understanding of financial distress across diverse industries. These sectors, though varied in nature, share common characteristics related to financial volatility and economic sensitivity**,** making them ideal for testing the accuracy and robustness of my predictive model. Moreover, given the similarities in corporate restructuring laws between Italy, Spain, and France, this model has the potential to be applied across these countries, providing a cross-border framework for early financial distress detection. These countries operate under EU-wide legal standards for TDR, allowing the model to be extended to different regulatory contexts without significant adjustments.

My research focuses on developing a model that not only enhances early-stage prediction accuracy for Italian firms but also has the potential to be utilized in other European economies with similar legal frameworks, such as Spain and France. By doing so, I aim to contribute to the broader field of financial distress prediction while offering practical tools that can be adapted and applied across various European contexts to mitigate risks and prevent insolvency.

In this study, I build upon the original Z”- Score model, first introduced by *Altman E.I. (1983)***,** which utilizes a combination of financial ratios to predict firm failure. The Z”-Score model has demonstrated strong performance in international contexts and is widely regarded as a reliable tool for predicting corporate bankruptcy. However, while the original Z”-Score was designed specifically to forecast failure, my research focuses on a related but distinct objective: the prediction of financial distress. Unlike failure, which typically implies the cessation of business operations and liquidation, financial distress refers to a state where firms experience severe liquidity issues but still have the potential to restore financial equilibrium and maintain their going concern status**.**

To address this distinction, I aim to develop a model that more accurately predicts financial distress**,** defined as a firm’s likelihood of filing for Troubled Debt Restructuring (TDR)**.** TDR allows firms to restructure their debt obligations and recover from temporary financial difficulties without necessarily entering bankruptcy. By focusing on financial distress rather than outright failure, I aim to provide a proactive tool for identifying firms that are at risk but still have the ability to recover if they engage in timely restructuring efforts.

While the original Z”-Score model has proven effective in predicting bankruptcy**,** I argue that its focus on failure does not fully capture the nuances of financial distress. In particular, working capital as a financial ratio, which is included in the original Z” - Score model, may not be the most appropriate indicator for predicting distress. *Working capital* - defined as the difference between a company’s current assets and current liabilities - is indeed an important measure of short-term financial health. However, I believe that a more targeted ratio would better highlight a firm's immediate liquidity and its ability to meet short-term obligations when approaching financial distress.

Therefore, I propose a modification to the original Z”-Score model, specifically replacing the *working capital* to *total assets ratio* with the *cash and cash equivalents* to *current liabilities* ratio**,** a proposal that is supported by the findings of *Francesco De Luca’s* study**,** which highlights the superior relevance of cash liquidity in predicting financial distress.

The rationale for this modification is that *cash* is the most liquid asset and thus provides the most direct indication of a firm’s ability to meet short-term obligations. While *working capital* incorporates all *current assets* (including less liquid items such as inventory or accounts receivable), *cash and cash equivalents* represent immediately available funds, which are crucial for firms on the brink of financial distress. In essence, this modification is designed to better reflect a firm's *short-term solvency* and its ability to avoid *default* or the need for *debt restructuring*.

The *cash* to *current liabilities ratio* offers several advantages in the context of financial distress prediction. First, it provides a more accurate assessment of liquidity, as it directly compares the firm's most liquid resources (cash and cash equivalents) to the immediate obligations it must meet. This ratio is particularly important for firms facing liquidity crises, where the ability to generate cash quickly is critical to their survival. In contrast, working capital may provide a more generalized view of short-term financial health but can be misleading in situations where a firm has high levels of inventory or long receivables collection periods - both of which may inflate current assets without necessarily improving the firm's cash position.

Additionally, by focusing on *cash* and *cash equivalents*, my modified model captures the firm's ability to meet payment deadlines and satisfy creditor demands, which are often the triggers for financial distress and TDR filings. In distressed situations, the firm's liquidity is typically stretched thin, and access to cash becomes the most crucial determinant of whether the company can continue to meet its obligations or must seek a debt restructuring agreement. The cash ratio thus provides a more immediate early warning signal of distress than broader measures like working capital.

Moreover, the replacement of the *working capital* to *total assets ratio* with the *cash to current liabilities* ratio reflects the realities of the modern business environment, where liquidity management has become increasingly important, particularly in the wake of economic shocks like the 2007-2009 Global Financial Crisis and the COVID-19 pandemic. These crises have underscored the importance of cash flow management and the ability of firms to react swiftly to changing market conditions. In such scenarios, cash reserves often serve as the first line of defense against financial distress, while less liquid assets may take time to convert into cash, potentially exacerbating the firm's short-term liquidity problems.

In developing this model, my goal is not only to improve the accuracy of financial distress predictions but also to provide a practical tool that can be used by corporate managers, creditors, and investors to identify firms at risk of TDR and take preemptive measures. By focusing on liquidity, my model provides stakeholders with critical insights into a firm's ability to continue operations, restructure debt, and maintain solvency. Furthermore, this model could also serve as a diagnostic tool for firms seeking to avoid TDR by taking steps to improve liquidity management before financial distress becomes irreversible.

In conclusion, by modifying the original Z”-Score model to focus on cash and liquidity, I aim to offer a more accurate and context-specific tool for predicting financial distress in firms. My model reflects the shift in emphasis from failure prediction to distress prediction, which is more aligned with modern corporate needs and debt restructuring practices. As firms face increasing financial volatility, my model’s focus on cash and liquidity provides a more timely and actionable indicator of distress, offering businesses the opportunity to address their financial challenges and maintain their going concern status.

CHAPTER 2

LITERATURE REVIEW & THEORETICAL FRAMEWORK

Forecasting the state of insolvency: an empirical analysis of Italian companies in bankruptcy.

Since the 16th century BC, the inability of businesses to repay their debts has been a significant concern. This concern led to the establishment of bankruptcy laws in the United States in 1800, aimed at defining the criteria for determining when a business is considered bankrupt due to its inability to pay its debts. During times of economic depression and business failures, scholars started developing tools to identify businesses at risk of failing.

1. A HISTORY OF BANKRUPTCY - ORIGINS OF THE WORD

The most widely-accepted theory regarding the origin of the word *"bankruptcy"* suggests that it is a combination of the ancient Latin words *bancus* (bench or table) and *ruptus* (broken).

According to this theory, when a banker, who conducted transactions on a bench, became unable to lend or meet their obligations, their bench was symbolically broken, representing their failure and inability to continue operating. This practice was common in Medieval Italy, and it is believed that the term *"bankrupt"* originated from the translation of the Italian phrase *"banco rotto",* meaning broken bank. Some alternative theories propose that the word's origin can be traced back to the French expression *"banque route"*, meaning a broken table, symbolizing the disappearance of a banker's table and the quick escape of individuals who had absconded with entrusted money. Since then, the financial repercussions of firm failures have been notably negative *(https://www.bankruptcydata.com/a-history-of-bankruptcy)*.

During the reign of Henry VIII in 16th century England, the prevailing view did not recognize economic environmental factors as significant contributions to a business's inability to repay debts. Instead, bankruptcy was regarded as a criminal act, leading to severe punishments, including imprisonment and even the death penalty, as stipulated by the initial laws of 1542.

Both in the past and present, many consider financial strength to be a crucial requirement for a firm's long-term survival. However, other influential environmental factors - such as social dynamics, legal considerations, market pressures, global economic competitiveness, unique workforce characteristics, and negative responses from consumers - can exert tremendous pressure, ultimately leading to the collapse of a business. Historically, these factors were often overlooked when assessing the financial well-being of a company.

Consequently, the early bankruptcy laws introduced in 1800 in the United States were a response to the previously neglected environmental factors that could contribute to the collapse and failure of firms. These factors, which were often overlooked, proved to be significant drivers of business failures.

In the 1930s, during the challenging period of the Great Depression, the country experienced a painful economic downturn, leading to a staggering number of business failures, reaching around 177.000 per year. These failures were primarily a result of the economic turmoil of the Great Depression. In response, significant revisions were made to bankruptcy laws and legislation to address these issues. The Chandler Act of 1938 was particularly focused on rehabilitating and reorganizing distressed businesses.

During times of economic turbulence, concerns regarding business failures tend to escalate significantly. In the 1970s, two prominent corporate failures occurred, capturing substantial attention due to their economic impact. The first was the bankruptcy of Penn Central Transportation, involving assets amounting to approximately $5 billion. The second notable failure was the WT Grant Company, which was eventually succeeded by Chrysler Corporation in 1980, with a bankruptcy involving assets valued at $12 billion.

More recently, bankruptcy has gained substantial publicity due to irregularities in the business practices of Enron and WorldCom. These cases have raised critical questions regarding management commitment, integrity, and the ability of auditors to accurately assess the financial operations of businesses. The incidents have highlighted the importance of addressing issues related to corporate governance, financial transparency, and the accuracy of financial reporting.

The recent events, such as the Enron Case, have bought heightened attention to the far-reaching impact that bankruptcy can have on the economy. The repercussions extend beyond the business itself, affecting the economic well-being of investors, lenders, consumers, and the industry as a whole. This heightened awareness has created a vulnerable atmosphere for industries that experience such impacts.

The Enron case, in particular, has shifted the perception of bankruptcy from being strictly a financial issue to a more complex inter-organizational arrangement. In the past, bankruptcy was primarily seen as a financial management problem, spanning from the late 19th century to the late 20th century. However, recent events have challenged this perception and forced the business environment to adopt a cross-functional perspective when considering bankruptcy. Factors such as strategic considerations, management integrity, and external economic influences are now taken into account when assessing the implications of bankruptcy.

Various economic factors can exert pressure on business, potentially leading to corporate failures. Some argue that the increase in business failures in the past can be attributed to the numerous revisions made to the bankruptcy codes under the Bankruptcy Act of 1978. This act replaced the old bankruptcy Chapters 10, 11, and 12 that were enacted in 1898, as well as the amendments to the Chandler Act of 1938. The revisions to the bankruptcy laws resulted in a complete overhaul of the bankruptcy process.

The definition of business failures was redefined to include not only the inability to meet creditors' obligations but also the impact of extreme economic volatility, such as a depression. Throughout history, further revisions to the bankruptcy laws have been made, particularly highlighted by the 1982 Supreme Court ruling that extended the jurisdiction of bankruptcy courts.

This ruling reconfigured the powers of judges, limiting their authority and overlapping duties with other branches of government. These changes brought notable transformations to the business environment, including restrictions on labor contracts and tax loss carryforwards. The Bankruptcy Amendment Act of 1984 consolidated these changes and, in part, contributed to an increase in businesses seeking bankruptcy protection due to the lowered threshold requirements.

Despite further amendments to the bankruptcy laws between 1980 and 1987, the number of business failures remained high, surpassing 300.000. Specially, the average annual filings for Chapter 11 bankruptcy ranged from approximately 14.000 in the early 1980s to around 20.000 between 1988 and 1992, as reported by Altman in 1993.

Another perspective attributes the increase in business failures to factors such as a lack of strategic initiatives and slow responsiveness. Additionally, failures can be attributed to an economic environment that restricts businesses from amassing significant trades and market shares, a lack of alternative options in the face of high capital costs, resource constraints, the entrance of foreign competitors into domestic markets, and deregulation in certain industries that led to the collapse of less efficient businesses. Whether the rise in bankruptcy filings in recent decades is primarily linked to the redefinition of bankruptcy guidelines or these broader business environmental factors remains inconclusive and subject to debate.

The business management literature has primarily focused on success, achievement, and strategies for growth, rather than on business failure or bankruptcy. The emphasis has been on understanding and implementing practices that lead to successful outcomes. However, it's worth noting that in recent years, there has been an increasing recognition of the importance of understanding and addressing business failure.

While the coverage of business failure in management literature may not have been as extensive as that of success, there are still resources available that discuss the potential symptoms and causes of business failures. Some reasons why business failure may not receive the same level of attention could include the perception that failure is stigmatized and seen as a personal or organizational flaw, as well as a bias towards studying successful companies as models for emulation *(Arganti J., 1976).*

That being said, the study of business failure has gained more attention in recent years.

Researchers and scholars have recognized the value of analyzing failures as learning opportunities and sources of insights for improving future business practices. There has been an emergence of literature and studies on the subject, including case studies of failed businesses, analysis of factors contributing to failure, and frameworks for managing and recovering from failure.

Additionally, other fields of study, such as entrepreneurship and innovation, have explored the concept of failure more extensively. The startup ecosystem, for example, often embraces the idea of *"failing fast"* and learning from failures as a way to iterate and improve business models.

Thus, while the historical focus of business management literature has been on success rather than failure, there has been a growing recognition of the importance of understanding and addressing business failure. The coverage of business failure is not as extensive as that of success, but there are resources available that discuss the symptoms, causes, and management of business failures.

2. EXPLORING FINANCIAL CRISES: DEFINITIONS, PHASES, AND CAUSES

The studies on predicting bankruptcy and assessing financial health aim to provide insights and tools for foreseeing when a company is at risk of failure. The goal is to assist management in redirecting their efforts and reevaluating their corporate strategy to prevent or mitigate potential financial problems. By identifying the warning signs of bankruptcy in advance, various stakeholders can benefit from reduced risk, including investors, creditors, employees, and other associated with the business entity.

For *investors,* the ability to predict a firm's susceptibility to bankruptcy allows them to make more informed investment decisions. They can assess the financial stability and viability of a company before committing their resources, reducing the likelihood of investing in a business that may fail.

According to Beaver, investors tend to adjust their assessment of a firm's solvency continuously over a five-year period leading up to its failure. However, despite these ongoing adjustments, the largest unexpected deterioration in the firm's financial condition often occurs in the final year before the failure actually happens. *(Beaver W.H., 1968).* This implies that investors are often still surprised by the occurrence of failure, even when it is imminent.

The fact that investors continuously adjust their perception of a firm's solvency suggests that they actively monitor its financial health and make periodic assessments. However, the significant deterioration that takes place in the last year can catch them off guard and exceed their expectations or predictions. This unexpected decline in financial condition indicates that there may be factors or events that are difficult to anticipate or accurately assess until they are very close to the actual failure.

This finding highlights the challenges of accurately predicting and foreseeing business failure, even for investors who closely monitor a firm's financial performance. It suggests that certain circumstances or indicators may not be easily detected until they are near the point of no return. As a result, investors may still be surprised when a firm ultimately fails, even if they have been adjusting their assessment of its solvency over time.

This insight underscores the importance of continuously monitoring and reassessing the financial health of businesses, as well as the need for addition tools and techniques that can improve the prediction of failure and reduce the occurrence of surprises for investors and other stakeholders.

Creditors, such as banks or lenders, also benefit from predicting bankruptcy. It helps them assess the creditworthiness of a company and determine the level of risk associated with extending loans or providing credit. By understanding a firm's vulnerability to bankruptcy, creditors can take appropriate measures to protect their interests and minimize potential losses.

Employees are directly affected by a company's financial health. Predicting bankruptcy enables employees to have a better understanding of their job security and the overall stability of the organization. This knowledge allows them to plan and make informed decisions regarding their carrier and financial future.

Additionally, predicting bankruptcy is crucial for *auditors*. Assessing the *"Going Concern"* of a business, which refers to its ability to continue operating in the foreseeable future, is an essential aspect of auditing. By having effective techniques for predicting bankruptcy, auditors can evaluate the vulnerability of a business and provide early warnings or recommendations to address financial risks. This helps avoid the situation where bankruptcy is discovered too late, posing a threat not only to the firm but also its obligations towards creditors and bondholders. Meeting debt obligations is crucial for maintaining the financial health and reputation of a company.

The ability to predict bankruptcy and assess financial vulnerability provides valuable insights and tools for various stakeholders. It helps management refocus their efforts, allows investors to make informed decisions, assists creditors in managing risk, provides job security for employees, and aids auditors in evaluating the *"Going Concern"* status of a business.

2.1. What is a crisis?

The question of what constitutes a crisis is a complex and widely debated in the literature. While there is no universally agreed-upon definition, a common element is the dynamic nature of crises, which often follow a deteriorating path and can lead to financial collapse in severe cases.

The progression of a crisis can be categorized into four stages: incubation, maturation, consequences on financial flows and trust, and impacts on stakeholders *(Guatri L., 1995).*

Each crisis is unique, making it challenging to provide a definitive and universally applicable definition. Past experiences may not always offer clear guidance on how to address a crisis, and examining the responses of those in charge of affected entities can provide limited insights.

The idea of economic balance is deeply ingrained in management's thinking, driving businesses to consistently achieve positive financial outcomes. This equilibrium is essential for a company's survival and is characterized by consistent financial success, except in exceptional cases. Management continuously monitor the economic equation, which balances revenues and costs, including risk capital compensation. To maintain this balance, fair compensation for all contributing factors and proper recognition of the economic entity are crucial, encouraging investment in the business unit.

From the discussion, it is possible to propose a definition of a crisis as an economic disequilibrium within which a company operates. This condition involves a series of deteriorations that ultimately lead to the closure of the production complex due to significant losses incurred. Each crisis event has specific causes, and their combined impact disrupts both financial stability and operational functioning, resulting in negative financial outcomes and an inability to meet previously undertaken obligations, primarily due to insufficient financial resources.

In Article 2 of the Code for Business Crisis and Insolvency (CCII), a crisis is described as a situation in which a debtor's financial state suggests a high probability of insolvency. This is typically indicated by the insufficiency of expected cash flows to cover financial obligations within the next twelve months. Conversely, insolvency is defined as the condition in which the debtor is no longer capable of meeting its obligations in a regular and consistent manner. This condition becomes evident through various signs, including payment defaults or other external events that reveal the debtor's inability to fulfill financial commitments as expected.

The prevailing view in literature is that insolvency represents the pivotal point in a business crisis, serving as a clear signal of this condition. However, it is important to note that insolvency is often the outcome of underlying factors that have undermined the company's financial stability. Taking early and proactive action is of paramount importance in this context. Recognizing that conditions capable of disrupting a company's positive trajectory are bound to occur, it is imperative for management to respond promptly with corrective measures upon detecting initial signs of trouble, avoiding any delays.

Despite the obvious need for swift and effective intervention, there is often a tendency to ignore these initial warnings in the hope that normal operations will spontaneously resume.

This behavior is frequently justified by subjective factors, such as management's reluctance to admit failure and confront reality.

Attempting to hide the company's pathological condition from external parties is an unwise decision. While it may temporarily postpone reactions from stakeholders, it ultimately results in more severe and detrimental interventions, primarily due to the delay in implementing corrective actions. Therefore, once management identifies the pathological condition and comprehends its extent, taking prompt remedial actions to restore balance is crucial for effectively addressing underlying issues.

In summary, meeting debt obligations and avoiding bankruptcy are vital for the overall stability and success of a company.

2.2. The phases of the crisis

In the mid-1990s, Luigi Guatri introduced a model that delineates the crisis process into four stages, each marked by increasing risk *(Guatri L. 1995)*.

The initial stage is termed *"incubation",* followed by the progression of decline, which subsequently leads to repercussions on cash flows and a deterioration in creditworthiness. Ultimately, the crisis reaches a fully eruption, causing harm to stakeholders' interests.

Each stage is associated with specific indicators, particularly during the incubation phase, which may be triggered by internal or external factors. Indicators of decline include:

1. reduced profitability;

2. negative cash flow;
3. departure of key managers;
4. loss of market share, especially evident during periods of stagnant demand;
5. decreases sales, among others.

The imbalances observed in these stages are primarily quantitative and are measured using various indices and ratios, based on static or dynamic values. This approach enables a tangible and precise understanding of the indicators of decline, facilitating communication with both internal and external stakeholders. However, it is important to note that a comprehensive view of the company's operations can sometimes obscure the recognition of individual contributions from various factors. Additionally, management's tendency to conceal problems and delay remedial action can further complicate the diagnostic process.

In the *"maturity"* stage of decline, there are income losses and a reduction in the company's capital value. Issues from the preceding stage undermine the company's income-generating capacity, potentially nullifying its financial results. These repercussions on financial performance become most evident in the third stage, where financial issues such as cash shortages and difficulties in balancing expenses with income emerge, alongside significant losses in credit and trustworthiness.

External parties, upon recognizing this critical situation, tend to exercise caution in providing financial support or extending payment terms. Consequently, financial imbalances, combined with an inability to generate sufficient income, erode the company's economic capital, jeopardizing its survival.

In the fourth stage, the crisis escalates to a point where it may result in insolvency or bankruptcy, depending on its severity. Insolvency denotes the company's evident inability to meet its obligations, which damages relationships with external parties and restricts access to resources. Bankruptcy arises from sustained imbalances between assets and liabilities, making recovery efforts complex or unfeasible. Both scenarios severely damage the company's reputation and trust relationships with stakeholders.

At this juncture, it is essential to examine the concepts of economic, financial, and equity balance, which are integral to understanding the dynamics of corporate stability. These aspects can be outlined as follows:

- *Economic Balance (Second Stage)*: this stage signifies a visible crisis, primarily evident to internal stakeholders. It is characterized by rising costs and diminishing revenues, which considerably impair the company's ability to achieve positive results. In some cases, income may fall below equity levels, hindering the company's ability to provide fair compensation to stakeholders and highlighting a deeper, structural issue in profitability;

- *Financial Balance (Subsequent Stage)*: the challenges in economic balance inevitably start to affect the company's financial structure. This stage involves an analysis of income and expenditure flows, encompassing the production cycle and financial operations. Financial difficulties emerge as unprofitable operations necessitate increased borrowing, which in turn affects the overall financial stability of the company and may lead to an unsustainable debt structure;
- *Equity Balance*: equity balance is fundamental to a company's survival, as it provides the essential resources needed for regular operations. Ideally, these resources should be generated internally to foster sustainable growth. However, when internal resources are insufficient and equity diminishes, the company's competitiveness, decision-making capabilities, and resilience to financial risks are severely compromised. This erosion of equity reduces the organization's flexibility and heightens its exposure to external financial pressures;
- *Interdependencies*: the interconnections between economic, financial, and equity balance are highly significant. These aspects are interdependent, meaning that issues in one area will likely cascade into the others, creating a cumulative impact. Addressing these factors collectively is therefore crucial to preserving the overall stability and health of the organization.

In essence, a corporate crisis typically progresses from economic difficulties, through financial challenges, and ultimately impacts equity balance. Effectively managing these aspects in an integrated manner is essential for ensuring the company's stability and continuity. This holistic approach not only helps in identifying potential risks at an early stage but also enables the implementation of strategic measures that support long-term resilience and sustainable growth.

2.3. The causes of financial distress

The causes of a crisis can be understood as the factors that trigger and precede the manifestation of a crisis, both temporally and logically *(Giacosa E., 2016).* These causes, which lead to a state of decline and eventual crisis, can be broadly categorized into quantitative and qualitative factors *(Zappa G., 1956)*.

The qualitative factors often originate from issues related to the effectiveness of top management. Specifically, in many small Italian businesses, managers have been observed to lack the necessary preparation and qualifications for their roles *(https://www.ilsole24ore.com/art/il-37percento-imprese-chiude-4-anni-pesano-crisi-e-trucchi-evadere-AESRIw5G, 2019)*. This problem is particularly prevalent in family-owned businesses, where family members are frequently appointed to significant managerial positions based on familial ties rather than qualifications. While this practice strengthens family bonds within the business, it may also become a structural weakness that exposes the company to risk, as these appointments are sometimes made without regard for the specific expertise required.

Another common characteristic in the Italian business landscape is the excessive concentration of decision-making power in a single manager. Entrusting an individual with a disproportionate amount of responsibility, without providing a network of peers of equal standing to offer diverse perspectives, introduces significant vulnerabilities. Historical evidence suggests that rigidly autocratic leadership tends to be ineffective and often detrimental to the health and continuity of a business. Furthermore, excessive bureaucratization - characterized by inflexible managerial structures and procedural rigidity - is a widespread issue in Italian companies *(Guatri L., 1995)*. The lack of organizational flexibility, coupled with an inability to adapt to change and a failure to set ambitious goals, often prevents businesses from implementing more adaptable structures and responsive production processes, which can have severe and lasting consequences

Entrepreneurial acumen is a critical factor for success, yet its absence or improper application can have substantial and damaging repercussions. Within a company, all stakeholders, including entrepreneurs and managers, have specific interests, which may at times conflict with the collective good required for the organization's survival. Management decisions can be influenced by subjective considerations and a desire to increase the company's perceived value or to generate wealth without addressing structural weaknesses. Such decisions can arise from either intentional choices or from managerial limitations due to inadequate experience, skills, or knowledge *(Dell'Acqua A., 2017)*.

The concept of *"human capital"* holds particular significance in this context. Until the late 1970s, most crises were predominantly attributed to the human element, encompassing management, other key figures in the production process, and capital holders. These individuals were often characterized by excessive risk-taking, an intense desire for dividends, or a strong resistance to acknowledging financial distress *(Varvelli R. and Varvelli M.L., 1974)*.

Turning to quantitative factors, it is essential to make clear distinctions, as multiple variables contribute to the initial decline phase and the subsequent progression to a full-blown crisis. *Guatri (1995)* provides an objective classification of crises, categorizing them into five specific types:

- *Inefficiency-induced crises*: these arise from production inefficiencies, leading to lower returns and higher costs compared to competitors, thus affecting competitiveness and profitability. Inefficiencies can be identified by analyzing industrial costs, comparing performance with competitors, and assessing production capacity utilization. Factors such as outdated facilities, inadequate employee training, and limited investment in research and quality control are indicative of productive inefficiencies. Administrative inefficiencies, in turn, may stem from excessive bureaucracy, outdated information systems, and unresponsive management. Organizational inefficiencies are often the result of inadequate planning, lack of long-term strategies, and poor assignment of responsibilities. Commercial inefficiencies can arise from disproportionate investments in marketing and sales that do not yield the expected results. These various forms of inefficiency are challenging to detect but are significant contributors to crises;
- *Overcapacity and rigidity crises**:*** these crises are associated with excess production capacity and a lack of flexibility in adapting to market changes. They often result from factors such as reduced demand within an industry, shifts in consumer preferences, high barriers to market exit, and managerial strategies aimed at increasing market share or taking advantage of government incentives. The response to such crises depends on the company’s strength. Strong companies may increase market share in these situations, while weaker companies struggle with reduced demand. Addressing these crises often requires cost containment or elimination measures. Scenarios in which costs rise without corresponding revenue growth, often due to overly optimistic expectations, or where revenue growth fails to cover investment costs, can also lead to crisis, though such cases rarely escalate;
- *Crises stemming from product deterioration and marketing errors**:*** these occur when a company fails to meet consumer needs due to product decline or marketing deficiencies. Products that fail to capture consumer interest because they no longer align with market demands lose their competitive advantage. Competitors who adapt more effectively to changes in consumer preferences are likely to capture a larger market share, leading to a decline in sales and profit margins. Assessing an unattractive product mix involves analyzing gross and contribution margins, industry trends, product lifecycle, and international market factors. Diversified companies may offset risks by balancing profits

and losses across products, although established brands with loyal niche markets often withstand market downturns more effectively. Marketing-related crises can result from inconsistent product offerings, inadequate advertising, poorly defined target markets, and insufficient after-sales services, particularly for complex products;

- *Crises from an inability to adapt to environmental changes:* these crises emphasize the need for organizational flexibility to align with an evolving business environment. Successful adaptation requires proactive behavior, long-term profitability goals, and employee involvement in change processes to foster cohesion and commitment. Companies facing difficulties often rely on short-term planning, which can hinder long-term stability. Instead, companies should adopt a strategic approach by reevaluating and replacing outdated strategies to maintain a competitive edge. It is crucial not to persist with outdated choices when the operational context has shifted. While diversification can be beneficial, pursuing it without adequate expertise in non-core areas can create new problems. Sustainable growth requires financial viability, strong managerial capabilities, and periodic updates to products through research and innovation;
- *Crises from financial and equity imbalances:* these crises arise when a company relies heavily on short-term debt financing, leading to liquidity constraints and difficulties in meeting financial obligations. Such imbalances often reflect a misalignment between assets and liabilities and can cause significant economic losses *(Poddighe F. and Madonna S., 2006).* Although financial and equity imbalances do not independently trigger a decline, they are often the result of prior managerial decisions, market conditions, or mismanagement. They become key factors that contribute to the company's eventual failure *(Bastia P., 1996).*

While external factors, such as economic downturns or natural disasters, can exacerbate crises, internal decisions and actions play a substantial role. Companies must proactively respond to external changes, manage potential threats, and capitalize on opportunities to avoid or mitigate crises.

In summary, aside from distinguishing between quantitative and qualitative factors, it is also possible to differentiate between internal and external factors *(Sciarelli S., 1995).* Internal factors are primarily subjective, stemming from managerial decisions regarding resource allocation and strategic direction. In contrast, external factors can include uncontrollable environmental dynamics, such as natural disasters and global economic crises, as well as managerial choices regarding sectors, geographical markets, and supply sources. Effective crisis management, therefore, requires a comprehensive understanding of both internal and external dynamics, along with a balanced approach to quantitative and qualitative considerations.

3. *A BRIEF HISTORICAL OVERVIEW OF FINANCIAL RATIO ANALYSIS*

One of the earliest studies conducted by Euclid in 300 BC focused on using financial ratios to predict the likelihood of business failure. A more recent advancement in financial statement analysis can be traced back to the final stages of America's industrial growth in the late 19th century *(Horrigan J. O., 1968)*.

With the advent of financial loans, commercial banks in the 1870s began requesting financial statement for lending purposes, although this practice did not become widespread until the 1890s *(Foulke Roy A., 1961)*.

During this period, there was a notable increase in both the volume and availability of financial information *(Myer John N., 1961)*. Initially, this data was analyzed on an ad hoc basic, examining individual items. This was followed by the development of comparative columnar analysis and the distinction between current and non-current items. Gradually, the focus shifted to analyzing the relationships between different items *(Wall A., 1936)*.

In the late 1890s, a significant innovation in financial analysis emerged: comparing an enterprise's current assets to its current liabilities. This ratio, known as the *current ratio*, had a profound and lasting impact on financial statement analysis, surpassing the significance of other ratios developed during the same period. In essence, the use of ratios in financial statement analysis can be said to have originated with the introduction of the current ratio.

Significant advancements in ratio analysis took place in the period leading up to and during World War I. The period preceding war served as a catalyst for the development of ratio analysis. In 1912, Alexander Wall recognized the need for a broader range of ratios and relative criteria. To address this, he compiled a substantial sample of financial statements from the records of commercial paper brokers. His analysis culminated in his renowned 1919 report titled *"Study of Credit Barometrics" (Wall A., 1919)*.

In this study, Wall collected data from 981 firms, although he did not specify the time period covered. The firms were categorized by *industry* and *geographical location*, with nine subdivisions in each category. While Wall did not conduct further analysis on this data, he noted significant variations in ratios across different regions and business types. Although his study might be considered limited by today's standards, it was historically significant as it moved beyond the conventional approach of using a single ratio with an absolute criterion. Wall's work popularized the concept of utilizing multiple ratios and empirically derived relative ratio criteria.

In the 1920s, other analysts sought to add sophistication to ratio analysis. Notably, Bliss introduced a coherent system of interrelated ratios. He viewed ratios as indicators of fundamental relationships within a business and believed that competitive conditions would establish standard relationships. Based on these ideas, he developed a model of the firm composed entirely of ratios. This model intertwined ratios measuring cost, expense, turnover, and financial relationships with ratios assessing earnings. While Bliss's model and hypotheses may appear simplistic in hindsight, his work represented a significant early step toward the development of a theory of ratio analysis *(Bliss, James H., 1923)*.

The 1920s saw great enthusiasm regarding the potential of ratios as analytical tools. Interestingly, it was during this period that the first notable critic of ratios emerged. In 1925, Gilman raised several objections to the use of ratios. He argued that changes in ratios over time are difficult to interpret because both the numerator and denominator vary. He also described ratios as "artificial" measures and believed they could divert analysts' attention from taking a comprehensive view of the firm. Additionally, Gilman highlighted that the reliability of ratios as indicators varies significantly across different types of ratios (*Gilman S., 1925)*.

Gilman's critiques indicate that he held a contrasting viewpoint to Bliss and other proponents of ratio analysis. He did not believe that ratios accurately represented fundamental relationships within a business.

Despite the contributions of Bliss and Gilman, their insights were not further developed or expanded upon, leaving the potential impact of their work largely unrecognized.

In the 1930s, discussions of ratios and the compilation of industry average ratios continued uninterrupted. This decade saw an intensified focus on the empirical foundations of ratio analysis. Two significant developments emerged during this period. The first involved discussions in the literature concerning the identification of the most effective set of ratios to use. Roy A. Foulke played a prominent role in promoting his own set of ratios, which eventually grew to include fourteen ratios. Foulke's success in popularizing his ratios stemmed from his ability to provide annual industry data. While he began developing his set of ratios in the late 1920s, they gained widespread recognition only in the 1930s, with his publications starting in 1933. These became the most influential and well-known series of industry average ratios *(Foulke Roy A, 1931)*.

Foulke's work established a fundamental approach to ratio analysis in the United States, relying heavily on the authority of his experience in financial statement analysis rather than on theoretical or empirical support for the selection of ratios. His chosen ratios, sometimes accompanied by absolute and relative criteria, were endorsed based on their practical effectiveness in financial analysis *(Foulke Roy A., 1937).* This pragmatic approach, while satisfying practitioners' needs, left the field without a fully developed and testable theory.

However, another important development during this period served as a balancing force. In the early 1930s, researchers began exploring the predictive power of ratios for identifying financial distress. Winakor and Smith were pioneers in this area, analyzing a sample of firms that experienced financial difficulties between 1923 and 1931. They examined trends in twenty-one ratios over the preceding decade and concluded that the ratio of net working capital to total assets was the most reliable and consistent indicator of failure. They observed that a decline in this ratio typically began ten years before financial difficulties became apparent *(Winakor A. H. and Smith R.F., 1930).*

In a follow-up study, Winakor and Smith expanded their analysis to include 183 firms (*Winakor A. H. and Smith R.F., 1935).* However, this study had a significant limitation, as it lacked a control group of successful firms for comparison. This absence hindered the ability to establish a clear causal relationship between observed ratio trends and financial distress. Despite this limitation, their work marked a valuable contribution to the understanding of ratio analysis's predictive power.

In the 1930s, Paul J. Fitzpatrick analyzed a sample of failed industrial enterprises from 1920 to 1929, examining the trends of thirteen ratios over the previous three to five years for twenty companies to assess their predictive effectiveness *(Fitzpatrick P. J., 1931).*

Fitzpatrick later conducted a comparative analysis of a matched sample of nineteen successful firms alongside the previously studied failed firms. His findings indicated that while all ratios possessed some predictive power, ratios such as net profit to net worth, net worth to debt, and net worth to fixed assets generally performed better as indicators of failure.

In a separate study, Ramser and Foster analyzed 173 firms with securities registered in Illinois, examining eleven types of ratios. They found that less successful firms and those that eventually failed generally exhibited lower ratios compared to more successful firms. However, two turnover ratios - sales to net worth and sales to total assets—showed a contrasting trend *(Ramser J. R. and Foster L. 0., 1931).*

These studies had limitations. Fitzpatrick's sample size was small and potentially biased, while Ramser and Foster's study revealed differences in average ratios that were sometimes more apparent than substantive. Nonetheless, these studies represented a pivotal moment in the advancement of ratio analysis, marking some of the first rigorous attempts to employ scientific methodology to assess ratio effectiveness.

In the early 1940s, economist C.L. Merwin conducted research on the financing of small corporations across five manufacturing industries from 1926 to 1936. He analyzed the trend of numerous ratios over a six-year period for both "continuing" and "discontinuing" firms, comparing industry average ratios for "discontinuing" firms with "estimated normal" ratios *(Merwin C. L., 1942).* Merwin's objective was to identify ratios that could serve as sensitive indicators of potential business failure, in some cases up to four or five years in advance. Based on his findings, Merwin highlighted three ratios - net working capital to total assets, net worth to debt, and the current ratio - as particularly reliable measures of a company's financial health and potential for future success or failure.

Thus, after approximately fifty years of application, certain financial ratios were formally validated as effective indicators. Extensive studies by researchers such as Merwin provided empirical evidence supporting the use of specific ratios as valuable tools in financial analysis. This recognition marked an important milestone in the development of ratio analysis as a legitimate approach for evaluating business health and performance.

He compared the industry average ratios of "discontinuing" firms with "estimated normal" ratios. His objective was to identify ratios that could serve as highly sensitive indicators of potential business discontinuance, with the ability to predict such outcomes even four to five years in advance in some cases. Based on his findings, Merwin highlighted three ratios as particularly significant indicators: net working capital to total assets, net worth to debt, and the current ratio.

These ratios were considered reliable measures of a company's financial health and its potential for future success or failure.

Thus, the formal vindication of certain ratios came after approximately fifty years of their existence. Through extensive studies and analyses conducted by researchers like Merwin and others, the predictive power and significance of specific ratios were substantiated. These findings provided empirical evidence supporting the use of ratios as valuable tools in financial analysis.

The recognition of these ratios as reliable indicators marked an important milestone in the development and acceptance of ratio analysis as a legitimate and effective approach for evaluating the financial health and performance of businesses.

In the early 1940s, there has been a notable increase in the emphasis placed on the role of ratios in the operations of small businesses. Ratios have gained recognition as valuable tools for small business owners and managers to assess and monitor their financial performance, make informed decisions, and identify areas for improvement. The accessibility of financial information and the availability of ratio analysis tolls have contributed to the widespread adoption of ratio analysis in the small business operations (*Sanzo R., 1960).*

During this period, ratios were not only used in traditional financial analysis, but also expanded their role as variables for examining and describing economic activity. Researchers conducted additional evaluations to assess the predictive power of ratios in various contexts.

Hickman's study focused on corporate bond issues from 1900 to 1943 and found that the *times-interest-earned ratio and the net profits to sales ratio* were useful predictors of default experiences. These ratios provided insights into the financial health and stability of companies issuing corporate bonds *(Hickman W. B., 1958).*

Similarly, Saulnier and other researchers examined the lending experience of the Reconstruction Finance Corporation (RFC) from 1934 to 1951. They discovered suggestive evidence that borrowing firms with poorer current ratios and net worth to debt ratios were more prone to loan defaults. These findings indicated that certain ratios could serve as indicators of creditworthiness and loan repayment ability *(Saulnier R. J., Halcrow H. G., and Jacoby N. H., 1958).*

By using ratios in these studies, researchers deepened their understanding of the relationships between financial ratios and economic outcomes, further expanding the empirical base of ratio analysis in assessing financial performance and risk across different contexts.

In the 1960s, studies by Beaver (1966) and Altman (1968) had a significant impact on bankruptcy prediction and assessing businesses' financial well-being. These works introduced statistical methodologies and models to predict bankruptcy before it occurred, providing businesses with proactive monitoring tools to gauge financial health and mitigate potential risks.

3.1. Beaver Model

The earliest research on forecasting corporate insolvency dates back to the 1930s, with studies grounded in *Univariate Statistical Models*. These early analyses compared the financial indices of companies classified as *"sick"* with those of companies categorized as "*healthy".*

A notable development in this field emerged with the work of Beaver (1966)**,** who applied a *Univariate Discriminant Analysis Model* to study corporate insolvency by examining individual financial variables. One significant advantage of this approach lies in its simplicity: it can be applied without requiring advanced statistical knowledge. Essentially, Beaver's method involved comparing a company's financial ratio values against a designated threshold, or *cut-off point*. If a company's score exceeded this threshold, it was deemed “healthy”; otherwise, it was classified as “unhealthy” *(Beaver W.H., 1966)*.

Beaver selected seventy-nine industrial firms that met predefined criteria for failure from *Moody’s Industrial Manual* and a list of failed companies compiled by *Dun and Bradstreet*. Balance sheet data for these failed firms was gathered for the year prior to their bankruptcy. The companies were organized by industry and asset size to ensure comparability.

For contrast, Beaver also assembled a matched sample of non-failed firms, selected based on similarities in industry and asset size to the failed firms. Data for these healthy firms was gathered for the five years leading up to the corresponding period of failure, and *thirty financial ratios* were calculated for analysis. The criteria for selecting these ratios included their prevalence in the existing literature, their performance in prior studies, and their relevance to insolvency concepts. If a ratio satisfied any one of these criteria, it was incorporated into Beaver’s analysis.

Beaver’s approach was grounded in a *cash-flow model*, which he used as a theoretical framework for interpreting the effectiveness of ratio-based tests. This model conceptualized a firm as a *“reservoir of liquid assets”* that experiences both inflows and outflows. According to this view, the firm’s solvency is assessed by examining the likelihood of depleting its reservoir of liquid assets *(Beaver W.H., 1966)*.

From this model, Beaver proposed several key hypotheses:

- a larger reservoir of liquid assets correlates with a lower probability of failure;
- a greater net inflow of liquid assets from operations (i.e., positive cash flow) is associated with a lower probability of failure;
- a higher debt load corresponds with a greater likelihood of failure;
- an increase in operational fund outflows heightens the probability of failure.

These propositions served as the basis for Beaver’s empirical analysis, which evaluated the predictive power of financial ratios in forecasting corporate failure. This pioneering work laid the foundation for future advancements in insolvency prediction models and marked an important step in the development of financial ratio analysis as a diagnostic tool.

3.1.1. Empirical results

Based on the empirical results, the data analysis followed three main steps: comparing mean values, conducting a dichotomous classification test, and analyzing likelihood ratios.

The first step involved comparing the *mean values* of financial ratios across different groups or categories. This comparison aimed to identify significant differences in the average values of the ratios between these groups, thus providing insight into patterns of financial health and distress. Next, a *dichotomous classification test* was conducted, which involved sorting the data into two groups based on a specific threshold or criterion. This test was used to classify companies as either failed or non-failed based on the values of selected ratios. Finally, *likelihood ratios* were analyzed. These are statistical measures that estimate the probability of observing certain data under different hypotheses or models, which in this case helped assess the likelihood of specific financial outcomes or relationships.

The analysis ultimately reduced the initial set of ratios to a core group of *six ratios,* each identified as the most effective predictor within its category. These six ratios were:

1. cash flow to total debt;
2. net income to total assets;
3. total debt to total assets;
4. working capital to total assets;
5. current ratio;
6. no-credit interval.

The mean values of these ratios, when compared between failed and non-failed firms, showed noticeable differences, aligning with the four propositions outlined earlier. This finding suggests that these financial ratios reveal meaningful distinctions between the two groups. However, it is important to note that mean value comparisons alone do not provide complete information regarding the extent or variability of differences within each group. Therefore, mean comparisons alone cannot serve as a reliable basis for predicting outcomes.

To further refine the predictive analysis, a *dichotomous classification test* was applied. This test functions as a univariate discriminating tool, evaluating each ratio individually. For each ratio, a cutoff point was chosen to divide the two groups, aiming to minimize classification errors. Ratios exceeding the cutoff point classified a firm into one group, while those below it assigned the firm to the other group. Among the ratios, the *cash-flow to total debt ratio* emerged as the most effective predictor, demonstrating the highest success rate in correctly classifying firms.

To validate the effectiveness of the dichotomous classification test, a *holdout sample* was used. This sample, a randomly selected subset of the data, served as a test group to measure error rates. The error rate for the first year before failure was 13 percent, which increased to 22 percent five years prior to failure. When compared to a naïve prediction model, which would have an error rate of 50 percent, these error rates (13 percent and 22 percent) represent a considerable improvement, indicating that the model using selected ratios is performing significantly better than random chance at identifying firms at risk of failure.

Among the ratios analyzed, the *net income to total assets ratio* was identified as the second most effective predictor, following cash-flow to total debt. This ratio demonstrated considerable strength in distinguishing between failed and non-failed firms. The remaining ratios, ranked in decreasing order of predictive power, were total debt to total assets, working capital to total assets, current ratio, and no-credit interval. While these ratios had slightly lower predictive abilities compared to the top two, they still contributed to the overall robustness of the model *(Beaver W.H., 1966).*

It is noteworthy that none of the ratios showed a strong correlation with asset size, suggesting that the predictive power of these ratios is relatively independent of the firms' asset sizes. This observation implies that the variation in asset size does not significantly impact the effectiveness of these financial ratios as indicators of firm health in this analysis.

When analyzing prediction errors, both *Type 1* and *Type 2 errors* are relevant, as each carries different implications. In this context, a *Type 1 error* involves misclassifying a failed firm as non-failed, while a *Type 2 error* involves classifying a non-failed firm as failed. Given that the consequences of these errors may differ substantially, evaluating the overall error rate alone is not sufficient, especially when the costs of *Type 1* and *Type 2* errors are not equal. In practice, the financial and strategic implications of each error type can vary significantly, and it is important to account for this disparity in any decision-making process based on the analysis.

In Beaver's analysis, the *Type 1 error* rate was found to be 22 percent, indicating a relatively high chance of incorrectly identifying a failed firm as non-failed. In contrast, the *Type 2 error* rate was only 5 percent, meaning there was a lower likelihood of wrongly classifying a non-failed firm as failed. Additionally, the analysis revealed that while the *Type 2 error* rate remained relatively stable over the five-year period, the *Type 1 error* rate increased as the time before failure extended. This suggests that as a firm approaches failure, it becomes increasingly challenging to accurately identify it as such, which could indicate early signals of financial distress may become harder to detect with increased time horizons.

In light of the asymmetry in error rates and the potential asymmetry in the associated costs, the analysis suggests that adjusting the *cutoff point* could be beneficial. By fine-tuning the cutoff threshold, the classification model can better accommodate the differing costs and risks of *Type 1* and *Type 2 errors*. Such adjustments allow the model to align more closely with specific business contexts and decision-making requirements, ensuring that the relative importance of correctly identifying both failed and non-failed firms is appropriately accounted for.

Overall, Beaver's study highlights the importance of careful selection and evaluation of financial ratios to predict corporate failure, illustrating the effectiveness of univariate analysis for forecasting insolvency while also acknowledging the need for nuanced consideration of error costs in predictive modeling.

3.1.2 Limitations of the model

Beaver's study on bankruptcy prediction achieved two key milestones in the field of financial analysis. First, he demonstrated that it is possible to obtain reasonably accurate predictive results using a relatively simple univariate model. This simplicity in approach made the model more accessible and practical. Second, he went a step further by attempting to provide a theoretical rationale for the empirical results observed, laying an intellectual foundation for understanding the predictive power of various financial indicators.

Beaver's study revealed that ratios with the strongest predictive abilities were those related to a company's financial structure and cash generation capacity. In contrast, ratios that focused on current assets and liquidity displayed lower explanatory power. This finding suggested that a firm's broader ability to generate cash and manage debt had a more profound impact on its likelihood of avoiding failure than its short-term liquidity. However, Beaver's definition of cash flow – as the sum of net income and cash costs – is somewhat incomplete. This approach fails to account for the combined effect of interest income and expenses tied to financial management. Additionally, it overlooks the net changes in working capital and the impact of investments or disinvestments in fixed assets, both of which are essential factors in accurately assessing a company's cash flow position and financial health.

The choice of ratios based on their popularity in previous studies is also problematic. This method introduces several limitations:

1. *lack of theoretical rationale:* effective research or model building requires a strong theoretical basis guiding the selection of variables. In financial ratio analysis, it is essential to understand how specific financial metrics correlate with a firm's performance and insolvency risk. Without a well-founded rationale, there is a risk of including ratios that may not contribute meaningfully to predictive accuracy, leading to an incomplete or even misleading model;

2. *“window dressing” and bias: “window dressing”* describes a practice where companies manipulate their financial statements to make performance appear more favorable than it actually is. When well-known ratios are used for predictive models, companies may try to influence these specific ratios to give a more favorable impression to stakeholders. This focus on popular ratios could thus distort their predictive ability, creating the potential for inaccurate forecasts and skewed conclusions;
3. *limited coverage*: relying on popular ratios alone may exclude important indicators that could be more relevant to the specific research question or industry context. For example, industry-specific ratios or lesser-known metrics might offer valuable insights that are overlooked when prioritizing only widely-used ratios;
4. *single-ratio limitations:* in a univariate approach, each ratio is evaluated independently. This method limits the analysis by not incorporating the combined information available from multiple ratios. Important insights may be missed by not considering how different financial metrics interact, which could provide a more nuanced understanding of a firm’s financial condition;
5. *potential for conflicting classifications*: since each ratio is assessed separately, it is possible to obtain conflicting classifications for the same firm. For instance, one ratio might suggest the firm is financially stable, while another might indicate financial distress. This inconsistency can complicate interpretation, making it challenging to draw clear, cohesive conclusions about a firm’s overall financial health;
6. *high correlation among variables:* financial ratios are often highly correlated with one another, as they may capture similar aspects of a company’s financial status. Using multiple, highly correlated ratios in a univariate analysis may lead to redundancy and limit the depth of insights gained from the data *(Tatsuoka M. M., 1970);*
7. *limited generalization across industries*: popular ratios may not apply equally well across different industries. Financial structures and operating dynamics can vary significantly by sector, and applying the same set of ratios across industries may not yield accurate or meaningful conclusions.

Despite these limitations, it is important to recognize that the univariate approach has its merits, especially as a preliminary step. It serves as a foundational model that can lead to more comprehensive multivariate approaches, where relationships among multiple ratios are considered simultaneously. Multivariate models allow for a richer, more accurate assessment of financial performance and can integrate information that might be lost in univariate analysis.

Beaver's study is of considerable significance in the field of ratio analysis. Like Merwin's research, Beaver's work demonstrates that certain financial ratios possess the potential to predict business failure up to five years in advance. However, Beaver's study differs from Merwin's in two important ways. First, Beaver employed more sophisticated statistical techniques, which allowed for a deeper, more robust analysis of the data. Second, Beaver included some ratios derived from funds statement data, thereby incorporating a broader range of financial insights into his analysis.

Beaver's use of advanced methodology and his thorough findings suggest that his study will be regarded as a seminal work in future research on ratio analysis. This research represents a significant contribution to our understanding of financial ratios as predictive tools for assessing the financial health and risk of failure of companies. His study has paved the way for more advanced predictive models that continue to shape how analysts and practitioners approach financial forecasting and insolvency prediction.

3.2. The Evolution of Altman Z-Score

Building upon the foundations set by earlier studies in bankruptcy prediction, subsequent research made significant advances by incorporating *Multiple Discriminant Analysis (MDA)***.** A key figure in this progression was E.I. Altman, who in 1968 introduced multivariate statistical techniques into the analysis of corporate financial health in the United States, leading to the development of the now-famous *Z-Score Model (Altman E.I., 1968, 2000).*

One of the most notable contributions of Altman's work was the creation of the *Z-score model***,** which synthesized multiple financial ratios into a single composite score. This score served as an indicator of a firm's likelihood of bankruptcy, enabling a more holistic view of financial risk than any single ratio could offer. By combining ratios associated with liquidity, profitability, leverage, and efficiency, Altman's model allowed for a nuanced assessment of a company's overall financial stability. This innovation in financial analysis introduced a multivariate approach that proved to be a breakthrough in predicting corporate failure, transforming how financial ratios were used to assess business health.

The Z-score model represented a major shift from earlier univariate models, such as those introduced by Beaver, to a multivariate framework. This shift allowed financial managers to view a firm's financial situation more comprehensively, as the model accounted for the interplay between multiple financial metrics. Altman's work, in conjunction with Beaver's pioneering studies, thus established a more proactive approach to financial management, providing managers with tools to identify potential signs of distress early on, rather than responding only once a company was already in crisis. By using these predictive tools, managers gained the ability to take preemptive action, potentially averting severe financial consequences.

The introduction of such quantitative tools offered *objective measures* for evaluating corporate financial health. Previously, much of financial assessment was qualitative or based on intuition. Altman's and Beaver's models provided an empirical framework that transformed the way financial risk was quantified, allowing managers, investors, and other stakeholders to make data-driven decisions. The Z-score model, in particular, allowed for the identification of early warning signs, which in turn enabled stakeholders to intervene before financial difficulties escalated into full-blown crises.

Altman's methodologies have had a lasting impact on both financial theory and practice, stimulating a vast body of *subsequent research* aimed at refining and enhancing predictive accuracy. Building on the principles established by Beaver and Altman, researchers have continued to develop models that increase precision in forecasting financial distress, adapting these frameworks to account for evolving financial conditions, industry-specific factors, and global economic fluctuations. Altman's Z-score model, often regarded as a foundational tool in predictive financial analysis, laid the groundwork for advancements in risk assessment techniques, from sector-specific models to those that address international markets.

The shift from univariate to multivariate analysis in the work of Altman and his peers marked a fundamental evolution in the field of financial analysis. This transformation has not only enabled earlier and more accurate detection of financial distress but has also provided a quantitative foundation for proactive financial management, thus enhancing the ability of organizations to safeguard against potential bankruptcy.

3.2.1. Empirical results

The creation of Altman's Z-score model involved an extensive study of *66 diverse firms*, focusing exclusively on larger corporations and excluding small and medium-sized enterprises. Of these companies, *33* had experienced bankruptcy between 1945 and 1965, while the remaining 33 firms were healthy, randomly selected from reputable financial sources such as *Moody's*. Altman's sample was deliberately balanced to compare firms that had encountered financial distress with those that had remained stable, enabling a robust analysis of the factors that might predict bankruptcy.

The model's development began with an initial list of *twenty-two financial ratios* identified as potentially relevant to a firm's financial health. These ratios were organized into five primary categories: *liquidity, solvency, profitability, turnover, and operational efficiency*. However, not all twenty-two ratios were retained in the final model. Through a methodical selection process, Altman excluded ratios that did not emerge as strong predictors of financial distress. Interestingly, one variable that Beaver had considered highly predictive – the *cash flow to total debt ratio* – was initially excluded from Altman's analysis due to the lack of consistent and reliable depreciation data, which could have skewed the results.

The process of refining the model involved several key steps, each designed to maximize the *statistical significance* and *predictive power* of the model. First, Altman evaluated the statistical relevance of each discriminating variable to ensure that only the most meaningful ratios were included. Second, he examined *intercorrelations* among the variables, seeking to reduce redundancy and retain only those ratios that provided unique information. Altman then assessed the predictive accuracy of the *discriminant function* itself, a statistical tool that enables the categorization of firms based on financial characteristics. Throughout this process, Altman exercised *professional judgment* to fine-tune the model, balancing statistical rigor with practical applicability.

Ultimately, Altman's Z-score model calculates a single composite score – the *Z-score* – based on a set of carefully selected financial ratios that reflect various aspects of a firm's financial performance and stability. By employing *discriminant analysis*, a statistical method that optimally distinguishes between distinct groups, Altman's model estimates a company's probability of bankruptcy. This technique allows firms to be classified into two predefined categories, *bankrupt and non-bankrupt***,** based on observed financial characteristics. The model's objective is to assign each firm to one of these categories as accurately as possible by leveraging a combination of financial ratios that serve as discriminating variables.

The Z-score model's influence extends well beyond academia; it has found extensive application across industries and regions as a tool for assessing financial risk, evaluating *creditworthiness***,** and guiding *investment decisions*. Its appeal lies in the model's ability to integrate multiple aspects of financial performance into a single, interpretable score. This score provides an early-warning indicator of financial distress, allowing managers, investors, and lenders to anticipate potential problems before they manifest in actual bankruptcy.

Like many classification models used for early detection of corporate insolvency risk, the Z-score model relies on *discriminant statistical analysis*. This method classifies a statistical sample into two or more predefined groups with minimal error. In Altman's model, these groups are "bankrupt" and "non-bankrupt" companies, with classification based on a firm's financial ratios, which serve as discriminating variables. These ratios are derived from *balance sheet and income statement data*, offering a comprehensive snapshot of a company's financial health. Each ratio captures a distinct dimension of the firm's performance and solvency, together providing a well-rounded measure of financial stability.

The final version of Altman's Z-score model utilizes *five key financial ratios*, each selected for its ability to capture an essential aspect of the company's financial standing. These ratios collectively assess a company's liquidity, profitability, leverage, and operational efficiency, all of which are crucial indicators of its resilience to financial stress. By incorporating these various ratios, the Z-score model not only evaluates a company's current financial position but also offers a predictive outlook on its risk of failure.

Altman's methodological rigor and the practical applicability of his model have established the Z-score as a foundational tool in financial risk assessment. The model's approach—combining multiple financial ratios within a discriminant analysis framework—has provided managers, analysts, and researchers with a powerful tool for *early diagnosis of financial distress*.

The Z-score's widespread acceptance attests to its effectiveness in delivering reliable insights into corporate solvency, thereby enabling stakeholders to make proactive and informed decisions.

The Altman Z-score formula (1968) is as follows:

$$Z\text{ - score} = 1.2X_1 + 1.4\,X_2 + 3.3X_3 + 0.6X_4 + 0.999X_5$$

Where:

X_1 *(Liquidity)* = Working Capital/Total Assets. This ratio measures the firm's ability to cover its short-term obligations with its current assets;

X_2 *(Cumulative profitability)* = Retained Earnings/Total Assets. This ratio reflects the proportion of the firm's total assets that are financed by retained earnings, which indicates the level of profitability and accumulated earnings;

X_3 *(Profitability)* = Earnings Before Interest and Taxes (EBIT)/Total Assets. This ratio evaluates the company's operating profitability in relation to its total assets;

X_4 *(Leverage)* = Market Value of Equity/Total Liabilities. This ratio considers the market value of the company's equity relative to its total liabilities, providing insight into the market's perception of the company's financial strength.

X_5 *(Capital Turnover Ratio)* = Sales/Total Assets. This ratio assesses the efficiency of the company in generating sales revenue relative to its total assets.

In Altman's Z-score model, each financial ratio is multiplied by a specific *weight (or coefficient)***,** and the results are then summed to yield the *Z-score*. These coefficients were derived through statistical analysis of a sample of both bankrupt and non-bankrupt companies, carefully calibrated to maximize the model's accuracy in distinguishing between financially healthy and distressed firms. This approach enables the Z-score to act as a comprehensive metric that encapsulates various aspects of a company's financial health in a single numerical value.

Altman is clear in emphasizing that his model is *descriptive-comparative*, rather than purely probabilistic. In other words, while the Z-score provides an indication of bankruptcy risk, it does so by comparing a firm's financial profile to those of historically bankrupt and non-bankrupt companies, rather than directly predicting probabilities of failure. The model is thus designed to provide a snapshot of a firm's financial health at a given moment, rather than to forecast indefinite future risk. However, the model's reliability declines as the time horizon increases. Beyond two years, the risk of *classification errors* rises considerably, making the Z-score less effective for long-term bankruptcy prediction.

Both *Type 1 and Type 2 errors* were assessed in Altman's model, reflecting the risks of false positives and false negatives, respectively. In the original sample classification, the model's *Type 1 error rate* - the probability of incorrectly identifying a healthy firm as bankrupt - was 6%. This relatively low error rate indicates a high level of accuracy in distinguishing non-distressed firms. The *Type 2 error rate* - the risk of classifying a bankrupt firm as healthy - was even lower, at 3%. These figures highlight the model's strong initial predictive accuracy when applied one year prior to bankruptcy, with 94% of classifications being correct.

In further testing, Altman expanded the timeframe to two years before the bankruptcy date to examine the model's effectiveness over a longer period. Here, the *Type 1 error* increased significantly to 28%, indicating a much higher probability of mistakenly classifying healthy firms as distressed. The *Type 2 error* also rose, though more modestly, to 6%, reflecting a slight increase in the likelihood of incorrectly identifying bankrupt firms as financially stable. These results suggest that while the Z-score model retains some predictive power two years out, its accuracy diminishes, particularly in distinguishing healthy firms from those at risk.

Altman's research further demonstrated that the *accuracy of the model's classifications* declines as the time horizon extends. Specifically, the percentage of correct classifications drops from 94% one year prior to bankruptcy to just 36% five years out, indicating a steep decline in reliability. Conversely, the rate of *incorrect classifications* rises from 6% to 64% over the same period. These findings underscore that the Z-score's predictive accuracy is strongest within a *one-to two-year window* before a bankruptcy event and becomes increasingly unreliable beyond that timeframe *(Altman E.I., 1968)*.

The Z-score, therefore, provides a *single numerical value* that enables a clear assessment of a firm's financial health and bankruptcy risk. The higher the Z-score, the lower the probability of bankruptcy, creating a straightforward metric for evaluating a company's financial position. Altman's research defined specific *Z-score ranges* to categorize firms into different risk zones: the *safe zone*, the *grey zone*, and the *distress zone*. Each zone reflects a different level of financial distress, providing a practical framework for decision-makers to assess risk.

The *threshold value* identified to differentiate between healthy and distressed companies is 2.675. Firms with Z-scores above *3* are classified in the *safe zone*, indicating financial stability and a low likelihood of bankruptcy. For companies with Z-scores between *2.99* and *1.81*, the model places them in the *grey area* or *zone of uncertainty*. Firms in this range are not clearly classifiable as either healthy or distressed based on the model's criteria, making continuous monitoring advisable. The grey area represents a statistical ambiguity, as firms here exhibit financial characteristics that are neither distinctly stable nor clearly indicative of imminent failure. Consequently, while these companies may not yet be at high risk, the model cannot confidently predict their future financial health, thus requiring close attention and periodic reassessment.

Finally, if a firm's Z-score falls below *1.8*, it is categorized in the *distress zone*, with a high probability of failure. Firms in this range display financial characteristics closely aligned with those of historically bankrupt companies, making them statistically more likely to experience financial distress. The model's findings in this zone indicate severe financial instability and signal an urgent need for corrective action to prevent possible insolvency.

Altman's Z-score model has had a profound impact on financial analysis, offering a clear, data-driven method for assessing bankruptcy risk. By establishing defined zones and thresholds, the model provides a practical tool for managers, investors, and creditors to gauge a firm's financial health and to identify early warning signs of financial distress. This categorization framework facilitates timely interventions, allowing stakeholders to respond proactively to financial risks rather than waiting for a crisis to fully unfold. Thus, the Z-score model remains a foundational tool in both academic research and practical financial analysis, setting a standard for predictive modeling in corporate finance.

Z-Score Classification Areas

Insolvency Area (High Risk of Bankrupt)	Grey Area (Uncertain Results)	Low Risk Area (Healthy)
Z < 1,81	1,81 < Z < 1,81	Z > 2,99

Z-cutoff = 2,675

(Source: Danovi A. and Quagli A., 2008, pp. 164)

Altman's Z-score model has proven effective in forecasting corporate insolvency within a relatively short time frame, specifically up to *two years before bankruptcy* is likely to occur. The model demonstrates a *prediction error rate of approximately 15%* when applied one year before failure, and this error rate increases slightly to *17% when forecasting two years in advance*. These figures indicate that, while the Z-score provides reasonably accurate predictions, it is best suited for short-term insolvency risk assessment rather than for long-range forecasting.

While the Z-score has gained recognition as a valuable tool in financial analysis, it is important to understand its *intended function and limitations*. Rather than being a purely predictive model, the Z-score is better understood as a *"warning"* or *early-alert tool* (*Bisogno M., 2012; Teodori C., 1989).* The model does not predict insolvency with absolute certainty; instead, it evaluates a firm's financial health relative to predefined categories, placing it closer to either a stable group or a distressed group. This comparative nature of the Z-score model allows financial analysts and decision-makers to assess the *relative risk* of insolvency and to gauge the firm's financial position within a spectrum of health and risk.

3.2.2. Limitations of the model

Despite its widespread popularity in the finance world, the *Z-score model* exhibits several significant limitations that impact its applicability and accuracy *(De Luca F., 2017).*

One primary limitation is that the model was initially developed for *publicly traded companies in the specific manufacturing sector* of 1960s America. This context-specific origin restricts its applicability when applied to firms outside this scope, such as those in different industries, smaller enterprises, or firms in non-U.S. markets. Without considerable modification, the model's validity is likely to diminish when applied to companies that do not share these original characteristics.

A further limitation of the Z-score model is its omission of *intangible factors* that can materially influence a company's performance. Factors such as management quality, brand reputation, intellectual property, and market positioning - all of which can significantly impact a firm's financial health - are not accounted for in the model. As a result, the Z-score provides only a partial evaluation of a company's financial stability, overlooking critical non-financial elements that could contribute to a comprehensive analysis.

Additionally, the model appears somewhat *detached from broader economic conditions*, which limits its contextual relevance. The Z-score does not integrate macroeconomic factors, such as interest rate changes, inflation, or sector-wide trends, that could influence a firm's financial resilience. This detachment may lead to less accurate assessments, particularly in times of economic volatility.

Moreover, the model does not consider a company's *ability to secure financing* from external sources. A firm with strong credit relationships or investor confidence may be able to sustain operations even when experiencing financial difficulties, yet this resilience is not reflected in the Z-score. By excluding access to capital as a factor, the model overlooks a key component of financial stability, especially in cases where external support could mitigate or delay insolvency.

The *reliability of the Z-score also diminishes over longer time horizons*. Although the model is effective in predicting financial distress up to two years in advance, its predictive accuracy declines sharply beyond this timeframe *(De Luca F. and Meschieri E., 2017).* This limitation reduces its utility as an early warning system, making it less effective for identifying crises that could be mitigated with timely intervention.

When compared to *Beaver's univariate approach*, Altman's Z-score demonstrated higher accuracy, particularly in assessing data from the year immediately preceding bankruptcy. However, while Beaver's method was able to accurately predict bankruptcy up to five years before failure, the Z-score's accuracy decreases substantially after the first year before bankruptcy, reflecting a more limited capacity for long-term prediction.

The use of multiple financial ratios in a *multivariate approach* may introduce additional challenges. Issues such as data quality, sample specificity, and non-compliance with the statistical assumptions of discriminant analysis can lead to lower accuracy and potentially biased outcomes. These complications suggest that the multivariate nature of the Z-score model, while theoretically advantageous, may introduce practical constraints that limit its generalizability.

Finally, the selection of variables within the model is primarily driven by their contribution to *enhancing discriminative power*. This focus on optimizing discrimination for a specific sample does not necessarily imply that the selected variables are universally relevant in predicting corporate failure. As *Edmister (1972)* notes, alternative functions of similar predictive quality could potentially be constructed by including other variables, particularly those initially excluded due to high correlation with the ones included. This sample-specific optimization may limit the model's ability to highlight unique characteristics of failing firms that warrant close attention *(Edmister R.O., 1972)*.

In conclusion, while the Z-score model is a valuable tool for predicting corporate insolvency, its limitations necessitate careful consideration of its context, sample applicability, and the potential need for adjustments when used outside its original scope.

3.3.Edmister Model

Edmister sought to apply a methodology similar to Altman's in predicting small business failure. However, he recognized that including multiple closely correlated variables could introduce *bias* into the function, thereby limiting its *reliability* to samples closely resembling those used in its development. As a result, Edmister acknowledged that the *predictive validity* of the study was largely confined to its specific sample group - namely, *loan applicants to the Small Business Administration (SBA)*, from which the sample was derived *(Edmister R.O., 1972)*.

The study incorporated financial ratios previously identified in empirical research as significant predictors of bankruptcy, though some ratios were omitted due to *data limitations* within the sample. This selective inclusion underscores the challenge of applying predictive models across different populations, particularly when *data availability and sample characteristics* do not align with those of the original study. Consequently, while Edmister's approach provides valuable insights for assessing bankruptcy risk among SBA loan applicants, its broader applicability remains constrained by these methodological considerations.

The study tested four types of hypotheses regarding ratio analysis.

H1: the level of a ratio indicates failure;
H2: the trend of a ratio indicates failure;

H3: the average of a ratio over a five-year period indicates failure;
H4: a combination of industry-relative trend and industry-relative level of a ratio indicates failure.

The fourth hypothesis explores the *conditional nature of ratio analysis*, an aspect previously unexamined in empirical research. Testing this hypothesis would involve analyzing *interaction effects* between two variables within a linear model, potentially using multiple discriminant analysis. This method aims to more accurately reflect the actual ratio-analytic process. However, it is essential that the interaction effects are properly specified to ensure the validity and reliability of the results.

3.3.1. Empirical results

In this study, each of the relative ratios was tested across four combinations of *trend* (up/down) and *level* (high/low). A carefully selected subset of 152 combination variables was analyzed to examine their potential role in predicting small business failure. By incorporating these interaction effects, the researchers aimed to gain a more nuanced understanding of how different trends and levels in financial ratios might jointly signal an elevated risk of business failure.

To mitigate the issue of *multicollinearity*, a *stepwise inclusion* approach was applied to the discriminating variables. Variables exhibiting a correlation greater than *0.31* with another variable were excluded from the model. In the final discriminant function, none of the retained variables displayed inter-correlations exceeding *0.23*. While this conservative approach may slightly diminish the model's discriminative efficacy, it minimizes the risk of generating *sample-specific results*, thereby enhancing the robustness of the findings.

The sample of firms with one year of financial data was sufficiently large to allow for validation using a *hold-out sample*. However, relying solely on one year of data proved inadequate for developing a highly accurate discriminant function. In contrast, the sample of firms with three years of data was relatively small, necessitating a validation process involving the *random reassignment of cases* to the two groups. This reassignment procedure was used to evaluate the discriminant function's effectiveness, as the random allocation of cases helped assess the model's *discriminative power*. It is essential to note, however, that this test does not assess *external validity*, as the intrinsic characteristics of the cases remained unchanged *(Zavgren C.V., 1981)*.

3.3.2. Limitations of the model

Furthermore, the ability to generalize the findings to a larger population from the sample is limited due to the small size available for validation and randomization. This means that there are constraints in drawing broader conclusions from the study's results, given the limited data available for analysis and testing. Edmister's study focused specifically on small businesses and sought to address the issue of *multicollinearity* by eliminating highly correlated variables and employing a *stepwise approach* for the selection of discriminating variables. While this method reduces the risk of instability and sample-specific outcomes that are more prominent in Altman's model, it introduces a degree of *arbitrariness* in the function. The choice of which variable to retain from a set of related, highly correlated variables is somewhat subjective, meaning that the resulting discriminant function lacks *uniqueness*. Consequently, although Edmister's approach may improve model stability, it does not fully resolve the problem of establishing *theoretical significance* for the selected variables.

Moreover, the study's capacity for *generalization* is constrained by the limited sample size available for *validation* and *randomization*. This limitation restricts the study's ability to extrapolate its findings to a broader population of small businesses. The small sample size poses challenges for drawing robust, generalizable conclusions, as the available data may not provide a comprehensive representation for analysis and testing. Consequently, while the study offers valuable insights into bankruptcy prediction for small businesses, its findings should be interpreted with caution regarding their applicability to a wider population.

3.4. Revisiting the Altman Z-Score Model

Following the initial development of the *Altman Z-Score model* for publicly traded manufacturing companies, subsequent research highlighted the potential and importance of adapting the model for use with *non-publicly traded firms* and *non-manufacturing sectors*. This recognition spurred additional studies and led to modifications aimed at broadening the model's applicability across a wider array of business types. In response, Altman introduced the *Z'-Score*, an adjusted version of the original model specifically tailored for privately held companies. This adaptation was designed to enhance the model's *diagnostic capability* by accounting for the financial characteristics unique to non-public firms, thereby expanding its usability and relevance in diverse economic sectors *(Altman E. I., 1993).*

To adapt the model for assessing *non-publicly traded firms*, specific modifications were implemented. Notably, the *fourth variable (X4)*, which originally utilized the market value of equity, was recalculated using the *book value of equity* instead. Additionally, new weights were assigned to each variable to reflect the financial characteristics of privately held companies more accurately. These adjustments resulted in the formulation of the *Z'-Score*, expressed by the following function (1983), which aims to enhance the model's applicability and diagnostic accuracy for firms outside the public market:

$$Z'\text{- score} = 0.717X_1 + 0.847X_2 + 3.107X_3 + 0.420X_4 + 0.998X_5$$

In the adapted model for *non-publicly traded companies*, the *cutoff point* was retained at *2.67*, but the *"grey zone" of uncertainty* now extends from *1.23* to *2.90*. Despite these modifications, the model's *predictive capability* remained almost identical to that of the original 1968 version. Specifically, the accuracy in correctly identifying *bankrupt firms* (Type I accuracy) decreased slightly, achieving *91%* compared to *94%* in the classic model. In contrast, the accuracy in identifying *healthy firms* (Type II accuracy) remained consistent at *97%*, mirroring the results of the original model.

As the model continued to evolve, further adjustments were made to broaden its applicability to *non-manufacturing firms*. One key variable that exhibited significant variation across industries was the *sales-to-total-assets ratio*. To address this sectoral sensitivity, Altman, Hartzell, and Peck introduced the *Z''-Score model* in 1995, providing an additional adaptation designed to enhance the model's relevance and predictive reliability across diverse industries *(Altman E. I., Hartzell J., and Peck M., 1995)*.

The Z''-Score model retains the same variables as the Z'-Score, with the exception of the *sales-to-total-assets ratio (X5)*. This adjustment was implemented to minimize *industry-specific biases* within the function, thereby enhancing its applicability across various sectors. As a result, the *weighting coefficients* assigned to each variable in the Z''-Score take on different values, reflecting this recalibration for broader and more accurate predictive use:

$$Z''\text{- score} = 6.56X_1 + 3.26X_2 + 6.72X_3 + 1.05X_4$$

This iteration of the model can also be adapted for evaluating the financial health of *non-U.S. firms*. Altman, Hartzell, and Peck extended the applicability of the Z''-Score model to companies operating in *emerging markets*, with a particular focus on *Mexican firms*. To calculate the Z''-

Score for companies in emerging economies - referred to as the *EM-Score* - the authors recommend adding a *constant value of +3.25* to the computed score. This adjustment standardizes the scores and ensures that bonds rated *D* (indicating default) are assigned a score of *0*.

In this modified model, the *"grey zone"* for the Z-Score spans from *1.10* to *2.90*. Firms within this range occupy a financially uncertain position, necessitating further examination to accurately determine their financial stability. This adaptation allows for a more nuanced and contextually relevant assessment of companies in emerging economies, enhancing the model's ability to predict financial distress or stability in these environments.

It is noteworthy that, although the Z-score model was originally designed for *publicly traded manufacturing firms*, its framework has been successfully applied and adapted to other industries and contexts over time. However, the model's *predictive accuracy* is contingent upon the quality and reliability of financial data and the relevance of the model to the specific industry and circumstances of the firm under evaluation.

4. LITERATURE OVERVIEW and HYPOTHESIS DEVELOPMENT SINCE the 1980s

Within the academic literature, considerable attention has been directed toward the *prediction of corporate financial distress and failure* as a critical aspect of corporate finance. This focus stems from a recognition that corporate failure has *adverse consequences* not only for the affected company but also for a broad array of stakeholders, including investors, employees, creditors, and the wider economy *(Cultrera L. and Bredart X., 2016).*

Researchers and scholars have extensively investigated this area to *understand the dynamics, factors, and indicators* associated with corporate financial distress, acknowledging the substantial impact these elements exert on both a company's internal operations and the broader network of stakeholders involved. The study of *corporate distress and failure prediction* is essential due to its wide-ranging implications, underscoring the need for a thorough understanding to mitigate risks and support sustainable financial health within the corporate sector.

In this context, a robust body of literature has emerged, offering a variety of *predictive models* developed by both academic researchers and practitioners. These models leverage diverse factors and data sources to enhance predictive accuracy. *Accounting information* has been particularly central to this research domain, with foundational contributions focused on identifying financial metrics and ratios that serve as early indicators of distress. Such research not only informs risk management practices but also contributes to the development of more resilient financial structures within the corporate landscape *(Altman E.I., 1968; Altman E.I. et al., 2017; Beaver*

*W.H., 1966; De Luca F. and Mechieri E., 2017; Dewaelheyns N. and Van Hulle C.,2006; Gupta J. et al., 2018; Li X. et al., 2020; Mselmi N. et a*l., 2017*; Ohlson J.A., 1980; Ruxanda G. et al., 2018; Zavgren C.V.,1985; Zmijewski M.E., 1984).*

Collectively, these models make significant contributions to the literature by presenting a range of perspectives and *methodological approaches* for predicting financial distress based on *accounting information*. Researchers frequently engage in comparative analyses of these models to assess their *effectiveness across various contexts* and to identify the most reliable predictive approaches. Such evaluations not only enhance the *theoretical framework* for understanding financial distress but also provide valuable insights into the *practical applicability* of different models within distinct economic and industrial settings.

4.1.Ohlson model (1980)

The *Ohlson model*, also known as the *Ohlson O-score*, represents a substantial contribution to financial analysis, particularly in predicting the *likelihood of financial distress* or bankruptcy. By focusing on *accounting variables*, the model provides a robust assessment of a company's probability of encountering financial difficulties. Its effectiveness in offering insights into a firm's financial health has rendered it a valuable tool for *investors, financial analysts, and researchers* aiming to anticipate and mitigate corporate risk.

The model’s utility extends beyond distress prediction to *equity valuation*, where its reliance on accounting metrics facilitates a more nuanced analysis of a company's *intrinsic value*. This applicability underscores its broader relevance within the field of corporate finance.

The dataset used in Ohlson’s study, covering the years 1970-1976, distinguishes this research from prior studies. Unlike many analyses that rely on data from *Moody’s Manual*, Ohlson’s study utilizes *financial statements (10-Ks)* filed at the time, including data from *105 bankrupt* and *2,058 non-bankrupt firms***.** This approach allows for consideration of the *timing of bankruptcy relative to financial statement releases*, an important yet often neglected factor in earlier studies. By focusing on this aspect, Ohlson’s research addresses a critical methodological gap in previous literature.

The study identifies *four statistically significant factors* that influence the probability of failure within one year: *company size, financial structure metrics, performance metrics, and current liquidity metrics*. Additionally, it suggests that previous studies may have *overstated predictive accuracy*, especially when predictors were derived from financial statements issued after a bankruptcy event. This insight calls for a more cautious interpretation of earlier research findings.

While the Ohlson model has notable strengths, it also exhibits certain limitations. The *prediction error rate* reported in Ohlson's study is higher than that observed in Altman's original study (1968) and other pre-1970 studies, although it aligns with error rates found in more contemporary research (*Altman E.I., 1968)*. Moreover, the simplicity of the models in this study enhances their practicality but excludes *market transaction (price) data*, which may restrict the model's predictive capabilities.

Ohlson's research also raises important questions regarding the *practical significance of bankruptcy forecasting*, highlighting the inherent challenges in justifying the utility of such predictions. Rather than making definitive claims about the relative effectiveness of various predictive systems, the study presents *descriptive statistics* focused on estimated prediction error rates. This emphasis reflects Ohlson's cautious approach and signals the necessity of addressing *statistical and methodological complexities* in developing reliable datasets for bankrupt firms - a dimension that has often been overlooked in the existing literature *(Ohlson J.A., 1980).*

4.2. Zmijewski model (1984)

The *Zmijewski model*, developed by Robert A. Zmijewski in 1984 and commonly referred to as the Z-score, is a prominent tool in predicting financial distress by assessing a company's likelihood of facing financial challenges or bankruptcy through key financial ratios. Known for its high accuracy in distress prediction, the model assigns lower Z-scores to companies with elevated risk levels. Its *accessibility* - stemming from reliance on data available from basic financial statements - has made it a widely adopted tool among financial analysts across diverse industries. The Zmijewski model is instrumental in *evaluating financial health*, providing early indicators of potential distress, and enabling *proactive risk management*.

A notable feature of the Zmijewski model is its exploration of *two types of biases* often encountered in financial distress studies: *choice-based sample bias* and *sample selection bias*. The first, choice-based sample bias, arises when the dependent variable is observed before sample selection, leading to biased parameters and probability estimates. The second, sample selection bias, occurs when only observations with complete data are included, resulting in non-random impacts on parameter estimates.

Zmijewski argues that *non-random sample selection* could exaggerate the effectiveness of the Z-score model in predicting financial distress. To mitigate this issue, he introduces a *probit* *approach***.** In response to choice-based sample bias, probit estimates are compared with *adjusted probit assessments* across various samples. Findings indicate that unadjusted probit results are indeed biased, but this bias diminishes as sample composition approaches that of the broader population and is ultimately eliminated through adjustment. Nevertheless, this bias does not significantly affect *statistical inferences*.

Sample selection bias is evaluated by comparing probit estimates with *bivariate probit assessments***.** While some bias persists, it similarly does not substantially impact statistical inferences. Adjusted methodologies offer refined estimates of sample probability distributions, which are essential for precise probability estimation or for evaluating the effects of independent variables. However, qualitatively, these adjustments do not diverge significantly from random sampling assumptions, influencing only *individual group error rates* rather than overarching conclusions.

Despite its utility, the Zmijewski model has certain limitations. It is predicated on the *assumption of a linear relationship* between financial ratios and the probability of financial distress, which may not align with the inherently nonlinear nature of financial dynamics, thus potentially impacting its accuracy. Moreover, the model may lack sensitivity to *industry-specific nuances***,** reducing its precision in certain sectors.

Additionally, the model does not fully account for *macroeconomic factors* such as economic cycles, industry trends, or regulatory shifts, which could introduce complexities beyond the model's scope. The *accuracy* of the Zmijewski model is also contingent on the availability, quality, and timeliness of financial data; incomplete or outdated data may impair its predictive effectiveness.

Another consideration is the model's sensitivity to *accounting policies*. Variations in accounting practices can affect the comparability of financial ratios across firms, which may undermine the model's reliability. Furthermore, the Zmijewski model's primary focus on financial metrics means it does not explicitly incorporate *non-financial factors* such as management quality or market perception, which could also influence a company's risk profile.

To maintain its relevance, the Zmijewski model requires *continuous validation and adaptation* to accommodate shifts in the economic landscape and evolving financial practices *(Zmijewski M.E., 1984).*

4.3.Zavgren model

Developed by C.V. Zavgren in 1985, the *Zavgren model* has established itself as a valuable tool in financial analysis, particularly for predicting *financial distress* and assessing a company's *vulnerability to bankruptcy*. Focusing on specific *accounting ratios* and *financial indicators*, this model has shown considerable success in evaluating a firm's financial health and resilience in the face of potential financial challenges.

The Zavgren model excels by providing a comprehensive assessment of a company's financial standing. By concentrating on critical accounting ratios and financial metrics, it enables analysts and researchers to adopt a proactive approach to risk management and strategic decision-making. Furthermore, the model offers a context-sensitive analysis of financial risk, making it adaptable to a wide range of industries. This versatility enhances its utility, offering users a more refined tool for conducting in-depth financial analyses that extend beyond generic evaluations.

In its development, Zavgren employed *logit* and *probit models* to estimate the probability of business failure, utilizing this probability as a metric of financial risk. The models demonstrated *statistical significance* at a confidence level exceeding 99%, effectively distinguishing between distressed and financially stable firms over a five-year horizon. *Information-theoretic measures* validated the model's robustness, with a notable increase in information content averaging 18% for failing firms and 16% for non-failing firms over the five years following initial assessment.

The model's *classification and prediction error rates* were favorable, particularly in *temporal generalizability tests*, where it outperformed other models. Notably, *efficiency ratios* emerged as the most significant predictors in the long term, suggesting challenges in adapting asset utilization in the short term. Contrary to expectations, *profitability* did not serve as a distinguishing factor. Additionally, the *acid test ratio* showed high statistical significance with a negative coefficient in later years, underscoring the critical importance of meeting current obligations to avoid bankruptcy. Early liquidity measures also revealed that failing firms tended to prioritize liquidity over productivity, while higher debt levels consistently differentiated struggling firms from their healthier counterparts.

While the Zavgren model has notable strengths, it is not without limitations. It operates under the assumption that specific *accounting ratios universally signal financial distress*. However, the complex financial dynamics inherent to various industries may not be fully captured by this model, potentially restricting its *accuracy* in sectors with distinct financial characteristics.

Another limitation is the model's *sensitivity to economic conditions*. Fluctuations in the economic landscape and industry-specific trends can impact the model's predictive accuracy, necessitating caution in interpreting results, particularly during periods of significant economic volatility. Continuous validation and careful contextual application are essential to maintain the model's effectiveness and relevance in dynamic economic environments *(Zavgren C.V.,1985).*

4.4.Dewaelheyns and Van Hulle model (2006)

The *Dewaelheyns and Van Hulle model,* introduced in 2006, represents a significant contribution to financial analysis, particularly within the area of *predicting financial distress*. This model likely integrates specific *accounting variables* and *financial ratios* to evaluate a company's financial stability and assess its susceptibility to distress. Unlike many traditional models, this study highlights the often-overlooked impact of *intra-group relationships* in Continental Europe, an aspect frequently neglected in bankruptcy prediction models. By drawing upon insights from literature on bankruptcy prediction, internal capital markets, and business groups, Dewaelheyns and Van Hulle illustrate that group dynamics - such as *group size, current performance, leverage*, and *liquidity* - significantly influence the informational value of accounting ratios used to predict bankruptcy for individual entities within the group.

Their findings reveal that incorporating group-related data enhances the predictive power of the model compared to conventional benchmarks, especially when including key financial metrics of the group or ultimate corporate owner. The study concludes that financially stable groups tend to support weaker subsidiaries, enhancing their survival prospects. This behavior suggests motives beyond agency theory, such as reinforcing intra-group relationships to secure external financing guarantees and uphold the group's reputation *(Dewaelheyns N. and Van Hulle C., 2006).*

This approach underscores the unique contributions of various **f***inancial distress prediction models* and highlights the considerations that researchers make in selecting or tailoring models to specific applications. Research into financial distress prediction has evolved to encompass the *interplay between accounting metrics and the probability of financial distress*, as well as the incorporation of additional variables.

In addition to accounting-based models, there has been substantial investigation into *stock market information* as a predictor of financial distress, with notable studies by *Bharath S.T.* and *Shumway T. (2008), Duffie D. et al. (2007),* and *Vassalou M. and Xing Y. (2004)*. Recognizing the interconnectedness of accounting and market-based factors, scholars have pursued a *holistic approach,* combining both types of variables, as demonstrated by *Shumway T. (2001), Campbell J.Y. et al. (2008), and Vo D.H. et al. (2019).* Further expanding this scope, recent research has introduced *macroeconomic variables* alongside accounting and market indicators *(Vo D.H. et al., 2019),* as well as *regulatory variables*, as seen in *Fernandez-Gamez M.A. et al. (2020).*

Beyond traditional metrics, researchers have explored additional dimensions such as *network-based metrics (Liu J. et al., 2019), corporate governance (Chen C.C. et al., 2020; Ragab Y.M. and Saleh M.A., 2021),* and auditing variables *(Munoz-Izquierdo N. et al., 2020).* This comprehensive approach demonstrates the dynamic and evolving nature of corporate financial distress and failure prediction research, with scholars continually refining and expanding models to enhance predictive precision and applicability.

In *2017, De Luca F. and Meschieri E.* applied *Multivariate Discriminant Analysis (MDA)* specifically to accounting ratios to develop a *"prewarning*" model for financial distress prediction. This study focused on Italian firms listed on the Milan Stock Exchange, particularly those that had undergone Article 182-bis restructuring agreements from 2003 to 2012. Using MDA, they established a discriminant function based on seven accounting ratios, validated as effective indicators of financial distress, to calculate the probability of *Troubled Debt Restructuring (TDR)***.** Additionally, they introduced the *M-index*, an indicator derived from historical data to illustrate trends in financial stability (*De Luca F. and Meschieri E., 2017).*

Other researchers, such as Altman E.I. et al. (2017), broadened the analytical scope by incorporating *logistic regression* alongside MDA. In contrast to a strict focus on accounting ratios, they integrated variables such as *year of bankruptcy, firm size, age, industry, and country***.** This broader approach generally enhanced model performance, though it also highlighted country-specific variations. While the original *Z''-Score model* proved effective at an international level, the study suggests that *country-specific adjustments* enhance classification accuracy for both European and non-European contexts.

Within the domains of *finance and accounting*, failure prediction models function as critical *risk assessment tools* across various applications. Altman's work emphasizes that while failure prediction modeling is highly valuable, it may be unnecessary to build a predictive model if it is not central to the research question. A well-validated, general model that performs consistently across different countries is highly desirable, and empirical testing demonstrates that both the original Z''-Score model and its re-estimated versions are effective and easy to interpret globally. Future research directions should consider refinements to these models, including *alternative methodologies* (e.g., panel data analysis), new variables (e.g., macroeconomic data), and validation using data from other regions, particularly *emerging markets (Altman E.I. et al., 2017).*

Recent studies, such as those by *Apergis N. et al. (2019)* and *Charalambakis E.C. and Garrett I. (2019),* have advanced this field by incorporating both *traditional accounting variables* and *logit models* - statistical techniques that predict binary outcomes like financial distress. These studies reflect an ongoing shift towards comprehensive modeling frameworks that integrate diverse variables and methods, aiming to capture the complex dynamics of financial distress prediction more effectively.

4.5. Apergis N. et all (2019)

The researchers concentrate on *predicting financial distress* and examine the interconnected relationships among financial distress, performance, employment, and *research and development (R&D) investment* within *multinational corporations (MNCs)***.** Utilizing *conditional logit and hazard models* along with a *conditional mixed-process model***,** they aim to generate consistent and efficient estimates of financial distress. The study's findings indicate that financial distress precipitates contractions in *organizational performance***,** *employment levels***,** and *R&D investment***.** Notably, the impact on R&D can be mitigated through the adoption of *risk-hedging strategies***.** Significant *country-specific variations* are observed, with MNCs in Canada, Israel, and the U.S. showing positive outcomes from risk-hedging practices. Additionally, the study underscores the detrimental effects of pre-existing financial distress on employment, particularly in Canada, the U.K., the Netherlands, and the U.S.

When considering the *policy implications* of these findings for MNCs, it is essential to acknowledge the *model's limitations*. A critical limitation involves the potential *heterogeneity across sectors* within MNCs, which may lead to interpretative bias if all units are treated as homogeneous entities. This study also highlights the need for access to comprehensive *cross-country datasets* to enable more precise analysis and comparisons across different national contexts.

In conclusion, the findings suggest that MNCs perform *distinct roles* across various countries, impacting employment and R&D investments in diverse ways during periods of financial distress. This study provides valuable insights for policymakers, emphasizing the need to account for country-specific factors and sectoral differences when evaluating the resilience of MNCs in times of financial instability *(Apergis N. et al., 2019).*

4.6. Charalambakis E.C. and Garrett I. (2019)

Leveraging a substantial dataset of nearly 31,000 Greek private firms from 2003 to 2011, this research rigorously examines the determinants of financial distress in corporations. By employing a *multiperiod logit model*, the study identifies several key factors influencing the likelihood of financial distress, including *profitability, leverage, retained earnings-to-total assets ratio, firm size, liquidity ratio, export status, dividend payment behavior*, and the *real GDP growth rate***.** The model demonstrates a high classification accuracy, correctly identifying 88% of firms that went bankrupt during the Greek debt crisis as having a high likelihood of financial distress. Additionally, the research highlights that the *impact of these variables varies by firm size***,** particularly when firms are categorized into small and medium-sized enterprises based on employee count.

The model's performance is robust across both in-sample and out-of-sample evaluations**,** effectively predicting firms at risk of financial distress over both *short-term and extended time horizons* of two to three years. This predictive accuracy underscores the model's practical utility and reliability in anticipating financial distress over varying periods *(Charalambakis E.C. and Garrett I., 2019).*

In the broader field of financial distress prediction and business failure analysis**,** researchers have adopted diverse modeling methodologies to deepen their understanding of the factors influencing corporate insolvency.

Firstly, some scholars utilize *discrete hazard models*, drawing on frameworks established by *Campbell J.Y. et al. (2008)* and *Shumway T. (2001).* These models are well-suited for time-to-event data analysis**,** enabling the integration of both accounting and market variables to offer a comprehensive view of the dynamic elements contributing to financial distress.

Secondly, *logit regression* remains a prominent approach, as evidenced in studies by *Hernandez Tinoco M. and Wilson N. (2013),* and *Vo et al. (2019).* This technique combines *accounting, stock market, and macroeconomic variables* to construct predictive models, thereby enhancing the precision of financial distress forecasts by capturing multiple facets of a firm's financial and market performance.

Another sophisticated approach involves *multilevel logistic modeling*, as employed by *Fernandez-Gamez et al. (2020).* This model accommodates multiple layers of variables - including *accounting, macroeconomic, and regulatory factors* - to capture the complex interplay among various influences that may collectively heighten the risk of financial distress.

Finally, researchers such as *Munoz-Izquierdo N. et al. (2020)* have applied *classical logistic regression models* with a specific focus on accounting and auditing variables. This traditional approach enables the prediction of financial distress likelihood based solely on fundamental financial indicators.

Collectively, these studies illustrate the rich array of statistical models utilized to decipher and predict financial distress within corporations. Each modeling approach presents unique advantages, contributing to a more *refined and comprehensive understanding of the multifaceted dynamics* associated with corporate failure. These diverse methodologies enrich the field, fostering insights that support early intervention and strategic decision-making to mitigate financial risk in the corporate landscape.

5. ADVANCEMENTS IN FINANCIAL DISTRESS PREDICTION: BEYOND STATISTICAL MODELS

In addition to traditional statistical models, recent years have seen a substantial increase in the use of *artificial intelligence (AI)* and *machine learning (ML)* techniques for predicting bankruptcy and financial distress. *Empirical studies* provide evidence that *ML-based approaches* frequently outperform traditional statistical methods in predictive accuracy and robustness. This shift underscores the growing recognition of *AI* and *ML's potential* to handle complex, non-linear relationships in financial data, thus offering enhanced precision in forecasting financial distress.

5.1.Barboza F. et al. (2017)

This study explores the application of machine learning (ML) models in predicting corporate bankruptcy and financial distress, emphasizing the comparative advantages of ML techniques over traditional statistical methods. While the research acknowledges the enduring relevance of established models such as those developed by Altman and Ohlson - valued for their simplicity and structured frameworks - it reveals a significant improvement in predictive accuracy when ML approaches are employed. Traditional models, like Multiple Discriminant Analysis (MDA) and Logistic Regression (LR), achieve accuracy rates ranging from 52% to 77%. In contrast, ML models demonstrate a marked enhancement, with accuracy levels reaching between 71% and 87%.

The findings underscore the practical strengths of ML models, particularly their ability to produce high predictive accuracy even when applied to raw data, as well as their effective integration of metrics reflecting variable growth rates and temporal changes. These capabilities make ML models particularly well-suited for capturing the dynamic nature of financial distress indicators, providing a more nuanced approach to forecasting corporate failure.

However, the study also addresses the limitations inherent in ML models. One notable challenge is their difficulty in managing non-separable datasets, which can complicate model training and diminish interpretability. Additionally, the substantial computational resources and processing time required for some ML algorithms highlight a practical constraint, particularly for large datasets or time-sensitive applications.

In reviewing the existing literature, the study identifies certain gaps, such as the limited focus on feature selection and the insufficient attention to varying classification costs across different bankruptcy prediction models. It recommends further research in these areas, particularly through the incorporation of macroeconomic variables to enrich the models' predictive capacity. Furthermore, it advocates for a thorough analysis of overfitting, a recurrent issue in ML methodologies that can compromise model generalizability. These proposed directions underscore a broader research agenda aimed at refining ML approaches for more robust and reliable bankruptcy prediction models *(Barboza F. et al., 2017)*.

5.2. Zelenkov Y. et al. (2017)

The effectiveness of this approach is grounded in its integration of novel indicators that encompass a broad spectrum of factors, including the external environment, economic conditions, and management characteristics, in addition to traditional financial ratios. This multi-dimensional perspective significantly improves predictive accuracy compared to models that rely solely on financial metrics. Notably, the study emphasizes feature selection - a critical element for enhancing model performance - by utilizing genetic algorithms.

The combination of genetic algorithms with standard classification models has demonstrated substantial effectiveness, achieving superior accuracy and error balance compared to other methodologies. Although computationally intensive, this approach's adaptability across diverse contexts underscores its practical advantages.

The model was tested on a well-balanced dataset of Russian firms, consisting of 456 bankrupt and 456 solvent companies, using 55 features. Results indicate that the proposed method outperforms alternative models, achieving an accuracy rate of 0.934 and a well-balanced precision-recall ratio. The model showed high sensitivity in detecting bankrupt firms *(recall = 0.953)* and a strong ability to correctly identify non-bankrupt firms *(precision = 0.910)*. Furthermore, the model's feature selection capabilities were validated: removing features selected by less than 50% of classifiers in the ensemble led to improved performance metrics, suggesting a refined approach to identifying truly relevant features.

In summary, the proposed method not only capitalizes on the strengths of traditional classifiers but also mitigates their limitations, offering a more robust framework for business decision-making and risk assessment *(Zelenkov Y. et al., 2017).*

5.3.Cao Y. et al. (2022)

In this study, researchers advance a Bayesian network framework to predict the probability of firm bankruptcy, employing a series of refined methodologies to improve both the accuracy and interpretability of the model. Key to this enhancement is the application of the LASSO (Least Absolute Shrinkage and Selection Operator) technique, which aids in selecting the most relevant variables from a broader set of potential predictors, thereby refining the network's focus on critical factors. The Bayesian network is subsequently constructed and parameterized through the EM (Expectation-Maximization) algorithm, an iterative approach that optimizes the model’s parameters to better capture underlying probability distributions.

Empirical analysis reveals that this Bayesian network model performs remarkably well, with its predictive accuracy only slightly below that of a more intricate Deep Neural Network (DNN) comprising three hidden layers. While the DNN achieves slightly higher accuracy, the Bayesian network offers notable advantages in transparency and interpretability, both highly valued in financial prediction models. The network’s architecture is designed to clarify the relationships among variables, effectively illuminating the process by which conditional default probabilities are derived from the selected features. This transparency makes it possible for users, including investors and policymakers, to comprehend the logic driving the model’s decisions, thereby supporting more informed financial and regulatory actions.

This model’s combination of interpretability and accuracy represents a significant advancement toward developing machine learning models that are not only robust but also accessible to decision-makers who rely on clear explanations for predictive insights. The model’s broad applicability and relevance extend beyond technical users, offering value to a range of stakeholders interested in assessing bankruptcy risk through an approach that aligns with principles of explainable AI.

However, the study also identifies limitations in the current model, particularly regarding its fixed structure. The Bayesian network’s topology remains static over time, which restricts its ability to adapt to changing economic conditions or new patterns in financial data. This rigidity may ultimately reduce the model’s effectiveness in dynamically evolving environments, where shifts in macroeconomic indicators or sector-specific trends could impact the variables that drive financial distress. Future research could address this by exploring methods for dynamically adjusting the Bayesian network's topology to enhance its flexibility and relevance. Additionally, incorporating a hazard model could offer a more comprehensive view of bankruptcy risk over time, contributing to a deeper understanding of the factors that influence a firm's likelihood of failure in various economic contexts *(Cao Y. et al., 2022).*

In a broader context, research consistently demonstrates that artificial intelligence (AI)-based models generally surpass traditional statistical methods in predictive performance. This trend highlights the potential of AI for advancing financial modeling practices by integrating complex, multidimensional data and adapting to non-linear relationships among variables. The success of this Bayesian network model further reinforces the value of AI techniques, suggesting that continued innovations in interpretable and adaptive AI models could significantly benefit the field of financial prediction.

5.4. Dube F. et al. (2023)

This study investigates the application of artificial intelligence (AI), specifically Artificial Neural Networks (ANN), in predicting financial distress among companies, focusing on both financial services and manufacturing firms listed on the Johannesburg Stock Exchange (JSE) over a period spanning from 2000 to 2019. Artificial Neural Networks, inspired by the neural structures of the human brain, serve as computational models designed to learn from data and identify patterns that may be too complex or non-linear for traditional statistical methods. Leveraging this capacity, the ANN models in this study were meticulously developed and tested to assess their ability to anticipate financial distress within a corporate setting.

The results are particularly compelling, demonstrating notable predictive accuracy: the models achieved an accuracy rate of 81.03% for firms in the financial services sector, and an even higher rate of 96.60% for those in the manufacturing sector. Such high accuracy levels underscore the efficacy of ANN models in consistently identifying firms at risk of financial distress, suggesting that ANNs are particularly well-suited for recognizing the diverse and dynamic factors that contribute to financial vulnerability across industries.

A distinctive feature of this study is its emphasis on the long-term predictive capability of ANN models, capable of forecasting financial distress as far as five years before a firm might be formally classified as distressed. This early-warning capability is invaluable, offering stakeholders - including company management, investors, and financial regulators - the opportunity to identify financial challenges well before they escalate. By providing this critical lead time, the model not only enables stakeholders to intervene and implement mitigation strategies, but it also supports proactive risk management approaches that can significantly alter the financial trajectory of a firm.

The contributions of this study are substantial, extending to both theoretical and practical dimensions. Theoretically, the research emphasizes the utility of AI models, particularly ANNs, in addressing complex forecasting challenges within the financial sector. Traditional financial models often struggle to capture the intricate relationships and non-linear patterns within financial data, but the ANN model's adaptive structure allows it to learn from the data without predefined assumptions, making it a powerful tool for predictive financial modeling. This theoretical advancement reinforces the potential for AI models to complement or even surpass traditional financial models in accuracy and adaptability.

On a practical level, the study establishes ANN as a viable early-detection tool with substantial implications for financial stability and risk management. By identifying financial distress well before it manifests in severe liquidity or solvency issues, the ANN model provides companies, investors, and policymakers with actionable insights. In a corporate setting, such a tool could be integrated into risk management frameworks, allowing companies to monitor financial health continuously and adjust strategies in response to early signs of distress. For investors, the model offers an analytical tool for assessing corporate risk, while for policymakers, it supports regulatory efforts by highlighting firms that may require closer monitoring.

Overall, this study positions ANN models as highly effective tools for forecasting financial distress and demonstrates their utility in delivering both accuracy and foresight in complex financial environments. By extending the practical and theoretical understanding of AI-driven predictive modeling, the research suggests that ANN models hold considerable promise for enhancing financial resilience and stability across multiple sectors *(Dube F. et al., 2023)*.

6. BEYOND FINANCIAL RATIOS: A BROADER PERSPECTIVE on FINANCIAL DISTRESS

Over the past decades, financial distress prediction models have undergone significant theoretical and methodological evolution. While early contributions to the field - most notably those of *Altman E.I. (1968)* and *Ohlson J.A. (1980)* - relied predominantly on firm - level financial indicators such as liquidity, leverage, and profitability, more recent scholarship has begun to question the sufficiency of purely quantitative, accounting-based frameworks. Indeed, although such models have proven useful in identifying financial imbalances, they often fail to capture the broader set of dynamics - both internal and external - that contribute to the emergence and escalation of corporate distress.

In this context, an important shift has taken place in the academic literature, as scholars increasingly advocate for a more holistic understanding of financial distress - one that accounts not only for numerical imbalances but also for the organisational and economic environments in which firms operate. Two dimensions, in particular, have emerged as essential to this expanded perspective: *behavioural and governance-related factors*, and *macroeconomic variables*.

On one hand, behavioural finance and organisational theory have drawn attention to the critical role played by managerial attitudes, cognitive biases, and decision-making styles in either mitigating or exacerbating financial vulnerability. Empirical evidence has shown that characteristics such as managerial overconfidence, excessive risk-taking, and weak governance structures may significantly impair a firm's ability to respond effectively to early signs of financial decline *(Malmendier U. & Tate G., 2005; Simons R., 1995; Altman E.I. et al., 2017).* These behavioural dynamics, while often qualitative in nature, can have measurable effects on a firm's financial trajectory and are increasingly viewed as necessary components in predictive modelling.

On the other hand, macroeconomic conditions represent a powerful exogenous influence on corporate health. Fluctuations in GDP growth, interest rates, inflation, and unemployment rates have been consistently linked to changes in firms' financial performance, liquidity availability, and overall survival prospects *(Carreira C. & Silva F., 2010; Charitou A. et al., 2004).* Periods of economic downturn tend to constrain consumer demand and tighten credit conditions, disproportionately affecting firms that are already financially fragile. Conversely, favourable macroeconomic environments may provide distressed firms with the breathing space necessary for operational or financial recovery.

Together, these two analytical lenses - behavioural and macroeconomic - suggest the need for a more nuanced and context-sensitive approach to distress prediction. Rather than viewing financial distress as merely the outcome of poor financial ratios, it should be understood as a *multifactorial process* shaped by managerial behaviour, organisational structure, and external economic conditions. This broader framework not only enhances explanatory power but also aligns predictive models more closely with the complexities of real-world corporate environments. The sections that follow will therefore explore in detail the role of these behavioural and macroeconomic determinants, drawing on recent empirical findings and theoretical developments in the field.

6.1. Behavioural and Governance Factors in Financial Distress

Within the evolving literature on financial distress prediction, one of the most salient and theoretically grounded developments is the growing attention to managerial behaviour as a determinant of corporate vulnerability. Traditional predictive models, based predominantly on firm-level financial ratios such as liquidity, leverage, and profitability, have increasingly been criticised for their inability to account for the behavioural dimensions of decision-making under uncertainty *(Altman E.I. et al., 2017).* This has led to a paradigm shift, whereby corporate distress is no longer viewed as a purely numerical imbalance, but rather as a process influenced by cognitive biases and psychological traits embedded in leadership practices.

A seminal contribution in this regard is provided by *Malmendier U. and Tate G. (2005),* who demonstrate that overconfident CEOs tend to overestimate returns, underestimate risks, and persist in overinvestment, particularly in contexts of financial constraint and external volatility. This managerial overconfidence distorts capital allocation and delays necessary adjustments, making firms more prone to distress *(Gervais S. et al., 2011).* Similarly, optimism and confirmation biases may impair the early recognition of negative trends and reinforce resistance to strategic change *(Bazerman M.H. & Tenbrunsel A.E., 2003).* These distortions are not isolated incidents, but rather recurrent cognitive patterns that amplify risk exposure if left unchecked.

These behavioural distortions are not episodic, but constitute recurring patterns of human cognition that amplify risk exposure when unchecked. This is especially evident in the phenomenon of *"escalation of commitment",* whereby managers double down on underperforming strategies due to sunk cost effects or reputational concerns, aggravating financial decline *(Staw B.M., 1976; Shefrin H., 2002).* These dynamics are particularly dangerous in high-uncertainty environments, where adaptive decision-making is most needed.

Governance structures play a critical moderating role in this context. Weak governance - characterised by insufficient board oversight, limited independence, or poorly designed control mechanisms - has been consistently linked to a higher probability of financial distress *(Jensen M.C., 1993; Agrawal A. & Knoeber C.R., 1996).* Conversely, sound governance frameworks, especially those that ensure separation between ownership and control, and enable active monitoring, reduce the scope for unchecked managerial bias and facilitate early corrective actions *(Simons R., 1995; Grabner I. & Moers F., 2013).*

These concerns are especially salient for SMEs and family businesses, which often display informal decision-making processes, limited accountability structures, and concentrated leadership. In these contexts, the behavioural traits of individual decision-makers can have a disproportionate effect on firm outcomes, increasing idiosyncratic risk and reducing the applicability of standard predictive models *(Chrisman J.J. et al., 2012).*

Recent empirical advances have started to operationalise these behavioural dimensions. For example, *Lee J. et al. (2020)* utilise natural language processing of earnings calls to identify excessive optimism in tone, which correlates with future performance shortfalls. Similarly, *Humphery-Jenner M. et al. (2016)* link narcissistic traits in CEOs - measured through linguistic self-referencing and remuneration structures - to lower firm resilience in distress contexts.

Despite these promising developments, integrating behavioural factors into prediction models remains methodologically challenging. Many traits, such as overconfidence or rigidity, are not directly observable and require proxy-based or subjective estimation, which complicates their inclusion in algorithmic tools *(Charalambakis E.C. & Garrett I., 2019).* Moreover, much of the literature has been developed within Anglo-American contexts, raising questions about the cross-cultural validity of these findings *(Hofstede G., 2001).*

In response to these concerns, scholars have begun to advocate for *integrated predictive frameworks* that combine traditional financial ratios with qualitative indicators related to management behaviour and governance quality *(Cao Y. et al., 2022; Dube F. et al., 2023).* Such models aim to offer a more nuanced and context-sensitive understanding of financial distress, particularly in increasingly complex and dynamic business environments. However, while these developments are promising, the existing literature in this area remains subject to several important *limitations***.**

First and foremost, behavioural and governance-related factors are inherently *difficult to quantify***.** Unlike balance sheet items or income statement ratios, variables such as overconfidence, strategic rigidity, or board ineffectiveness are not systematically recorded and often require subjective or proxy-based measurement. This complicates their integration into predictive algorithms and limits their comparability across studies *(Charalambakis E.C. & Garrett I., 2019).* Secondly, much of the empirical research on these themes has been conducted in specific national or institutional contexts - most often in the United States or Western Europe - raising questions about the *generalisability* of findings to other regions with different legal, cultural, and regulatory environments.

Finally, there is a tendency within the literature to treat behavioural and governance failures as secondary to financial deterioration, rather than as co-determinants or even antecedents of distress. This underestimates their role in the causal chain and obscures the potential value of early detection based on internal, non-financial signals.

Taken together, these limitations point to a pressing need for future research that not only theorises behavioural and governance factors but also finds innovative and empirically valid ways to incorporate them into predictive modelling. This includes the development of new indicators, the use of qualitative and mixed-method approaches, and the adoption of advanced techniques such as machine learning, which may be better suited to detect complex, non-linear patterns in human and institutional behaviour. By addressing these gaps, scholars can move closer to constructing truly multidimensional and actionable tools for identifying financial distress before it becomes irreversible.

In conclusion, while the integration of behavioural and governance dimensions into financial distress prediction represents a compelling avenue for enhancing both the explanatory and predictive power of current models, it also requires significant methodological and empirical innovation. Moving beyond traditional ratio-based diagnostics to include cognitive, structural, and institutional variables could yield more realistic and anticipatory frameworks. However, such evolution must proceed cautiously, with due attention to issues of measurement validity, cross-context applicability, and data availability. Only through interdisciplinary collaboration and methodological pluralism can the next generation of predictive models fully capture the complexity of financial distress and offer actionable insights for scholars, practitioners, and policymakers alike.

6.2. Macroeconomic Determinants of Corporate Distress

In recent years, a more sophisticated and multidimensional strand of literature has emerged, recognising that financial distress is not merely the result of firm-specific inefficiencies or deteriorating financial fundamentals, but rather the outcome of dynamic and often unpredictable interactions between internal vulnerabilities and broader macroeconomic forces. While foundational predictive models - such as those developed by *Altman E.I. (1968*) and *Ohlson J.A. (1980)* - laid important theoretical groundwork through the use of firm-level accounting indicators (e.g., liquidity, leverage, profitability), contemporary research increasingly argues that these static, ratio-based approaches provide only a partial view of the complex mechanisms underlying financial instability. In particular, the incorporation of macroeconomic variables such as GDP growth, interest rates, inflation, and unemployment has been shown to significantly enhance model robustness and contextual sensitivity *(Altman E.I. et al., 2010; Carreira C. & Silva F., 2010; Charitou A. et al., 2004).*

GDP growth is widely recognised as a key indicator of the macroeconomic environment in which firms operate. Positive GDP trends typically signal strong aggregate demand, enhanced consumer confidence, and improved credit availability - conditions that foster corporate stability and growth. Conversely, contractions in GDP, often observed during downturns, are strongly correlated with reduced revenues, liquidity stress, and heightened default risk *(Becchetti L. & Sierra J., 2003).* The global financial crisis of 2008 - 2009 serves as a notable example, as GDP declines across both advanced and emerging economies were accompanied by a sharp increase in corporate bankruptcies, illustrating the systemic impact of macroeconomic shocks (*Carreira C. & Silva F., 2010).*

Recent literature further highlights that the effect of GDP growth is not uniform across all firms. *Lee and Kim (2019)* demonstrate that macroeconomic fluctuations - such as those related to economic growth and energy costs - can have differentiated effects depending on firm size and sectoral exposure. Small and medium-sized enterprises (SMEs), which often face tighter financing constraints and operate with lower operational flexibility, are generally more vulnerable to GDP contractions than larger, more diversified corporations. This heterogeneity underscores the need for sector-adjusted and firm-specific sensitivity analysis in distress prediction models.

While GDP remains a valuable systemic indicator, its effectiveness is maximised when used alongside other financial and contextual variables. Its inclusion in hybrid models contributes to a more nuanced understanding of firm-level risk under varying macroeconomic conditions.

Interest rates are a fundamental macroeconomic variable with significant implications for firms' capital structures and financial stability. Elevated rates increase the cost of debt servicing, compress profit margins, and discourage investment - effects that are particularly severe for small and medium-sized enterprises (SMEs), due to their reliance on bank financing and limited access to capital markets (*Musso P. and Schiavo S.*, *2008)*

Campello M. et al. (2010) show that during periods of monetary tightening, the combined impact of higher interest expenses and restricted credit availability can trigger financial distress even in otherwise solvent firms.

Recent analyses underscore the disproportionate impact of rising interest rates on SMEs. A report by the *Federal Reserve (2023)* indicates that higher interest expenses can lead to firm distress and defaults, adversely affecting employment and investment *(Bräuning F. et al., 2023).* Similarly, a study by *Abecasis D. and Hill J. (2023)* estimates that higher rates will increase the interest burden for small businesses by just over 1 percentage point by 2024, from roughly 5.8%

of revenues in 2021 to around 7% in 2024. This escalation in debt servicing costs strains cash flows and limits the capacity for growth and investment.

Sector-specific dynamics further exacerbate this issue. Industries such as construction, real estate, and retail are acutely sensitive to interest rate fluctuations *(Dermine J., 2013)*. The *International Monetary Fund (2024)* reports mounting pressure in commercial real estate markets globally, as rising rates have driven down property values and increased default risks. Similarly, the *U.S. Government Accountability Office (2023)* reports that the commercial real estate market has experienced strains from the pandemic-related rise in remote and hybrid work, rising interest rates, and declining prices, particularly for office properties. These trends have made it harder for some property owners to repay their loans, increasing the likelihood of financial distress.

In sum, rising interest rates pose significant challenges to firms, especially SMEs and those in interest-sensitive sectors. The increased cost of borrowing can lead to financial distress, reduced investment, and higher default risks. Integrating interest rate considerations into financial distress prediction models is crucial for accurately assessing firm vulnerability in varying macroeconomic conditions.

Inflation exerts complex and, at times, contradictory influences on corporate financial health. While moderate inflation can erode the real value of fixed nominal liabilities, thereby easing debt burdens for some firms, unanticipated or volatile inflation introduces significant uncertainty in input pricing, wage costs, and demand forecasting *(Chen K.H. & Mahajan A., 2010)*. This uncertainty complicates strategic planning and can adversely affect profitability, particularly for firms unable to pass increased costs onto consumers. High inflation can also distort financial statement reliability, undermining the predictive value of conventional ratios used in distress modelling (*Koopman S.J. & Lucas A., 2005*). For instance, inflation may lead to overstatement of profits and asset values when historical cost accounting is employed, potentially misleading stakeholders about a firm's true financial position (RSM Global, 2022). These risks are especially salient in economies with structurally embedded inflationary pressures or weak monetary policy credibility (IASB, 1989).

Empirical studies have highlighted the challenges inflation poses to financial reporting. Research indicates that inflation can significantly distort balance sheet and income statement figures, making them less reflective of economic reality and thereby reducing their usefulness for investors' decision-making *(Binz T., Graham J.R. & Kubic M.D., 2023*). This distortion is particularly problematic in high-inflation environments, where traditional accounting measures may fail to capture the real economic value of assets and liabilities. Moreover, inflation can impact impairment assessments, as assumptions about future cash flows and discount rates become more volatile, leading to increased uncertainty in asset valuations *(Chen K.H. & Mahajan A., 2010)*.

To mitigate these challenges, accounting standards such as the *International Accounting Standard (IAS) 29* prescribe financial reporting in hyperinflationary economies, requiring entities to restate financial statements in terms of the measuring unit current at the end of the reporting period *(IASB, 1989)*. This approach aims to provide more meaningful information by adjusting for the effects of inflation, although its implementation can be complex and resource-intensive. Additionally, inflation accounting methods, such as constant purchasing power accounting, have been proposed to address the limitations of historical cost accounting in inflationary contexts *(RSM Global, 2022)*.

Unemployment rates, though often peripheral in firm-level analyses, serve as crucial macroeconomic indicators reflecting consumer spending power, labor market stability, and broader economic health. Elevated unemployment typically signifies economic distress, leading to diminished consumer demand, particularly in consumption-driven sectors such as hospitality, retail, and leisure. This reduction in demand can result in revenue volatility and underutilization of capacity, increasing the likelihood of financial deterioration at the firm level.

Moreover, high unemployment rates can indirectly influence corporate financial policies. *Agrawal A.K. and Matsa D.A. (2013)* found that firms tend to adopt more conservative financial strategies, such as reducing leverage, to mitigate the risk of financial distress that could lead to layoffs. This behavior underscores the interplay between labor market conditions and corporate financial decision-making.

Additionally, unemployment can serve as a proxy for systemic fragility, indicating potential spillover effects of economic downturns. For instance, during the 2008 financial crisis, rising unemployment rates were both a symptom and a catalyst of broader economic challenges, affecting consumer confidence and spending patterns, which in turn impacted corporate revenues and financial stability.

Despite the analytical relevance of these variables, the integration of macroeconomic indicators into financial distress prediction models is not without significant *limitations***,** both conceptual and empirical.

A key concern lies in *data aggregation bias***:** macroeconomic variables are typically reported at national or regional levels, whereas corporate distress manifests at the micro level. This discrepancy in data granularity reduces the explanatory power of such indicators, particularly for firms operating across sectors, geographies, or niche markets *(Laitinen E.K. & Laitinen T., 2000)*. Moreover, macroeconomic influences tend to exhibit *non-linear* and *lagged effects***,** often impacting firms not immediately but over successive quarters - dynamics which are difficult to capture in static or linear predictive frameworks commonly used in the literature.

Another persistent challenge is *data comparability across countries*. While macroeconomic indicators in advanced economies are typically available at high frequency and benefit from strong institutional reliability, the same cannot always be said for emerging markets. In many developing contexts, the availability, granularity, and standardisation of macro-level data remain insufficient, limiting the *portability* and *generalisability* of models across jurisdictions *(Dermine J., 2013)*. This creates significant barriers for cross-country analysis and undermines the predictive robustness of macro-integrated models in a global context.

Furthermore, the *endogeneity problem* presents a critical methodological hurdle. In many cases, firm-level financial distress may influence, rather than simply be influenced by, macroeconomic indicators. For example, rising corporate insolvencies can contribute to GDP contraction, declining investment, or increasing unemployment. If not adequately addressed, such bidirectional causality can bias empirical estimations and weaken the validity of predictive conclusions *(Kaminsky G.L. & Reinhart C.M., 1999)*.

While recent innovations in *computational modelling* - including machine learning and time-series econometrics - have improved the capacity to capture non-linear interactions and lag structures, the actual application of these methods remains limited in the financial distress literature. Most existing models continue to prioritise firm-level ratios and treat macroeconomic factors as control variables, rather than fully integrated predictors *(Altman E.I. et al., 2017; Charalambakis E.C. & Garrett I., 2019)*. As such, there remains a clear gap between theoretical acknowledgement of macroeconomic relevance and its practical incorporation into predictive frameworks.

In light of these limitations, there is growing scholarly consensus around the need for *hybrid, multi-level models* that synthesise firm-specific financial metrics with behavioural and macroeconomic data in a coherent, integrative manner. Such approaches would allow for more context-sensitive risk profiling and offer enhanced predictive accuracy, especially in volatile or systemically exposed sectors. Bridging the macro-micro divide is therefore not only a theoretical imperative but a practical necessity for policymakers, analysts, and financial institutions seeking to improve early warning systems and enhance the resilience of the corporate sector.

In conclusion, the macroeconomic environment plays a pivotal role in shaping corporate financial stability, both directly - through its impact on revenues, costs, and financing conditions - and indirectly, by influencing strategic behaviour, market confidence, and systemic dynamics. The effects of GDP fluctuations, interest rate movements, inflationary pressures, and unemployment levels are neither linear nor uniform across firms, but rather mediated by firm size, sectoral exposure, and financial structure. While integrating macroeconomic variables into distress prediction models enhances their contextual relevance, this integration is not without significant methodological challenges, including issues of data granularity, comparability, and endogeneity. Overcoming these limitations requires a shift towards hybrid, multi-level approaches that blend financial, behavioural, and macroeconomic data. Such models offer the most promising avenue for building early warning systems capable of capturing the complexity of financial distress in an increasingly interconnected and volatile economic landscape.

7.DISTINGUISHING CORPORATE FAILURE FROM FINANCIAL DISTRESS: A DETAILED ANALYSIS

In the existing body of research, corporate bankruptcy and financial distress are often treated as synonymous, despite the nuanced distinctions between these concepts. Many researchers employ terms such as financial distress, failure, default, and bankruptcy interchangeably when predicting financial difficulties on a global scale. However, financial distress encompasses a broader spectrum of challenges that a company may face, including difficulties in meeting financial obligations, while bankruptcy is a legal resolution to insolvency.

In their study, *Altman E.I. et al. (2017)* adopted a generalized approach, treating these terms as functionally similar events to create a universal predictive framework for financial distress across various contexts. This approach simplifies the model by assuming equivalence among diverse negative outcomes faced by companies, thereby yielding a uniform predictive tool for financial distress. However, this approach has been a point of contention, as other scholars argue that distinct legal, financial, and operational implications are associated with each term. These scholars stress the need to differentiate between financial distress and bankruptcy, suggesting that conflating these concepts may lead to oversimplified and potentially inaccurate predictive models *(Opler T.C. and Titman S., 1994; Andrade G. and Kaplan S.N., 1998).*

Critics of the broad categorization contend that it risks overlooking critical factors specific to each type of corporate failure, which may limit the precision of predictive models. They argue that financial distress - representing a wider array of financial difficulties a firm may encounter - does not inevitably lead to bankruptcy, a formal legal process involving the liquidation or restructuring of a firm’s assets. According to this perspective, recognizing the distinctions among these concepts facilitates the development of more precise risk models and enhances the accuracy of risk assessments. Tailoring models to specific types of financial challenges can improve understanding and lead to more effective strategies for managing financial distress and bankruptcy *(Gupta J. et al., 2018).*

Research by *Chen K. and Merville L.J. (1999)* reinforces this argument, showing that financial distress may prompt corrective measures such as restructuring without necessarily resulting in formal bankruptcy. Furthermore, studies by *Campbell J. Y., Hilscher J., and Szilagyi J. (2008)* demonstrate that distinguishing between distress and bankruptcy allows for more accurate assessments, particularly in corporate credit risk pricing. This differentiation is critical for creditors, investors, and policymakers aiming to implement risk management strategies responsive to the specific dynamics of financial distress and bankruptcy (*(Gupta R., 2020; Andrade G. & Kaplan S.N., 1998).*

In addition, other scholars emphasize the need to distinguish between various outcomes within financial distress prediction models. Recognizing distinctions between failure, survival, and alternative resolutions, such as acquisition, is argued to be essential for refining these models. *Astebro T. and Winter J.K. (2012)* argue that differentiating whether a firm will continue as a going concern or face acquisition is crucial in enhancing predictive frameworks. This perspective reflects the diverse trajectories a firm may follow when experiencing financial difficulties, thus increasing the precision and practical relevance of the models.

Further research supports this argument, showing that not all financially distressed firms proceed toward bankruptcy, with some pursuing restructuring or mergers as alternative outcomes *(Bruton, G. D., Filatotchev, I., & Wright, M., 2000).*

Laitinen E.K. (2005) emphasizes that incorporating a broader spectrum of outcomes into financial distress models - beyond a binary classification of failure or survival - improves risk management strategies. These nuanced approaches not only improve the predictive power of the models but also provide stakeholders, such as investors and creditors, with more actionable insights.

In the ongoing discourse on financial distress prediction, other researchers have made significant advancements by refining established models such as the Z-score model. For instance, *De Luca F. and Meschieri E. (2017*) enhanced the traditional Z-score model for predicting distress in Italian firms by adding the current ratio and quick ratio, resulting in a model that incorporates seven financial ratios. This augmented model was shown to provide more accurate predictions, demonstrating a strong link between declining accounting ratios and the likelihood of total debt restructuring (TDR), a critical indicator of financial distress. This aligns with earlier work by *Laitinen E.K. (1991)* and *Taffler R.J. (1983*), who argued that adding liquidity ratios, like the current and quick ratios, enhances the model's ability to detect early signs of distress.

These studies suggest that a comprehensive understanding of a firm's financial health - through a broader set of accounting ratios - can improve the predictiveness of financial distress models, particularly for debt restructuring scenarios *(Altman E.I., 2000)*.

Despite the substantial contributions of previous research, the literature still reveals significant gaps, particularly regarding the nuanced distinctions between corporate failure and financial distress. While various studies have addressed these terms, there remains a need for a comprehensive examination of their differences. This study seeks to bridge this gap by arguing that failure and financial distress are distinct concepts that require differentiated approaches within corporate finance.

Defining failure as the ultimate cessation of operations, as opposed to financial distress, which encompasses a broader range of challenges, allows for a more sophisticated understanding of how firms navigate financial difficulties. The study highlights that distinguishing these terms can enhance the precision of predictive models, particularly in assessing probabilities of outcomes such as restructuring, acquisition, or bankruptcy.

By clarifying the implications of conflating or distinguishing failure and financial distress, this research aims to enhance both theoretical frameworks and practical tools for risk assessment. Ultimately, this study contributes to a more nuanced framework for understanding corporate distress, offering valuable insights to stakeholders such as investors, creditors, and policymakers who rely on accurate predictions for informed decision-making.

8. *FINANCIAL CHALLENGES IN ITALY: LEGAL EVOLUTION AND DEBT RESTRUCTURING*

Following the global financial crisis of 2007–2009, many Italian businesses encountered severe financial challenges, mirroring a worldwide trend of corporate distress and large-scale debt restructuring. Companies across multiple sectors sought to renegotiate their financial obligations to maintain viability during this period. The Italian context is particularly notable due to the lack of a comprehensive legal and regulatory framework for managing debt restructuring at the outset of the crisis.

Specifically, Italian law, as reflected in the Civil Code, did not provide explicit provisions on how firms could effectively reorganize their debts when faced with financial distress. Likewise, the national accounting standards overseen by the *Organismo Italiano di Contabilità (OIC)* offered limited guidance for companies navigating restructuring processes. This absence of a well-defined legal and accounting framework created significant uncertainty for businesses, creditors, and financial institutions attempting to address complex restructuring needs during a time of heightened financial vulnerability.

This lack of formalized debt restructuring mechanisms sharply contrasted with international practices, where many countries had already established robust legal frameworks for addressing corporate distress. Consequently, Italian businesses faced a relative disadvantage, operating within a legal and financial environment unprepared for the scale of restructuring required in the post-crisis period. These regulatory gaps subsequently drove significant reforms aimed at aligning Italy's legal and accounting standards with international best practices, ultimately enhancing Italy's capacity to manage corporate distress more effectively in the years following the crisis.

In recent years, Italy has undergone substantial legislative reforms to create a more structured approach to corporate financial distress. Noteworthy legislative actions, such as Decree-Law No. 35 of March 14, 2005 (later converted into Law No. 80 on May 14, 2005) and Legislative Decree No. 5 of January 9, 2006, have played a pivotal role in redefining the legal framework, providing firms with more effective tools for addressing financial challenges. These reforms responded directly to the shortcomings exposed during the financial crisis, particularly the lack of clear legal guidelines for debt restructuring.

One of the most significant advancements within this reformed legal environment has been the introduction of restructuring agreements under Article 182-bis of the Bankruptcy Law *(Di Marzio F., 2006).* This mechanism enables financially distressed companies to negotiate debt restructuring agreements with creditors, offering a legally recognized path for financial reorganization without necessitating full bankruptcy proceedings. Article 182-bis thus provides a flexible, preventative tool that allows distressed companies to restructure their debts and continue operations, ensuring continuity while protecting creditors' rights.

These restructuring agreements represent a critical development in the Italian legal landscape, aligning it more closely with international practices and providing a viable alternative to liquidation. This legislative evolution reflects a broader shift towards facilitating corporate recovery and debt renegotiation rather than relying solely on bankruptcy as the inevitable outcome of financial distress. Alongside previous reforms, these legislative changes mark a turning point in Italy's approach to corporate distress, offering structured and predictable solutions for businesses in financial difficulties.

In addition, the recent introduction of the Italian accounting standard OIC 6 has further refined the legal and financial framework for corporate restructuring. OIC 6 details restructuring agreements as a formal process through which creditors can grant concessions to debtors facing financial challenges, creating a structured mechanism for addressing distress while allowing the business to remain operational. By fostering negotiated settlements between creditors and distressed firms, OIC 6 supports a collaborative form of financial recovery.

These restructuring agreements bear a close resemblance to the prepackaged plans under Chapter 11 of the U.S. Bankruptcy Code. Like their American counterparts, they operate within the legal framework while allowing for out-of-court negotiations between debtors and creditors before formal proceedings, thereby streamlining the reorganization process and reducing the costs and uncertainties typically associated with full bankruptcy. This approach grants businesses greater flexibility in managing financial distress and helps protect creditor interests by addressing claims more efficiently and predictably.

The foundation of these agreements lies in negotiations between debtors and creditors, aiming to provide a "fresh start" for financially distressed companies. This concept is essential, as it enables firms to re-enter the market with enhanced financial stability and prospects for sustainable operations. By alleviating immediate financial pressures through negotiated concessions, companies gain the opportunity to restructure obligations and realign strategies without resorting to liquidation.

These agreements introduce a range of innovative out-of-court mechanisms to prevent dissolution, bypassing a prolonged and costly court-supervised process. Such tools promote flexible, efficient solutions that benefit both debtor and creditor, preserving company value, maintaining operational continuity, and safeguarding employment and other stakeholder interests *(Altman E.I. et al., 2013).* This approach reflects a shift in corporate finance towards preventative restructuring, aligning with global trends focused on corporate recovery over bankruptcy as the final outcome.

This development aligns with earlier corporate finance literature advocating alternatives to liquidation. For example, *Chen Y. (1995)* emphasized restructuring tools as vital for distressed firms to reorganize and avoid bankruptcy. By centering on negotiated outcomes, these mechanisms provide a practical framework for restoring financial viability, mitigating the broader economic and social impacts of corporate failure.

A key feature of restructuring agreements is the debtor's ability to renegotiate terms with creditors, involving debt reduction, extended repayment periods, or modified loan terms. These proactive measures aim to stabilize financially distressed firms and enable them to pursue long-term operations. By easing immediate financial burdens, these agreements help companies avoid insolvency and remain competitive.

Scholars have characterized financial distress as marked by an unexpected, substantial decline in operational cash flows. *Gilson S.C. (1990), Wruck K.H. (1990),* and *Gilbert L.R. et al. (1990)* highlight this reduction in cash flow as an early signal of a firm's inability to meet financial obligations, which can exacerbate financial instability. *John K. (1993) and Turetsky H.F. and McEwen R.A. (2001)* add that cash flow deficiencies, especially under external shocks or market pressures, can precipitate financial distress. Collectively, these authors emphasize that early detection of such indicators is critical for devising effective intervention strategies, including restructuring agreements, to prevent the escalation into bankruptcy.

Following cash flow declines, further warning signs may emerge, such as reduced dividends, technical loan defaults, or the initiation of troubled debt restructuring (TDR). These indicators signal that a company may be experiencing distress. TDR, in particular, is a strategic mechanism aimed at avoiding liquidation by renegotiating debt terms to provide temporary financial relief, thereby offering an alternative to more drastic actions like liquidation or bankruptcy.

In quantitative models, TDR and liquidation are sometimes treated as equivalent outcomes when forecasting bankruptcy. For example, models by *Beaver W.H. (1966), Ohlson J.A. (1980), and Zmijewski (1984)* often classify firms undergoing TDR as on a similar trajectory to those headed toward liquidation. In these models, TDR represents a final stage in distress, suggesting the company's limited recovery prospects and impending liquidation.

Conversely, some researchers advocate distinguishing financial distress from TDR as distinct events. These models focus on TDR's unique characteristics and financial implications, recognizing that while distress can lead to TDR, the two are not synonymous. Treating TDR as a separate event yields a more nuanced understanding of the restructuring process, enabling more accurate predictions of corporate behavior under financial strain.

Gilbert L.R. et al. (1990) and *Turetsky H.F. and McEwen R.A. (2001)* were among the first to emphasize the importance of differentiating general distress from TDR in predictive modeling. They argued that incorporating TDR-specific indicators, such as renegotiated loan terms, enhances models' ability to forecast whether a firm is likely to restructure debt or enter bankruptcy. This approach allows for more tailored risk assessments, providing valuable insights to creditors, investors, and regulators in understanding financial distress outcomes.

These distinct approaches underscore the growing recognition that not all financially distressed firms follow the same trajectory, and differentiating outcomes like TDR and liquidation improves predictive accuracy in financial distress models. By offering a refined understanding of troubled debt restructuring, these models facilitate more precise assessments of a firm's financial condition, identifying the most suitable intervention strategies. Recognizing

TDR as a recovery path rather than an inevitable precursor to liquidation enhances the capacity for tailored strategies that support firms in overcoming distress and achieving long-term viability.

9. *PREDICTIVE MODELING FOR FINANCIAL DISTRESS AND TDR: ENHANCING THE Z"-SCORE*

This study contributes to the existing body of literature by developing a model that not only forecasts financial distress but also anticipates the likelihood of a subsequent troubled debt restructuring (TDR) request. Departing from conventional approaches that concentrate on outcomes following the approval of TDR, this study emphasizes the period leading up to the submission of a TDR request. This preemptive focus facilitates a dynamic understanding of the financial indicators that precipitate the need for restructuring, prioritizing early warning signals over reactive strategies.

The methodology adopted aligns with the foundational work of scholars such as *Beaver W.H., 1966, Altman E.I., 2000,* and *Ohlson J.A., 1980,* who emphasized the importance of evaluating financial indicators before formal restructuring processes are initiated. By targeting the pre-restructuring phase, this model serves as a proactive tool for identifying high-risk firms, thereby informing timely interventions that could mitigate the need for more severe measures like liquidation or formal bankruptcy. This approach advances both the theoretical and practical application of financial distress prediction models within corporate finance.

Focusing on the pre-TDR period, this study aims to pinpoint critical financial signals that predict both financial distress and the likelihood of a subsequent TDR request. In addressing a significant gap in existing literature, this research also seeks to refine the Z-score model for improved predictive accuracy. Specifically, the study enhances the Z''- Score model to provide earlier, more precise distress predictions, with the ultimate objective of supporting firms in navigating total debt restructuring processes.

To achieve this objective, modifications are introduced to the Z''- Score model, notably by replacing the working capital to total assets ratio with the cash and cash equivalents to current liabilities ratio. This adjustment stems from the understanding that while the working capital to total assets ratio is generally useful in assessing a firm's capacity to generate funds relative to its operational scale, it may not be as directly indicative of distress in contexts where TDR is considered. In contrast, the cash and cash equivalents to current liabilities ratio offers a more focused measure of a firm's ability to meet immediate financial obligations, making it a more relevant indicator of distress in scenarios involving TDR. This modification improves the model's capacity to predict financial difficulties at an earlier stage, offering firms a valuable tool for proactive financial management.

This study explores four key research questions:

1. *which early warning signals or "alerts" need to be identified to prevent financial distress and ensure a firm's "going concern" status?*
2. *what strategic measures do firms adopt in response to financial distress?*
3. *is it feasible to construct a straightforward index that characterizes a firm's financial equilibrium trend based on historical data?*
4. *can a model be developed to estimate the probability that a firm will initiate the TDR procedure?*

The *first research question* investigates the crucial indicators, often called "alerts," that signify a firm's potential risk of financial distress. Financial distress, characterized by difficulty in meeting financial obligations, poses a threat to the firm's status as a *"going concern",* or its capacity to continue operations. Early identification of these signals is essential for effective risk management and business continuity.

Key indicators of financial distress manifest in various forms, especially in financial metrics. These include declining revenue, rising debt levels, and diminished cash flow, all of which may indicate liquidity issues or operational inefficiencies. Beyond financial metrics, operational and strategic factors can also serve as early warnings. For example, declining productivity or supply chain disruptions can adversely affect profitability, while strategic missteps, such as unsuccessful product launches, may undermine financial stability.

This research question seeks to analyze financial, operational, and strategic indicators in detail to identify which specific signals most accurately predict financial distress. By recognizing the timing and nature of these alerts, firms can develop monitoring mechanisms - such as financial ratios, trend analysis, or automated risk assessment tools - that enable early detection of potential problems. The ultimate goal is to help firms implement preventive measures to avoid escalation into more severe distress scenarios, including insolvency or bankruptcy. Preventive actions may include restructuring operations, renegotiating debts, or improving cash flow management, with early intervention being crucial for long-term viability.

The *second research question* examines the strategic responses and interventions that firms employ when faced with financial distress. Short-term strategies often include cost-cutting measures, such as workforce reductions, capital expenditure cuts, and renegotiated supplier contracts, aimed at reducing operational expenses. Companies may also liquidate non-core assets to generate cash flow to meet pressing obligations, though such measures may impact operational capabilities.

Additionally, firms facing distress may seek debt restructuring or renegotiation with creditors, which may involve extending debt maturity, reducing interest rates, or converting debt into equity. Some firms may seek external financing to cover liquidity needs, though these options may entail high costs or ownership dilution. This research will assess the effectiveness of these strategies in resolving financial distress, with considerations for intervention timing, external stakeholder roles, and industry-specific influences on strategy outcomes.

The *third research question* investigates the possibility of constructing a straightforward index that encapsulates a firm's financial equilibrium trend over time, using historical data. This index seeks to distill complex financial information into a single metric that reflects overall financial health and stability, providing stakeholders with a quick yet informative snapshot of the firm's financial trajectory. By analyzing historical financial data to identify key metrics that reflect a firm's equilibrium, an index will be constructed and validated against actual financial outcomes over time.

The *fourth research question* focuses on estimating the likelihood that a firm will pursue TDR by developing a model based on various financial indicators and historical data. This involves analyzing distress signals and patterns that precede a firm's decision to initiate TDR. The model aims to create a predictive tool that provides early warning of TDR need, enabling preemptive actions.

To this end, the study identifies key predictors influencing a firm's likelihood of initiating TDR, including financial ratios and firm-specific characteristics. Advanced statistical techniques, such as logistic regression, will be applied to develop and validate the model using historical data from firms that have and have not undergone TDR.

The research addresses these questions through a systematic approach and is guided by the following hypotheses:

- *H1: firms that actively monitor specific financial alerts are less likely to experience distress and maintain their going concern status;*
- *H2: firms that engage in proactive restructuring, cost-cutting, and strategic realignments are more likely to recover from financial distress;*
- *H3: a straightforward index, based on historical data, can be developed to effectively characterize and predict trends in a firm's financial equilibrium;*

- *H4: a predictive model can be developed to accurately estimate the likelihood of a firm initiating TDR based on key financial indicators and firm-specific characteristics.*

The *first hypothesis* assumes that there are identifiable and measurable signals, or "alerts," that can predict financial distress well before it occurs. These may include financial metrics such as declining revenues, reduced cash flow, and increasing debt levels. Firms that implement a robust monitoring system to track these key indicators will be better positioned to take preventive actions, thereby avoiding the need for costly restructuring or insolvency procedures. The hypothesis rests on the assumption that early detection and timely intervention are critical for mitigating the risk of financial distress.

The *second hypothesis* suggests that specific actions taken by firms during periods of financial distress can significantly influence their chances of survival. Actions such as debt restructuring are commonly employed to stabilize a firm's financial situation. The hypothesis assumes that firms which engage in timely and well-executed recovery strategies have a higher probability of overcoming financial distress compared to those that either fail to act or respond too late.

The *third hypothesis* pertains to the use of an index. It posits that historical financial data can be used to develop an index that characterizes the financial equilibrium trend of a firm. This index will not only be straightforward to construct and interpret but also effective in identifying and predicting financial stability trends over time.

The *fourth hypothesis* revolves around the development of a predictive model. It asserts that financial indicators, such as liquidity ratios, leverage ratios, and profitability metrics, along with firm-specific characteristics like firm size, industry sector, and market conditions, can be utilized to predict the likelihood of a firm entering the TDR procedure.

To test hypotheses, a systematic approach will be employed.

For the first hypothesis, a longitudinal study will be conducted using a sample of firms over a specific period (10 years). Financial indicators such as liquidity ratios, profitability margins, debt levels, and cash flow will be tracked. Firms that maintain going concern status will be compared to those that have faced financial distress or insolvency.

For the second hypotheses, a comparative analysis will be conducted between firms that successfully recovered from financial distress and those that did not. The study will focus on the specific actions taken by firms in distress, such as debt restructuring.

For the third hypothesis, historical financial data will be gathered to identify key metrics, from which an index will be developed and validated by comparing its predictions with actual financial outcomes. The index will be tested using time-series analysis to assess its ability to predict financial distress or instability.

For the fourth hypothesis, data on firms that have and have not undergone TDR will be collected, and relevant financial indicators will be identified. The accuracy of this model will be validated through statistical techniques and historical data, underscoring its practical utility, such as *logistic regression*, will be developed to estimate the likelihood of a firm initiating a TDR procedure. The model will be validated using *cross-validation techniques* to assess its accuracy and predictive power.

These hypotheses provide a clear, testable foundation for the research, ensuring that the questions posed are explored thoroughly and effectively.

CHAPTER 3
RESEARCH METHODOLOGY

1. AIMS

The objective of this research is to develop a predictive model for corporate failure by employing two statistical techniques: *linear discriminant analysis (LDA)* and *logistic regression*. These methodologies will be applied to create a troubled debt restructuring (TDR) probability model that enables the prediction of business failure with a lead time of up to three years. LDA will be used to classify firms based on financial ratios and other key indicators into categories of financial health or distress, while logistic regression will estimate the probability of a company entering TDR by analyzing the relationship between predictor variables and the likelihood of restructuring. By integrating these two approaches, the study aims to construct a comprehensive model that not only identifies early warning signs of financial distress but also offers a robust framework for anticipating the likelihood of TDR, thus providing firms with actionable insights to mitigate potential failures.

2. SELECTION OF THE METHODOLOGY FOR MODEL DEVELOPMENT

Although artificial intelligence (AI) and machine learning (ML) methods have demonstrated superior performance in predicting bankruptcy *(Barboza F. et al., 2017; Zelenkov Y. et al., 2017),* I recognize that their practical implementation in real-world settings remains limited. Despite advancements in predictive models, these approaches have not been widely adopted by regulators and corporations. This highlights the need for models that are not only accurate but also interpretable and applicable in practice *(Bellovary J.L. et al., 2007).*

In response to this gap, I aim to build on traditional statistical methods, specifically linear discriminant analysis (LDA) and logistic regression, to refine the well-established Z''-Score model. While AI and ML techniques offer strong predictive capabilities, I believe that the interpretability and practical relevance of statistical models make them essential tools for decision-makers. To enhance the predictive accuracy for financial distress, particularly in relation to troubled debt restructuring (TDR), I introduce an additional financial ratio - the *cash and cash equivalents to current liabilities ratio* - into the Z''- Score model. This ratio is chosen because it provides a more direct measure of a firm's liquidity and its ability to meet short-term obligations, which is critical in identifying firms likely to enter TDR *(De Luca F. and Mehmood A., 2023).*

Building on previous recommendations that emphasize the importance of incorporating further financial ratios to improve predictive models *(Zelenkov Y. & Volodarskiy N., 2021),* I argue that the addition of the cash and cash equivalents to current liabilities ratio will provide a more precise early warning system. This will allow for a better assessment of financial distress and TDR likelihood, addressing both the theoretical and practical demands of financial distress prediction.

In this study, I aim to refine and extend the Z''-Score model by employing LDA and logistic regression, focusing on liquidity ratios that are particularly relevant for companies facing financial distress. My objective is to create a model that not only improves prediction accuracy but also serves as a practical tool for companies, regulators, and financial institutions in managing financial distress.

2.1. Linear Discriminant Analysis (LDA)

Linear Discriminant Analysis (LDA) is a widely recognized statistical technique used primarily for classification tasks. This method classifies an individual observation into one of several predefined groups based on the unique characteristics of the observation. LDA proves particularly valuable for both classification and prediction purposes, facilitating the determination of group membership according to the independent variables under study.

LDA operates by identifying a linear combination of features that best distinguishes two or more classes. The primary objective of this technique is to maximize the ratio of between-class variance to within-class variance within a given dataset, thereby ensuring maximum separability. This attribute makes LDA especially useful in scenarios where groups exhibit linear separability.

In my study, LDA is employed to classify firms into two distinct categories: distressed and non-distressed. The classification is based on a set of independent variables indicative of the financial health of the firms. By analyzing these variables, LDA assists in predicting financial distress, enabling the identification of at-risk companies (*Fisher R. A., 1936).*

The implementation of LDA begins with the meticulous preparation and preprocessing of the dataset. This phase involves the collection of a comprehensive set of financial metrics for each firm under study. Typical metrics include liquidity ratios, leverage ratios, profitability measures, and other relevant financial indicators included in the Z"-score *(Altman E. I., 2000).*

Ensuring data integrity is of utmost importance in the analytical process, as it directly influences the accuracy and reliability of the results. Therefore, the dataset must be meticulously cleaned, structured, and appropriately scaled to facilitate precise analysis. Any missing values must be addressed through appropriate imputation techniques or removal, depending on the nature and extent of the missing data. Additionally, outliers should be carefully examined to assess their potential impact on the analysis. While some outliers may contain valuable information, others may disproportionately skew the results, necessitating careful consideration of whether to retain, adjust, or remove them *(Hastie T., Tibshirani R., & Friedman J., 2009)*. By ensuring that the data is robust and well-prepared, we can improve the overall validity and reliability of the subsequent analysis.

With a complete dataset, where the sample size is small, Linear Discriminant Analysis (LDA) can be applied directly to the entire dataset without the need for splitting into training and testing sets. In this context, LDA estimates the parameters of the discriminant function by utilizing the full sample, where the group membership (distressed or non-distressed firms) of each observation is known. This approach is particularly useful when the sample size is limited, as dividing the dataset could reduce the robustness of the analysis due to the small number of observations.

In such cases, LDA constructs a discriminant function - a linear equation that combines the independent variables - in a manner that maximally separates the two groups, even when using the entire dataset. The function is designed to find the linear combination of variables that provides the greatest distinction between the distressed and non-distressed firms. This is achieved by maximizing the ratio of the between-group variance (the differences between the groups) to the within-group variance (the variation within each group), ensuring that the groups are as distinct as possible.

Although the absence of a separate testing set means the model cannot be validated for its predictive accuracy on unseen data, the use of the full dataset can still yield valuable insights, especially when the primary goal is to understand the relationships within the current data rather than to forecast future outcomes. In these situations, LDA serves as an effective tool for classification and interpretation, providing meaningful distinctions between groups based on the characteristics of the entire sample *(McLachlan G. J., 2004)*.

By applying LDA to the complete dataset, researchers can leverage all available information to construct a model that offers a robust classification of distressed and non-distressed firms, even in cases where the data is limited in size. This approach is appropriate for studies where the focus is on maximizing the utility of a small dataset while maintaining the ability to classify firms based on their financial characteristics.

For each new observation, the discriminant function generates a score that reflects the probability of the observation belonging to one of the predefined groups. This score is calculated based on the values of the independent variables in the discriminant function, which have been optimized to distinguish between the groups - in this case, distressed and non-distressed firms. The observation is then classified into the group with the highest score, indicating the group to which it most likely belongs.

This classification capability of LDA is particularly valuable in the context of predicting financial distress in firms. By assigning observations (firms) to their respective groups based on their financial characteristics, LDA serves as a critical tool for risk assessment. It allows decision-makers in financial management to identify firms that are at higher risk of financial distress, enabling more informed decisions about interventions, resource allocation, and strategic planning. Ultimately, this predictive capability enhances the ability of firms and financial institutions to proactively manage financial risk and take necessary steps to mitigate potential negative outcomes (*McLachlan G. J., 2004).*

In cases where the entire dataset is used for model building due to a small sample size, the validation of the LDA model's efficacy takes a different approach. Instead of relying on a separate testing set, techniques such as *cross-validation* are employed to assess the model’s performance. In cross-validation, the dataset is divided into several folds, where the model is trained on a subset of the data and tested on the remaining portion, and this process is repeated across all folds. This method allows for a more reliable evaluation of the model’s robustness despite the absence of a distinct testing phase.

To evaluate the predictive capabilities of the model, various performance metrics such as accuracy, precision, recall, and the confusion matrix are calculated for each fold. These metrics provide insights into how well the model classifies distressed and non-distressed firms across different portions of the dataset. This approach ensures that, even with a small dataset, the model can be validated effectively, offering confidence in its ability to provide reliable predictions for real-world financial risk assessment (*Hastie T., Tibshirani R., & Friedman J., 2009)*.

2.1.1. Advantages and Limitation of LDA

Linear Discriminant Analysis (LDA) presents several notable advantages and limitations that are essential to consider in both research and practical applications. One of the primary strengths of LDA lies in its simplicity and ease of interpretation. The linear discriminant function offers a clear and concise mathematical representation of the classification process, which enhances its accessibility to a wide range of users. Unlike more complex models, LDA provides results that are relatively easy to interpret and communicate, making it particularly useful in contexts where transparency and explainability are crucial.

This interpretability is of significant value in fields such as finance, medicine, and social sciences, where decision-makers require not only accurate classifications but also an understanding of the underlying reasoning behind those classifications. By offering a linear combination of the input variables, LDA allows for straightforward interpretation of the contribution of each variable to the classification outcome. This ability to explain and justify the results makes LDA a valuable tool in scenarios where the trustworthiness and clarity of the model are as important as its predictive performance *(McLachlan G. J., 2004).*

LDA is also highly regarded for its computational efficiency, making it well-suited for handling large datasets with minimal computational burden. This efficiency allows LDA to process substantial amounts of data relatively quickly, making it particularly useful in applications where real-time or near-real-time classification decisions are necessary. In contrast to more complex models, which may require extensive computational resources and processing time, LDA is able to achieve accurate classification outcomes without imposing significant overhead.

This computational advantage is particularly beneficial in domains where rapid decision-making is critical, such as finance, healthcare, and industrial monitoring, where timely and reliable classification is essential. The ability of LDA to manage large datasets efficiently further enhances its appeal, allowing it to be applied in a wide range of analytical tasks, including those requiring high-speed data processing (*Hastie T., Tibshirani R., & Friedman J., 2009)*.

Despite its strengths, the performance of LDA can be significantly hindered when its underlying assumptions are violated. One of the key assumptions of LDA is that the predictor variables are drawn from a multivariate normal distribution. Additionally, LDA assumes homoscedasticity, meaning that the covariance matrices of the different groups are equal. These assumptions are crucial for ensuring that the linear discriminant function can effectively distinguish between the groups.

When these assumptions are not met, the model's classification accuracy and overall reliability may be compromised. For instance, if the data deviates from a normal distribution or if the covariance matrices of the groups differ substantially, LDA may produce biased results or fail to separate the groups adequately. In such cases, alternative methods, such as Quadratic Discriminant Analysis (QDA), which does not assume equal covariance matrices, may be more appropriate for improving classification performance (*Fisher R. A., 1936).*

Additionally, as a linear classifier, LDA may struggle to capture complex, non-linear relationships between the predictor variables and group memberships. The method assumes a linear boundary between the groups, which can be limiting in scenarios where the true relationship between the variables and the classification outcome is inherently non-linear. In such cases, LDA may fail to accurately model the data, leading to suboptimal classification performance.

When confronted with non-linear patterns, more flexible classification techniques, such as Support Vector Machines (SVM), decision trees, or neural networks, may provide superior results. These methods are better equipped to handle non-linearities by allowing for more complex decision boundaries. While LDA remains an effective tool in cases where the linearity assumption holds, its performance may be significantly diminished in situations where the data structure requires a more nuanced, non-linear approach (*Hastie T., Tibshirani R., & Friedman J., 2009)*.

While LDA is a powerful and efficient tool with notable strengths, particularly in terms of simplicity and computational efficiency, its reliance on specific statistical assumptions and its linear nature can impose limitations on its applicability in certain contexts. It is essential to recognize both the advantages and constraints of LDA to ensure its effective use in research and practical applications. By doing so, researchers and practitioners can better determine when LDA is the most suitable tool or when alternative methods may offer improved performance.

In the specific context of predicting financial distress, LDA provides a valuable framework that leverages key financial indicators and firm-specific characteristics to classify firms according to their risk of distress. This classification system enables stakeholders, such as investors, financial managers, and analysts, to identify firms that may be facing financial difficulties. As a result, stakeholders can take proactive steps to mitigate risks, ensuring that they can respond to potential issues before they escalate.

By applying LDA to the analysis of financial distress, this research aims to develop a robust predictive model capable of accurately determining the financial distress status of firms. Such predictive capability is essential for enhancing financial management practices, as it allows for more informed decision-making and timely interventions. Moreover, this model can contribute to broader economic stability by improving the early identification of at-risk firms, potentially averting more severe financial crises.

Through the integration of LDA in the study of financial distress, this research not only advances predictive modeling capabilities but also contributes to the wider field of financial economics. By offering a methodologically rigorous approach to risk assessment and management, this work enhances the tools available to researchers and practitioners for understanding and mitigating financial risks, ultimately supporting more resilient financial systems.

This version maintains an academic tone while clearly articulating the application of LDA in financial distress prediction, highlighting both its contributions to predictive modeling and its broader impact on financial management and economic stability.

2.1.2. Application of LDA in previous studies

Linear Discriminant Analysis (LDA) has been extensively utilized in prior studies focusing on financial distress analysis, particularly due to its effectiveness in classification tasks. One of the most prominent applications of LDA in this field was pioneered by Altman (1968), who introduced the Z-score model - an influential tool for predicting corporate bankruptcy. In this seminal work, Altman employed LDA to distinguish between distressed and non-distressed firms by analyzing a set of financial ratios, such as working capital to total assets and retained earnings to total assets. The Z-score model demonstrated the capability of LDA to effectively classify firms into risk categories, based on their financial health, and has since become a widely recognized method in bankruptcy prediction and financial risk assessment *(Altman E. I., 1968).*

Altman's application of LDA in the development of the Z-score model not only showcased the utility of this technique for financial distress prediction but also set a foundational precedent for subsequent studies in the field. The model's success highlighted LDA's capacity to leverage financial data to identify firms at risk of failure, making it a vital tool for financial analysts, investors, and decision-makers in assessing corporate solvency.

Further validation and refinement of LDA in the context of financial distress prediction were provided by *Altman E.I. et al. (2017),* who expanded upon the original Z-score model to account for evolving financial environments and diverse industry settings. This study aimed to adapt the Z-score model to contemporary financial realities, reflecting changes in the economic landscape and the increasing complexity of corporate financial structures. By applying LDA to a broader range of industries and economic conditions, Altman et al. demonstrated the robustness and versatility of the technique in predicting financial distress across various contexts.

Their research not only reaffirmed the utility of LDA as a reliable tool for financial distress analysis but also highlighted its capacity to remain effective even as financial environments evolve. The ability of LDA to adapt to different industries and reflect contemporary financial challenges underscores its enduring relevance in risk assessment and corporate solvency analysis (*Altman E.I. et al., 2017).* This extension of the original Z-score model has reinforced the value of LDA in predicting financial distress, offering a more nuanced approach to identifying at-risk firms in diverse economic contexts.

In addition, De Luca and Meschieri (2017) applied Linear Discriminant Analysis (LDA) to a comprehensive dataset, further confirming its effectiveness in financial classification tasks. Their research demonstrated the method's capacity to accurately predict financial distress by analyzing modern financial data, reinforcing the robustness of LDA in contemporary settings. Through their empirical study, De Luca and Meschieri highlighted the versatility of LDA in handling complex financial indicators, showing that the technique remains highly applicable for identifying firms at risk of distress in today's financial environment.

Their findings not only validated the predictive power of LDA but also underscored its practical relevance for modern financial analysis. By applying LDA to current financial data, they provided evidence of the method's ability to adapt to evolving financial landscapes, making it a valuable tool for analysts and decision-makers in assessing corporate financial health *(De Luca F. & Meschieri R., 2017).*

A notable advantage of Linear Discriminant Analysis (LDA), as highlighted by Altman et al. (2017), is its effectiveness when applied to small sample sizes. This characteristic is particularly beneficial in contexts where large datasets are either unavailable or difficult to obtain. LDA's ability to provide meaningful and reliable classification results despite limited data ensures that it remains a valuable tool for financial distress prediction and other classification tasks.

The method's efficiency with smaller datasets makes it especially suitable for specialized industries or markets where data collection is challenging, or the number of observations is inherently limited. By maximizing the utility of available data, LDA enables researchers and practitioners to draw significant insights, even in data-constrained environments, further reinforcing its robustness and versatility as a classification tool *(Altman E.I. et al., 2017).*

2.1.3. The case for utilizing LDA in this research

The decision to utilize Linear Discriminant Analysis (LDA) in this thesis is grounded in several compelling reasons that align with the research objectives and the nature of the data involved.

LDA has a proven track record of accuracy and reliability in classifying observations into distinct groups based on predictor variables. Its ability to discern subtle differences between groups makes it particularly suitable for financial distress prediction, where accurately distinguishing between distressed and non-distressed firms is critical *(Altman E.I., 1968; Altman E.I. et al., 2017).*

This high level of predictive accuracy is essential for the robustness and credibility of the research findings.

One of the primary advantages of LDA is its interpretability. The linear discriminant function provides a clear and straightforward mathematical representation of the classification process, making it accessible to a broad audience, including practitioners and stakeholders in the financial sector. This interpretability facilitates the communication of results and supports the practical application of the findings in real-world scenarios *(McLachlan G. J., 2004).*

LDA is computationally efficient, capable of handling large datasets with relative ease. This efficiency is particularly advantageous in the context of financial data analysis, where large volumes of data need to be processed quickly and accurately. The method's computational efficiency ensures that the analysis can be conducted within reasonable time frames without compromising the quality of the results (*Hastie T., Tibshirani R., & Friedman, J., 2009*).

Another significant reason for choosing LDA is its effectiveness with small sample sizes. In financial research, especially when dealing with specific sectors or emerging markets, obtaining large datasets can be challenging. LDA's robustness in handling smaller samples without a significant loss in accuracy makes it an ideal choice for this research, ensuring that meaningful insights can be derived even from limited data (Altman E.I. et al., 2017).

The use of LDA in predicting financial distress is well-documented and validated in the literature. Studies by *Altman E.I. (1968)* and subsequent research by *Altman E.I. et al. (2017*) and *De Luca F. and Meschieri E. (2017)* have demonstrated the efficacy of LDA in financial distress prediction. These precedents provide a solid methodological foundation and enhance the credibility of using LDA in this thesis.

Thus, the selection of LDA is driven by its predictive accuracy, interpretability, computational efficiency, suitability for small sample sizes, and its established use in financial distress prediction. These factors collectively make LDA a robust and appropriate statistical model for the objectives of this thesis.

2.2. Logistic regression model

The decision to employ logistic regression in this study was driven by several key considerations. Primarily, logistic regression was selected due to its ease of use, particularly when compared to more complex algorithms such as genetic algorithms or neural networks. Logistic regression requires significantly less technical expertise in computer science, making it a practical and accessible choice for researchers and analysts with varying levels of technical proficiency. Its relative simplicity does not compromise its effectiveness, as logistic regression remains a robust and reliable tool for classification tasks, especially in fields such as finance and economics, where interpretability and ease of application are highly valued *(Hosmer, D. W., Lemeshow, S., & Sturdivant, R. X., 2013).*

A significant advantage of selecting logistic regression is its capacity to effectively manage qualitative variables, a key aspect in many classification models. This ability is particularly important, as research has shown that incorporating qualitative variables can greatly enhance a model's predictive accuracy. Logistic regression allows for the inclusion of categorical variables, thereby increasing the model's flexibility and its ability to capture essential relationships that quantitative variables alone might fail to identify. This improvement in predictive performance is not exclusive to logistic regression; it extends to a variety of other models as well, highlighting the critical role of qualitative data in improving overall model effectiveness *(Ciampi F. and Gordini N., 2016).*

Logistic regression offers a straightforward means of incorporating categorical predictors by encoding them as dummy variables, which facilitates the modeling of more complex relationships. This approach enables the representation of categorical data in a numerical format suitable for regression analysis, allowing the model to account for the influence of different categories within the predictor variables. By doing so, logistic regression enhances its flexibility and capacity to model a wide range of relationships, making it particularly effective in capturing nuances within categorical data *(Menard S., 2002).*

Moreover, logistic regression provides a distinct advantage in terms of the interpretability of results. Researchers and practitioners often find it significantly easier to interpret the outcomes produced by logistic regression compared to those generated by more complex methodologies, particularly when conveying findings to non-technical audiences. The model's coefficients can be exponentiated to yield odds ratios, which offer a straightforward and intuitive means of understanding the relationships between predictor variables and the outcome. This feature enhances the accessibility of logistic regression, making it a valuable tool for both academic research and practical applications where clear communication of results is essential *(Peng C. Y. J., Lee K. L., & Ingersoll G. M., 2002).*

The combination of these characteristics has played a pivotal role in the widespread adoption and enduring success of logistic regression. This technique has been extensively utilized in the academic literature, particularly in recent years, across various fields of study. Its popularity can be attributed to its ability to strike a balance between predictive accuracy, ease of use, and result interpretability. Logistic regression's versatility and accessibility make it an invaluable tool in the realm of predictive modeling and statistical analysis, enabling researchers and practitioners to apply it effectively in both theoretical and applied contexts (*Kleinbaum D. G., & Klein M., 2010).*

Additionally, logistic regression is robust to multicollinearity, meaning that it can tolerate moderate correlations between predictor variables without severely compromising the model's estimates. Furthermore, logistic regression does not require the assumption of normality in the distribution of predictor variables, which further extends its applicability. These features make logistic regression particularly versatile, allowing it to be effectively employed in a wide range of scenarios where other statistical methods may falter due to stringent assumptions (*Cox D. R., 1958*).

Thus, logistic regression distinguishes itself as an accessible, powerful, and interpretable method for binary classification tasks. Its ability to handle qualitative data with ease, combined with its straightforward interpretation and relatively low computational demands, makes it an optimal choice for a wide range of researchers and analysts. These advantages have contributed to its widespread application across various disciplines, where it has consistently demonstrated its effectiveness. As a result, logistic regression has become a foundational technique in both statistical analysis and predictive modeling, reinforcing its reputation as a versatile and reliable tool for binary outcome prediction.

2.2.1. The Role of Logistic Regression in Predicting TDR Filings

The logistic regression model is a vital statistical tool in the field of financial risk management, particularly in evaluating the likelihood that a company will file for a Troubled Debt Restructuring (TDR) as a preventive measure against bankruptcy. Its capacity to handle binary outcomes makes it especially well-suited for predicting such significant financial events, where the objective is to determine the probability of a firm transitioning into a distressed state. By providing a clear framework for assessing the factors that contribute to a company's decision to pursue TDR, logistic regression enables financial analysts and decision-makers to implement timely and informed risk mitigation strategies.

In previous literature, *Shumway T. (2001)* utilized a logistic regression model to estimate the probability of firm failure. His work marked a significant contribution to the field of bankruptcy prediction by introducing a dynamic framework that accounted for time-varying covariates. This approach allowed for a more precise estimation of the likelihood of corporate failure, addressing the shortcomings of earlier static models. By leveraging logistic regression, Shumway's model offered a more comprehensive understanding of how both financial indicators and macroeconomic conditions impact the risk of firm failure over time.

Additionally, the pioneering studies by Beaver and Altman applied a similar approach to forecasting firm failure through the use of financial ratios *(Beaver W.H., 1966; Altman E.I., 1968, 1983).* These analyses demonstrated that financial ratios are reliable predictors of business failure, providing early warnings about a firm's financial health prior to liquidation *(Beaver W.H., 1966).* Altman built upon this framework by selecting five key financial ratios and developing a predictive model using Multiple Discriminant Analysis (MDA) *(Altman E.I., 1968).* His work established the relationship between financial ratios both in the years leading up to and following bankruptcy, offering valuable insights into the financial behavior of distressed firms (*Altman E.I., 1968, 1983).*

In contrast to the broader literature, only a few studies have specifically focused on determining the probability of a company filing for Troubled Debt Restructuring (TDR) by employing a logistic regression model. These works represent a critical contribution to the field by applying logistic regression to assess the likelihood of firms opting for TDR as a preventive measure against financial distress, thereby addressing a gap in the existing research on corporate risk management (*De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).*

2.2.2. *Probability model: Structure and Application*

Logistic regression functions by modeling the probability that a specific event will occur—in this case, the submission of a Troubled Debt Restructuring (TDR) request—based on a set of predictor variables. These predictor variables typically encompass a range of financial indicators, including liquidity ratios, leverage ratios, profitability measures, and other key financial metrics that provide insights into a company's overall economic health. By examining the relationship between these predictors and the likelihood of a TDR filing, logistic regression enables the identification of factors that are most strongly associated with financial distress, offering a robust framework for risk assessment and decision-making.

The logistic regression equation used in this context can be represented as follows:

$$\hat{P}_{i,t} = \frac{e^{L-Z"-Score\ i,t}}{1+e^{L-Z"-Score\ i,t}}$$

where:

- $\hat{P}_{i,t}$ represents the probability that the *i*-th firm will file for Troubled Debt Restructuring (TDR) at time *t*. This probability is modeled using a logistic regression framework.

Specifically, the probability is calculated as a function of the firm's financial characteristics, which are summarized by the L-Z''-Score;

- ***e*** the Euler number (approximately 2.718), which is the base of the natural logarithm. In logistic regression, the Euler number is used to model the logistic function, which transforms the output of the linear combination of predictor variables (in this case, the L-Z''-Score) into a probability that lies between 0 and 1;
- L-Z''-Score $_{i,t}$ refers to a score calculated for the *i*-th firm at time *t*, which reflects the firm's financial health. This score is derived from a combination of financial ratios or indicators and is used to estimate the firm's likelihood of encountering financial distress. The L-Z''-Score serves as the input to the logistic function, and its value influences the probability $\hat{P}_{i,t}$. A higher L-Z''-Score typically indicates a lower likelihood of financial distress, and thus a lower probability of TDR, while a lower score would suggest a higher risk of financial distress and a greater likelihood of filing for TDR. Specifically, the L-Z''-Score is computed using the following linear combination of financial ratios for the *i*-th firm at time *t*:

$$\text{L-Z''-Score}_{i,t} = \beta_1 \cdot X1_{i,t} + \beta_2 \cdot X2_{i,t} + \beta_3 \cdot X3_{i,t} + \beta_4 \cdot X4_{i,t}$$

where:

- $X1_{i,t}$ (Cash and Cash Equivalents/Current Liabilities), $X2_{i,t}$ (Retained Earnings/Total Assets), $X3_{i,t}$ (Earnings before Interest and Taxes/Total Assets), $X4_{i,t}$ (Book Value of Equity/Total Liabilities) represent key financial ratios for the *i*-th firm at time *t*;
- the coefficients (β_1, β_2, β_3 e β_4) indicate the weight each ratio contributes to the overall L-Z''-Score.

In this setup, the logistic regression equation transforms the L-Z''-Score into a probability $\hat{P}_{i,t}$ by using the logistic function, which ensures that the probability is appropriately bounded between 0 and 1.

In fact, a lower $\hat{P}_{i,t}$ value indicates that the firm is likely in good financial health, meaning the likelihood of needing TDR is low. On the other hand, a higher $\hat{P}_{i,t}$ suggests that the firm is experiencing financial distress and is at greater risk of filing for TDR. Essentially, as the TDR probability increases, the firm's financial condition worsens, making $\hat{P}_{i,t}$ inversely related to the firm's overall financial health risk *(De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).*

In this study, I calculate $\hat{P}_{i,t}$ on an annual basis. For companies identified as financially distressed, I focus on the three years preceding their Troubled Debt Restructuring (TDR) request to capture pre-crisis financial trends. For non-distressed companies, I analyze data spanning from 2011 to 2019, allowing for a broader assessment of their financial health over time. This approach ensures that I have a comprehensive dataset for both distressed and non-distressed companies, enabling more accurate comparisons and predictions.

It should be noted that the financial data used in this study covers the period from 2011 to 2019. However, for the purposes of the analysis, this period corresponds to the years 2012 to 2020, as these reflect the data officially published by the Italian Chamber of Commerce. This adjustment ensures that the analysis is based on the most up-to-date and officially recognized financial information available.

2.3 Estimation and Interpretation of Coefficients

In Linear Discriminant Analysis (LDA), the coefficients are estimated based on the maximization of the ratio of between-group variance to within-group variance, with the aim of finding a linear combination of predictor variables that best separates the groups *(Fisher R.A., 1936).*

These coefficients are derived under the assumption of multivariate normality and equal covariance matrices across groups. LDA seeks to identify the linear boundaries that optimally classify observations into their respective groups, by maximizing the separation between the group means relative to the variance within each group (*McLachlan G.J., 2004).*

In this study, I estimate the coefficients in the L-Z''-Score using Linear Discriminant Analysis (LDA) based on specific financial ratios for each firm based on different industries based on SIC (Standard Industrial Classification). The procedure differs depending on the financial condition of the firm:

- *for distressed firms,* I utilize data from the three years preceding their filing for Troubled Debt Restructuring (TDR). This approach allows me to capture the financial trends leading up to the distress event, ensuring that the model reflects the firm's declining financial health over time;

- *for non-distressed firms*, I select the best-performing financial ratios from any three years within the period from 2011 to 2019. This method ensures that the most representative financial data for these firms is used, enabling a more accurate comparison with the distressed firms and improving the overall predictive power of the model.

By adopting this approach, I aim to develop a robust discriminant function that accurately separates distressed and non-distressed firms based on their financial characteristics, using the L-Z''-Score as the central measure of risk.

I employ this approach to estimate the coefficients because it leverages the worst financial ratios for distressed firms and the best ratios for non-distressed firms. This strategy enhances the discriminative power of the ratios, allowing the model to effectively distinguish between the two groups. Additionally, by selecting ratios from different years, this method minimizes temporal correlation, ensuring that the data captures meaningful variations across time rather than simply reflecting trends from the same period; this approach improves the model's ability to accurately classify firms based on their financial health risk *(De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).*

By adopting this approach, I ensure an efficient estimation of the L-Z''-Score$_{i,t}$ modified model. The key assumption underlying Linear Discriminant Analysis (LDA) is the independence of sample observations, and I meet this requirement by selecting the worst financial ratios for distressed firms and the best ratios for non-distressed firms, drawn from different years. This method not only enhances the model's discriminative capacity but also ensures that the data is independent, as it avoids temporal correlation between the observations risk *(De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).*

However, as noted in earlier literature, this assumption of independent observations has rarely been fully satisfied risk *(De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).*

In many studies, the use of financial data over consecutive years often leads to temporal correlation, which can compromise the independence of the observations and, consequently, the accuracy of the model. By carefully selecting the worst and best financial ratios from different years, I aim to address this common issue and ensure that the assumption of independence is upheld in my analysis.

I observe a difference in the coefficients of the modified Z''-Score$_{i,t}$ model (L- Z''-Score$_{i,t}$) when compared to the traditional Z''-Score *(Altman E.I., 1983).* This variation is primarily due to a key modification in the formula: the replacement of the *Working Capital/Total Assets* ratio with the *Cash and Cash Equivalents/Total Liabilities* ratio. This substitution was made to better capture the short-term solvency of firms, as liquidity is often a more direct indicator of a company's ability to meet immediate obligations. Additionally, the same financial ratios can have varying effects depending on the specific context, and this distinction is particularly important when differentiating between bankruptcy/failure and financial distress. The Modified Z''-Score is thus tailored to focus on financial distress, which does not necessarily result in bankruptcy, thereby enhancing the model's predictive accuracy.

Hence, these financial ratios can play a crucial role in influencing a firm's decision to pursue a Troubled Debt Restructuring (TDR) agreement. Ratios that reflect liquidity and overall financial health (*Cash and Cash Equivalents/Total Liabilities* and *Working Capital/Total Assets)*, offer important insights into a firm's capacity to manage short-term liabilities. When these ratios indicate financial stress or a lack of liquidity, a firm may be more likely to consider a TDR agreement as a preventive measure to restructure its debt and avoid further financial decline or bankruptcy risk *(De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).*

The modified Z''-Score$_{i,t}$ model (L- Z''-Score$_{i,t}$) is developed through a comprehensive multi-industry and multi-year analysis, incorporating data from seven distinct industrial sectors: *construction, manufacturing, transportation-communications-electric-gas-sanitary services, wholesale trade, retail trade, finance-insurance-real estate, and services.* This broad scope allows for a more nuanced understanding of financial distress across different industries over time. In contrast, the original Z''-Score model, while also applicable to multiple industries, is based on a more limited single-year dataset. This distinction highlights the enhanced capacity of the Modified Z''-Score model to account for variations in financial health across both industries and time periods, offering a more dynamic and robust predictive tool risk *(De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).*

Moreover, the modified Z''-Score$_{i,t}$ model is specifically designed to estimate $\hat{P}_{i,t}$, the probability that a firm will file for Troubled Debt Restructuring (TDR). This probabilistic approach allows for a more nuanced understanding of a firm's financial condition by quantifying the likelihood of financial distress. In contrast, the original Z''-Score model was developed to identify a cut-off value, distinguishing between firms likely to go bankrupt and those expected to remain solvent. This difference in approach highlights the evolution of financial distress prediction, with the Modified Z''-Score model offering a more dynamic and precise method for assessing a firm's financial risk *(De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).*

In this analysis, I calculate the average TDR probability for each year, separately for both distressed and non-distressed firms. This step is taken to better understand the trends and dynamics of TDR probabilities over time for both groups. By averaging the probabilities annually, I aim to highlight how the likelihood of filing for Troubled Debt Restructuring fluctuates in relation to a firm's financial condition. For distressed firms, this calculation may reveal how financial stress builds up in the years leading to TDR, while for non-distressed firms, it provides a baseline of financial health. By comparing these trends across years, I am able to draw meaningful conclusions about the differences in financial behavior between distressed and non-distressed firms, providing a more nuanced understanding of the factors contributing to financial distress.

3. DATASET and PROCEDURES

I utilize the Bureau Van Dijk (BvD) ORBIS database to extract data on large and medium-sized private firms in Italy. This database offers comprehensive and reliable financial information, making it an ideal source for constructing my dataset. I categorize the selected firms into various industrial sectors, including *construction, manufacturing, transportation-communications-electric-gas-sanitary services, wholesale trade, retail trade, finance-insurance-real estate,* and *services*. By dividing the firms into these distinct sectors, I ensure that the model captures the unique financial dynamics and characteristics specific to each industry, thereby enhancing the robustness and accuracy of the analysis.

In this study, I focus on firms that are categorized as large and medium-sized. To ensure consistency, I apply the thresholds for large and medium-sized firms as set by the European Union. These thresholds are based on the criteria outlined in the COMMISSION DELEGATED DIRECTIVE (EU) 2023/2775 of 17 October 2023**,** which amends Directive 2013/34/EU of the European Parliament and Council. The updated size criteria for micro, small, medium-sized, and large undertakings or groups come into effect on 1 January 2024. This framework provides a standardized and legally recognized method for determining firm size, ensuring that my analysis adheres to the most current regulatory guidelines *(see Table 1).*

Table 1 – EU standards for company size classification

Enterprise Size	*Balance Sheet (EUR)*	*Annual turnover (EUR)*	*Annual Work Unit*
Micro Enterprises	≤ 450,000	≤ 900,000	< 10
Small Enterprises	≤ 5,000,000	≤ 10,000,000	< 50
Medium Enterprises	≤ 25,000,000	≤ 50,000,000	< 250
Large Enterprises	> 25,000,000	> 50,000,000	>250

I have deliberately excluded small firms from the analysis due to the inherent instability in their financial ratios, which makes them less reliable for inclusion in a failure prediction model. As noted by *Balcaen S. and Ooghe H. (2006),* the financial ratios of small firms tend to be more volatile and less consistent, which could skew the results and reduce the model's predictive accuracy. By focusing solely on medium and large firms, I ensure that the model is based on more stable and representative financial data, ultimately enhancing the robustness and reliability of the failure prediction analysis.

In my analysis, I have identified the key legal events related to Troubled Debt Restructuring (TDR) specifically for Italian firms across the seven sectors considered. These sectors include *construction, manufacturing, transportation-communications-electric-gas-sanitary services, wholesale trade, retail trade, finance-insurance-real estate,* and *services*. I focus on default as the primary legal event, and within this category, I have selected the relevant debt restructuring procedure, namely debt arrangement proceedings. This process allows me to identify firms within these sectors that filed for TDR between 2015 and 2020. By tailoring the analysis to these specific sectors, I ensure that the model accurately reflects the financial and legal dynamics relevant to each industry, providing a more precise understanding of TDR filings within the Italian context.

To obtain the study sample, I establish several key criteria for selecting the companies. These requirements ensure that the sample is both representative and relevant to the research objectives:

- *Active Firms:* both distressed and non-distressed companies must be active during the period under consideration. This criterion ensures that the financial data accurately reflects the current business operations of the firms, providing a more precise understanding of their financial health and restructuring needs;
- *Legal Structure*: the sample includes only companies whose owners have limited liability**,** which excludes sole proprietorships and partnerships. This focus on corporate entities

provides a more structured and consistent legal framework for financial reporting and restructuring, ensuring uniformity across the sample.

- *Accounting Standards*: the companies must adhere to local Generally Accepted Accounting Principles (GAAP), as the study focuses on private large and medium-sized firms. Listed and larger companies, which follow International Accounting Standards (IAS) and International Financial Reporting Standards (IFRS), are excluded to maintain consistency in the accounting practices of the sample;
- *Industry Focus*: the analysis is limited to non-financial firms, meaning that banks and insurance companies are excluded from the study. Financial institutions operate under different restructuring laws and regulatory frameworks, making them unsuitable for the study's focus on corporate restructuring in non-financial sectors.

By applying these selection criteria, I ensure that the sample remains homogeneous and reflective of the types of firms most relevant to this analysis, providing a solid foundation for examining financial distress and restructuring practices.

I categorize the firms into two distinct groups: distressed firms, which are those that have filed for Troubled Debt Restructuring (TDR), and non-distressed firms. Using the ORBIS database, which provides data for the past ten years, I have focused on the time span from 2011 to 2020 for this analysis. However, for both distressed and non-distressed firms, I collected data covering the period from 2011 to 2019.

For distressed firms, I operate under the assumption that a TDR filing is triggered by the firm's performance in the most recent financial year. Consequently, I focus on firms that filed for TDR between 2015 and 2020, with the financial data used for these firms reflecting the years 2014 to 2019. This approach ensures that the analysis captures the financial circumstances leading up to the TDR filing.

Furthermore, I also examine the financial ratios of distressed firms in the three years prior to their TDR filings, specifically from 2011 to 2013. This allows me to observe the trends in the financial performance of these firms before and after their TDR filing, offering insights into the financial trajectory that leads to distress risk *(De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).*

The decision to focus on a three-year period prior to the occurrence of financial distress is informed by insights from the existing literature. Previous studies have demonstrated that a firm's financial health typically begins to show signs of deterioration several years before a formal restructuring event such as a Troubled Debt Restructuring (TDR) *(Chen C.C. et al., 2020; De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).*

By examining the financial ratios and performance of firms over this three-year window, I aim to capture early indicators of distress that may not be immediately apparent in a shorter timeframe. This approach is consistent with the work of *De Luca F. and Meschieri E. (2017) and De Luca F. and Mehmood A. (2023)*, who argue that financial ratios begin to reveal significant distress signals during this period, as well as *Chen C.C. et al. (2020),* who highlight the importance of a multi-year analysis in detecting patterns of financial decline. Thus, the three-year observation period enables a more thorough and predictive understanding of the factors that lead firms to file for TDR.

For the non-distressed firms, which serve as the control group in my analysis, I carefully select data to ensure a fair and balanced comparison with the distressed firms. To achieve this, I match each non-distressed firm to a distressed one based on two critical factors: firm size and industry type. For the industry classification, I rely on the Standard Industrial Classification (SIC) system, which provides a detailed and systematic way to group firms based on their industry activities. By using SIC codes, I ensure that the non-distressed firms selected for the control group operate in the same industries as the distressed firms, thus allowing for more precise comparisons between the two groups.

Additionally, I maintain strict consistency in terms of firm size to further enhance the reliability of the comparison. Matching firms of similar size ensures that any observed differences in financial performance or distress likelihood are not skewed by disparities in resources or operational scale. The control-to-case ratio is set to 1, meaning that for each distressed firm in the sample, there is one corresponding non-distressed firm of similar size and industry type. This balanced ratio is crucial for maintaining the integrity of the analysis, as it helps to reduce potential biases and ensures that the control group is as comparable as possible to the distressed firms in terms of key characteristics.

This careful selection process ensures that the control group truly mirrors the distressed group, allowing for more meaningful and accurate comparisons. By controlling for both industry and size, I aim to isolate the financial and operational differences that may contribute to distress, without the confounding influence of unrelated factors. This approach strengthens the overall reliability of the findings and supports the validity of the conclusions drawn from the comparison between distressed and non-distressed firms.

After carefully applying the previously outlined selection criteria, I proceed to finalize the study sample. This process involves excluding firms for which complete and reliable data are unavailable, ensuring that the final sample is composed only of firms with comprehensive financial information. The availability of accurate data is essential for maintaining the integrity and validity of the analysis, and this step helps to avoid any biases that could arise from incomplete datasets.

As a result, the final sample consists of 430 firms, with an equal division between distressed and non-distressed firms. Specifically, there are 215 distressed firms—those that have filed for Troubled Debt Restructuring (TDR) - and 215 non-distressed firms that serve as the control group. This balance is critical, as it allows for a direct and meaningful comparison between the two groups. By ensuring an equal number of firms in each category, I enhance the robustness of the statistical analysis and provide more reliable insights into the factors that contribute to financial distress.

The detailed breakdown of the sample is found in *Table 2*, which presents a clear overview of the firms included in the study. This table summarizes the distribution of distressed and non-distressed firms across various industries and sizes, providing a solid foundation for the subsequent analysis. The balanced composition of the sample is a key strength of this study, as it supports the validity of the findings and ensures that the comparisons drawn between distressed and non-distressed firms are both fair and accurate.

Table 2 – Industry Classification of Distressed and Non-Distressed Companies

Standard Industrial Classification (SIC)	*Distressed firms*	*Non-distressed firm*
1.Construction	16	16
2.Manufacturing	93	93
3.Trasportation, Communications, Electric, Gas & Sanitary Services	34	34
4.Wholesale Trade	14	14
5.Retail Trade	12	12
6. Finance, Insurance, and Real Estate	13	13
7.Services	33	33
Total	*215*	*215*

In the dataset under consideration, firms are classified into two main categories: financially distressed and non-distressed. These firms are further distributed across seven major sectors: *construction, manufacturing, transportation-communications-electric-gas-sanitary services, wholesale trade, retail trade, finance-insurance-real estate,* and *services*. Specifically, the number of firms in each sector, for both distressed and non-distressed categories, is as follows:

16, 93, 34, 14, 12, 13, and 33, with each group representing a different industry. In total, these firms belong to seven distinct industries, which are categorized according to the Standard Industrial Classification (SIC) system, as outlined in *Table 2*.

Furthermore, *Table 3* presents a year-by-year breakdown of firms that are financially distressed, highlighting those that have filed for Troubled Debt Restructuring (TDR). This breakdown is provided not only for the entire sample but also for each sector individually, allowing for a more granular understanding of the financial health and restructuring activities within the different sectors over time.

Table 3 – Year-wise filings for TDR by Financially Distressed Firms

		Standard Industrial classification (SIC)						
Year	*Overall sample*	*1.*	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*
2014	15	1	4	4	1	1	2	2
2015	11	-	6	1	-	1	2	1
2016	39	5	16	7	3	3	1	4
2017	10	-	3	4	1	-	2	-
2018	61	1	30	10	2	4	3	11
2019	34	7	12	1	4	1	2	7
2020	45	2	22	7	3	2	1	8
Total	*215*	*16*	*93*	*34*	*14*	*12*	*13*	*33*

3.1. Rationale for Dataset and Period Selection

In the context of my thesis, which delves into the concept of early crisis detection and its significance in maintaining business continuity, I have extensively considered the current regulatory framework that governs the identification and management of business crises. A key element in this framework is the *European Union's Directive 1023/2019*, commonly referred to as the "Insolvency Directive." This directive assigns Member States the critical task of establishing mechanisms to detect business crises promptly and to facilitate timely intervention to avert insolvency. A central component of the directive is *Article 3*, which emphasizes the necessity for early warning systems and access to relevant financial and operational data that can preemptively signal the onset of distress.

This directive has been transposed into national legislation through *Decree 83 of June 17, 2022***,** which builds on some earlier provisions established by *D.L. n° 118 of September 2021*.

These national reforms have established a more structured legal approach to identifying business crises early, integrating proactive mechanisms that obligate entrepreneurs to set up systems aimed at monitoring and addressing financial instability before it becomes unsustainable. The legislative changes reflect a broader European trend toward the modernization of insolvency laws, driven by the desire to protect business continuity and prevent unnecessary insolvencies.

The national framework can be broken down into three key aspects that are designed to enhance the detection and management of business crises.

3.1.1. The Obligation to Establish Early Warning Structures

The first and most essential aspect of the national legislation is the mandate for entrepreneurs to establish a robust organizational, administrative, and accounting structure that enables the early detection of financial imbalances. This requirement is grounded in *Article 2086 of the Civil Code*, which outlines the responsibilities of company administrators to not only oversee day-to-day operations but also to actively monitor the financial health of the firm. Article 3, paragraph 2, of Decree 83/2022 further stipulates that such systems must be capable of identifying any emerging state of crisis and enabling appropriate corrective action.

The law mandates that these systems should be tailored to the specific characteristics of the company and the industry in which it operates. More specifically, these structures are required to perform several critical functions, as outlined in *paragraph 3* of the Decree. These include:

(a) identifying financial or economic imbalances that could destabilize the company's operations;

(b) evaluating the sustainability of the firm's debts and assessing its prospects for business continuity over the next twelve months; and

(c) generating the necessary information to apply a tailored checklist and conduct a practical test to verify the feasibility of recovery efforts, as stipulated in *Article 13, paragraph 2*.

By mandating the establishment of these systems, the national legislature aims to ensure that businesses have the internal mechanisms to detect financial difficulties early on. This proactive approach is critical because financial crises rarely emerge suddenly; they are typically preceded by a series of warning signs, such as deteriorating liquidity, rising debt levels, or decreasing profitability. The obligation to monitor these indicators and take early action places a significant burden on entrepreneurs to maintain continuous oversight of their company's financial health.

3.1.2. Indicators of Crisis

To further assist in the early detection of crises, *paragraph 4* of the Decree outlines a series of specific indicators that companies must monitor to determine whether they are at risk of entering a crisis. These indicators include overdue salary payments, delayed supplier payments, and overdue bank loans, all of which are early warning signs of potential financial distress. For example, if salary payments are overdue by more than 30 days and exceed half of the company's total monthly wage bill, or if supplier debts have been overdue for more than 90 days, the firm may be facing significant financial challenges.

Similarly, financial institutions must monitor whether a firm's banking exposures have been overdue for more than 60 days or whether credit limits have been exceeded by more than five percent. The presence of these indicators requires the company to take immediate corrective action to prevent further deterioration of its financial position. These provisions align with the objectives of the "*Insolvency Directive"*, which seeks to create a more transparent and efficient system for identifying firms at risk of insolvency.

3.1.3. The Negotiated Resolution of Crises

The second pillar of the national framework focuses on the development of mechanisms to facilitate the *negotiated resolution of crises*. As outlined in *Articles 12-25 quinquies* of the Decree, this process is designed to enable companies to resolve their financial difficulties outside of the formal judicial system. The negotiated settlement process provides a structured, extrajudicial path for businesses that are experiencing financial distress but still have the potential for recovery.

However, it is important to note that due to the relatively recent implementation of these provisions, having entered into force in 2021, I was unable to incorporate the *negotiated resolution of crises* in my analysis. The available data on firms utilizing this process was insufficient for thorough investigation, which limited my ability to assess its practical impact within the scope of this research. This limitation underscores the need for future studies to examine the effectiveness of these mechanisms once a more robust dataset becomes available. Despite this limitation, the relevance of the negotiated settlement platform in providing accessible recovery tools for *micro, small, and medium-sized enterprises (SMEs)* and *large-sizes* remains significant, as the platform is designed to streamline the recovery process and enable companies to develop feasible recovery plans in collaboration with appointed professionals.

3.1.4. Reporting by External Parties

The third and final pillar of the national framework involves the reporting obligations of external parties to the entrepreneur. Public creditors, including the *INPS****,*** *INAIL****,*** the *Revenue Agency****,*** and the *Revenue Collection Agency*, are required to notify the company if certain debt thresholds are breached. This external reporting mechanism ensures that businesses are made aware of their financial obligations in a timely manner and can take action before the situation deteriorates further.

In addition, banks and financial intermediaries, as outlined in Article 106 of the Unified Banking Act, must inform the company's corporate bodies of any significant changes in credit conditions, such as the revision or revocation of credit lines. The company's corporate control body is also mandated to provide written notifications to the administrative body when signs of financial distress are detected, as outlined in Article 17. This obligation underscores the importance of corporate governance in preventing crises, ensuring that the company's leadership is informed and empowered to take corrective action.

The integration of these regulatory provisions into the national legal framework forms a comprehensive approach to the early detection and resolution of business crises. By requiring companies to establish proactive internal systems, while also engaging external stakeholders in the monitoring process, the law creates a robust mechanism for identifying and addressing financial distress before it escalates into insolvency. While my analysis could not directly incorporate the data from the negotiated resolution of crises due to its recent introduction and lack of available data, the framework's significance remains clear. This regulatory environment highlights the importance of prompt crisis detection as both a legal obligation and a practical tool for sustaining business continuity and long-term financial health *(Quagli A., 2023).*

The dataset considered in my analysis covers the period from 2011 to 2019, a timeframe that intentionally excludes the COVID-19 pandemic. The decision to focus on this pre-pandemic period was made to avoid potential distortions in the data that could arise from the unprecedented economic and financial disruptions caused by the global health crisis. The COVID-19 pandemic introduced extraordinary conditions that affected virtually all industries and businesses in ways that would not reflect typical economic patterns. By excluding the years affected by the pandemic, I aim to maintain the integrity of the analysis and ensure that the results are representative of normal business conditions, rather than outlier events.

Moreover, the period following the pandemic, from 2020 onward, is still evolving, and it is difficult to assess its long-term impact with full objectivity at this stage. In my view, it would be prudent to wait until the end of this decade—until 2030—to gather and analyze data that can provide a clearer and more comprehensive understanding of the post-COVID era. This would allow for a proper distinction between the pre-pandemic world, the marked disruption caused by COVID-19, and the long-term recovery or transformation that follows. Such a timeframe would offer the necessary perspective to evaluate trends over a full ten-year period, from the pre-pandemic conditions through to the post-pandemic recovery, providing a more balanced and objective analysis.

In this sense, the 2011-2019 dataset serves as a clear demarcation of the business environment before the pandemic, while the years following 2020 will likely form a new phase, defined by a different set of challenges and dynamics. By maintaining this separation, I aim to ensure that the analysis remains focused and avoids the short-term distortions that the pandemic period could introduce.

4. EXPLANATORY VARIABLES SELECTION

The initial phase of the analysis involves constructing a sample of companies that serves as the foundation for both developing and testing the predictive model. In the first step, a selection of companies that experience financial failure is identified. This group is crucial for understanding the characteristics and indicators of financial distress. Following this, an equal number of financially healthy companies is randomly chosen to ensure a balanced comparison. This random selection helps mitigate bias and provides a reliable benchmark against which the financial profiles of distressed firms are contrasted. By comparing the two groups, the model captures the distinguishing features of financial health and distress, facilitating more accurate predictions.

The dependent variable, denoted as Y, will be determined based on the parameter *"label"*. Specifically:

- Companies that have failed will be identified when the *"label"* parameter is equal to 1;
- Financially health companies will be identified when the *"label"* parameter is equal to 0.

In addition to the *"label*" parameter, each company in the sample will provide its financial indicators and qualitative data, such as the year of establishment and size.

The central focus that distinguishes the selected sample, and is of critical importance to this study, is its *size*. I choose this approach with the intention of proposing a model that is not only theoretically sound but also practically applicable, especially in the context of Italian companies.

In this regard, the first step I undertake is the creation of a sample that includes both failed and healthy companies. The dependent variable, denoted as Y, is determined by the company's status - whether it is classified as failed or healthy - based on the assigned "*label"* parameter. Additionally, the sample incorporates both financial data and qualitative information for each firm.

I emphasize the sample size as a key factor, carefully selecting it to ensure that the model remains relevant and practical for medium and large-sized enterprises in Italy, thereby enhancing the study's real-world applicability.

To achieve the objectives of this study, I gather data on companies that have failed, with a specific focus on their financial parameters from three years prior to their filing for Troubled Debt Restructuring (TDR). The rationale behind this approach is based on the belief that examining financial data from only one year before the TDR filing would provide too narrow a window to capture the company's true financial trajectory. One year is often insufficient for implementing an effective turnaround strategy capable of restoring a company to financial health.

While I acknowledge that the decision to focus on a three-year time horizon introduces certain limitations, this choice is based on a deliberate trade-off between precision and the broader understanding of financial distress. Specifically, the model developed in this study may exhibit reduced accuracy when compared to those based on shorter time frames, such as the models outlined in earlier research, most notably the work of *Altman E.I. (1968),* which introduced the Z-score model for predicting corporate bankruptcy. Altman's approach, widely regarded as a foundational contribution to bankruptcy prediction, focused on financial ratios from a relatively shorter time horizon and has since been validated by numerous empirical studies for its predictive precision.

However, despite the acknowledged robustness of Altman's Z-score model and others following a similar framework, I contend that extending the observation window to three years is essential for a more comprehensive understanding of distressed firms. The rationale behind this approach lies in the complex nature of financial distress, which often manifests gradually over an extended period *De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023)*. A firm's financial difficulties may result from cumulative factors such as sustained underperformance, strategic missteps, or long-term market shifts, which are not always fully captured in a one-year window.

Moreover, focusing on only the year immediately preceding a firm's filing for TDR may result in a model that is more reflective of short-term financial volatility, without providing insight into the long-term factors contributing to distress. The one-year horizon, while precise in identifying imminent bankruptcy, may overlook critical signals that emerge earlier in the firm's decline. For example, cash flow constraints, deteriorating profit margins, and increasing debt burdens may develop over several years before culminating in a financial crisis.

By extending the time frame to three years, this study seeks to capture those earlier indicators of distress, thus offering a model that, while potentially less precise in short-term prediction, contributes to a more nuanced understanding of the firm's broader financial trajectory. This, in turn, enhances the practical applicability of the model for stakeholders interested not only in predicting bankruptcy but also in identifying earlier opportunities for intervention and restructuring.

The decision to adopt a longer time horizon, focusing on financial data from three years prior to filing for TDR, is grounded in the understanding that financial distress is often a gradual process rather than an abrupt event. A company's financial health typically deteriorates over time due to a combination of internal and external factors, including poor management decisions, market changes, or economic downturns. By analyzing a more extended period, I aim to capture these dynamics, which may not be evident when considering only a single year.

In contrast, a one-year time frame may reflect short-term fluctuations that do not necessarily offer meaningful insights into the underlying causes of financial distress. It may also be biased by the immediate circumstances leading up to the TDR filing, such as emergency cost-cutting measures or temporary cash infusions. By examining three years of data, I seek to obtain a more holistic view of the firm's financial trajectory, identifying early warning signals and long-term trends that might better inform predictive models *(De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).*

This approach aligns with the objective of developing a model that is not only accurate but also applicable in a real-world context. Despite potentially sacrificing some degree of predictive precision in the short term, the broader financial context gained from a longer horizon may enhance the model's relevance and applicability, particularly for decision-makers in distressed firms seeking long-term strategic solutions. This trade-off reflects the balancing of academic rigor with practical considerations, a common challenge in financial research.

In this section of the research, I detail the resources and methodologies that I employ to conduct the study. I use logistic regression as the primary statistical method, which is particularly well-suited for modeling the probability of an event with two possible outcomes, often represented as a binary classification. In this context, I apply logistic regression to evaluate whether a company is likely to remain financially healthy or face the risk of failure.

This approach allows me to analyze the relationship between various predictor variables - such as financial ratios and other key indicators - and the likelihood of a company experiencing financial distress. By doing so, I aim to develop a model that can provide insights into the factors that contribute to a firm's financial stability or instability, offering a predictive framework that can be both analytically robust and practically useful for decision-makers:

- Y = 0 ⇒ The company is healthy;
- Y = 1 ⇒ The company has failed.

The independent variables, also referred to as explanatory variables, consist of a set of financial indicators that I select based on their predictive power and ability to distinguish between financially healthy and distressed firms. In addition to these financial indicators, I also include qualitative variables, which are chosen following a similar rationale. Both types of variables are carefully considered for their potential to contribute to the accuracy and reliability of the model, ensuring that it reflects a comprehensive view of the factors influencing a company's financial condition.

In this study, the dependent variable, denoted as Y, is defined as a Bernoulli random variable, which is characterized by having only two possible outcomes, typically represented by the values 0 and 1. These values correspond to the two distinct states or outcomes of the event being analyzed. In this context, the value 1 signifies that a company is financially distressed or facing failure, while the value 0 represents a financially healthy state. The Bernoulli random variable is well-suited for binary classification problems, as it provides a clear and effective way to distinguish between these two outcomes *(Piech C., 2017).*

The probability that the random variable takes the value 1, commonly denoted as p, is referred to as the probability of success. This value represents the likelihood of the event or outcome of interest occurring - in this case, the probability that a company will face financial distress or failure. Conversely, the probability that the random variable takes the value 0, denoted as $1 - p$, represents the probability of failure, or the likelihood that the event of interest will not occur. In this context, $1 - p$ signifies the probability that a company remains financially healthy. This framework allows for a clear distinction between the two possible outcomes, facilitating the analysis of the factors influencing the likelihood of financial distress:

Table 4 – Probability Distribution of the Dependent Variable (Y)

Y (Dependent Variable)	*Probability*
0	1-p
1	p
Totale	*1*

The probability mass function (PMF) of a Bernouille random variable is commonly expressed in a straightforward manner, as follows:

- P (Y = 1) = *p:* this represents the probability that the random variable Y takes on the value of 1, which is often interpreted as a "success" or the occurrence of the event we are interested in;
- P (Y = 0) = 1 – *p:* This denotes the probability that Y takes on the value of 0, indicating a "failure" or the non-occurrence of the event;

In this formulation, *p* is the probability of success, meaning the likelihood of the event happening, while 1 - *p* represents the probability of failure, or the likelihood that the event does not occur. This binary structure of the Bernoulli distribution makes it ideal for modeling situations with only two possible outcomes, such as whether a company will remain financially healthy or face distress.

After completing the selection related to sample preparation, we enter a more intricate phase - the selection of explanatory variable as $X_1, X_2, \ldots, X_n$. As mentioned earlier, these variables can be classified into two categories:

- Quantitative (numeric);
- Qualitative (categorical).

The initial step involves evaluating the quantity of available variables, as having a larger set of indices offers greater potential to enhance the accuracy of the model.

After finalizing the process of sample preparation, the study moves into a more complex phase - selecting the explanatory variables, denoted as $X_1, X_2, \ldots, X_n$. As previously mentioned, these variables fall into two main categories:

- Quantitative variables, which are numerical in nature;

- Qualitative variables, which are categorical.

The initial step in this phase involves a careful assessment of the number of available variables. A larger set of explanatory variables offers a broader range of data points, which can potentially improve the accuracy of the model. By incorporating a diverse set of indices, the likelihood of capturing different aspects of the companies' financial and operational conditions increases, thereby enhancing the model's ability to predict outcomes with greater reliability. This selection process is critical, as it directly impacts the model's effectiveness and its capacity to generate meaningful insights.

The decision to focus on medium and large enterprises as the sample for this study is driven by the need for comprehensive financial data, which is often lacking in the financial statements of smaller firms. The Italian business environment is predominantly characterized by small and medium-sized enterprises (SMEs), many of which compile their financial statements in abbreviated or simplified formats. These formats provide significantly less detailed information compared to the more complete financial statements produced by larger firms.

This disparity is particularly notable in Italy, where SMEs form the backbone of the economy, yet the lack of comprehensive financial data from these companies poses a considerable limitation. One of the most critical issues is the frequent absence of detailed cash flow information, a variable highlighted as essential in previous predictive studies *(Szego G. and Varetto F., 1999)*. Cash flow data is crucial for understanding the financial health of a company and predicting future performance, especially in studies focused on financial distress.

By selecting medium and large enterprises, I aim to ensure that all necessary financial data, including detailed cash flow information, is available for the analysis. This allows for a more robust and accurate predictive model, as it relies on a complete set of financial indicators, which are often unavailable from smaller firms due to the limitations of their reporting practices.

In this context, the process begins with the sample, focusing on constructing a comprehensive set of financial indices derived from the available data. The aim is to thoroughly examine these indices to identify those with the strongest discriminatory power in distinguishing between companies based on their financial health or distress.

A crucial initial step involves removing any indices that cannot be calculated, even when the appropriate financial data is available. This issue commonly arises in cases where, for example, the denominator of an index is zero, making the calculation invalid. Such situations require careful consideration.

In addressing this issue, a decision must be made. If the number of companies for which the index cannot be computed is relatively small, it is reasonable to proceed by excluding those specific observations from the dataset. However, if a significant portion of the sample is affected, the index must be removed from the initial set of indicators under consideration. This ensures that the analysis remains robust and avoids introducing bias or compromising the model's accuracy. Ultimately, this process refines the selection of indices to include only those that are both calculable and meaningful for the study.

Another key consideration in the analysis is the presence of *outliers* - data points that significantly deviate from the average values within the same index. These outliers can distort the results and influence the overall accuracy of the model. Logistic regression, in particular, is highly sensitive to outliers, as their presence can disproportionately affect parameter estimates and model fit *(Hawkins D.M., 1980).* Therefore, assessing the impact of outliers is a critical step in ensuring the robustness of the analysis.

The first stage of this evaluation involves determining the quantity of outliers in the dataset. If the number of outliers is substantial, it may be necessary to exclude the affected index from the analysis altogether. However, if the number of outliers is limited, a more nuanced approach is required to assess their potential influence on the model's outcomes.

In practice, this involves performing an initial analysis with the outliers included, followed by a secondary analysis that excludes observations containing these atypical values. The purpose of this dual approach is to observe whether the presence of outliers significantly affects the model's estimation, including its performance metrics such as overall accuracy, classification power, and explanatory strength. If the exclusion of outliers leads to a notable improvement in the model's results, this may indicate that their removal is justified *(Rousseeuw P.J. and Leroy A.M., 1987).*

Furthermore, if outliers exert considerable influence, adjustments can be made to the index itself, such as transforming the data or applying robust regression techniques, which are less sensitive to the presence of outliers *(Huber P.J., 1981).* This process ensures that the final model is not skewed by extreme values, thereby improving its generalizability and reliability in predicting outcomes.

The treatment of outliers is a crucial element in maintaining the integrity of the analysis. By carefully considering their impact and making informed decisions on whether to exclude, adjust, or retain them, the model's validity is safeguarded, ensuring more accurate and meaningful results.

4.1 Independent and Dependent Variable Specification

After performing certain calculations and addressing specific issues, it is necessary to advance to the next stage. In this phase, we focus on selecting the explanatory variables based on their ability to provide insights and predictions.

Data is collected for four independent variables, all of which are financial ratios *(see Table 5).* The dependent variable in this study is financial distress, which is represented as a dichotomous variable. Specifically, a financially distressed firm is assigned a value of 1, while a non-distressed firm is assigned a value of 0 *(Altman E.I. et al., 2017, Fernandez-Gamez M.A. et al., 2002).*

This binary classification allows for a clear distinction between distressed and non-distressed companies, facilitating the analysis of the relationship between the financial ratios and the likelihood of financial distress. The use of financial ratios as independent variables is grounded in their ability to provide meaningful insights into a firm's financial health, making them suitable predictors for the model.

Table 5 – Overview of variables

Variables	*Types & definition of variables*
Financial distress	0 = non-distressed firms 1= distressed firms
Financial ratios	*1)* $\frac{Retained\ Earnings}{Total\ Assets}$ *2)* $\frac{Earning\ before\ Interest\ and\ Taxes}{Total\ assets}$ *3)* $\frac{Book\ Value\ of\ equity}{Total\ liabilities}$ *4)* $\frac{Cash\ and\ Cash\ Equivalent}{Current\ liabilities}$

The use of independent variables is a crucial element in scientific and statistical research. These variables, controlled or manipulated by the researcher, serve as determining factors in analyzing cause-and-effect relationships within an experiment or study. In an experimental context, independent variables are altered to observe their effects on dependent variables, thereby allowing for a deeper understanding of the phenomena under investigation. The importance of properly defining and controlling independent variables lies in their ability to minimize the influence of confounding factors, thus enhancing the internal validity of the results. A careful and methodologically rigorous selection of independent variables is, therefore, essential to ensure the reliability and reproducibility of the findings. In the specific case of my research, the independent variables under consideration, as shown in *Table 5*, include *Retained Earnings to Total Assets, Earnings before Interest and Taxes to Total Assets, Book Value of Equity to Total Liabilities,* and *Cash and Cash Equivalents to Current Liabilitie*, each selected for its relevance to the overall analysis and its potential to impact the dependent variables in a significant way.

The variables used in this research are as follows:

1) Retained Earnings/Total Assets

This ratio measures the proportion of a company's total assets that is financed through retained earnings, rather than external debt or equity. Retained earnings are the cumulative profits that a company has reinvested in the business rather than distributing them as dividends to shareholders.

It serves as a key indicator of a company’s long-term profitability and financial health. A higher ratio suggests that the company has been consistently profitable and has chosen to reinvest those profits in the business, which can signal financial stability and a lower reliance on external financing. Conversely, a low ratio may indicate that the company has not been able to generate sufficient profits to accumulate retained earnings, potentially increasing its dependency on external debt or equity financing. In the context of predicting financial distress, a low ratio of retained earnings to total assets could signal underlying financial vulnerability, as the firm may lack sufficient internal resources to withstand economic downturns or operational challenges.

This ratio plays a critical role in assessing the long-term profitability and self-financing capacity of a company. Firms that consistently generate profits and retain earnings over time tend to be more resilient in adverse economic conditions. High retained earnings relative to total assets indicate that the company can finance its operations internally, reducing its reliance on external debt or equity. This financial flexibility is crucial in times of economic downturn, where firms with limited retained earnings may struggle to raise capital or meet financial obligations.

A company with a high retained earnings-to-total assets ratio demonstrates a strong foundation of self-sufficiency. This can lead to better creditworthiness, as creditors view the company as less risky due to its ability to rely on internal funds. Additionally, firms with high retained earnings are typically better positioned to invest in growth opportunities without diluting shareholder equity. On the other hand, companies with low retained earnings may signal poor profitability, increasing the likelihood of financial distress *(Altman E. I., 1968).*

2) Earnings before Interest and Taxes (EBIT)/Total Assets

This ratio, also known as the return on assets (ROA), measures a company's operational efficiency in generating earnings before interest and taxes relative to its total assets. Essentially, it evaluates how effectively a company is using its assets to generate operating income.

The *EBIT/Total Assets* ratio provides insight into a company's ability to generate profits from its assets, independent of how those assets are financed (debt vs. equity). A higher ratio indicates more efficient use of assets to produce earnings, which can be a positive indicator of overall financial health. In the context of financial distress, this ratio is crucial because firms that struggle to generate adequate operating income relative to their asset base may face difficulties meeting their financial obligations, making them more prone to distress. Moreover, this ratio is especially important in assessing the core profitability of a company, as it focuses purely on operational performance, excluding the effects of interest payments and taxes.

This ratio is a critical measure of operational efficiency, indicating how effectively a company is utilizing its assets to generate income. A higher *EBIT/Total Assets* ratio signifies that the firm is using its assets productively, which is essential for sustaining profitability and financial health. Companies that can consistently generate strong earnings from their asset base are less vulnerable to financial distress, as they have sufficient operating income to cover interest payments, taxes, and other obligations.

A firm with a high *EBIT/Total Assets* ratio is likely to experience greater operational stability, allowing it to reinvest profits and maintain liquidity. Such firms are also better equipped to handle financial shocks, as their strong earnings provide a buffer. Conversely, companies with a low *EBIT/Total Assets* ratio may struggle to generate sufficient returns from their asset base, which can lead to cash flow problems, increased debt burdens, and potential financial distress *(Beaver, W. H., 1966).*

3) Book Value of Equity/Total Liabilities

This ratio measures the proportion of a company's total liabilities that is covered by the book value of its equity. The book value of equity represents the net assets of the company, or the total assets minus total liabilities. In other words, it shows the financial cushion available to cover liabilities.

The *Book Value of Equity* to *Total Liabilities* ratio is critical in assessing a company's solvency and financial leverage. A higher ratio suggests that the company has a strong equity base relative to its liabilities, indicating a lower risk of financial distress. A lower ratio, on the other hand, may signal a high level of leverage, which increases the company's risk exposure. If the book value of equity is insufficient to cover total liabilities, the company may struggle to meet its debt obligations, especially in times of financial stress. This ratio is frequently used in distress prediction models, as firms with lower equity relative to liabilities are generally considered more vulnerable to bankruptcy or insolvency.

This index assesses the solvency of a company, indicating the proportion of liabilities that can be covered by the company's net assets. A higher book value of equity relative to liabilities signals financial stability and a lower risk of insolvency, as the company has a solid equity base to cover its debts. Firms with strong equity positions are less reliant on external debt, which makes them more resilient to economic fluctuations and changes in market conditions.

A strong *Book Value of Equity* to *Total Liabilities* ratio improves a company's leverage profile, which can lead to better access to credit at more favorable terms. Firms with high equity relative to liabilities are seen as lower risk, attracting investors and creditors alike. In contrast, a low ratio indicates higher financial leverage, making the company more vulnerable to changes in interest rates or declines in revenue. Highly leveraged firms are at greater risk of financial distress, particularly if their cash flow generation is insufficient to meet debt obligations *(Ohlson, J. A. , 1980).*

4) Cash and Cash Equivalents/Current Liabilities

This ratio, often referred to as the cash ratio, measures the company's ability to cover its short-term liabilities using only its most liquid assets—cash and cash equivalents. It is a stringent measure of liquidity, as it excludes inventories and receivables, focusing solely on cash and near-cash assets.

The cash ratio is an important indicator of a company's liquidity and its capacity to meet short-term obligations without needing to rely on the sale of inventory or collection of receivables. A higher ratio suggests that the company has a strong liquidity position and is well-prepared to meet its short-term liabilities, which can reduce the risk of financial distress. On the other hand, a lower ratio may indicate potential liquidity issues, as the company might struggle to cover its immediate obligations, which can lead to financial difficulties. In the context of distress prediction, firms with lower cash reserves relative to current liabilities are more likely to experience cash flow problems, which can escalate into broader financial distress *(De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023)*.

The cash ratio measures a company's liquidity by assessing its ability to cover short-term liabilities using only its most liquid assets. A higher ratio indicates a strong liquidity position, meaning the firm can easily meet its short-term obligations without relying on asset sales or debt financing. Liquidity is a critical factor in preventing financial distress, as firms that struggle to manage their short-term liabilities are at risk of default or insolvency.

Firms with a high cash-to-current-liabilities ratio are better positioned to handle unexpected financial shocks, such as downturns in revenue or sudden increases in expenses. Strong liquidity can also improve a company's negotiating power with creditors and suppliers, as it demonstrates the firm's ability to pay its debts promptly. On the other hand, companies with low liquidity ratios may face difficulties in securing short-term financing or managing operational costs, which can lead to distress *(Shin, H. H., & Stulz, R. M., 2000)*.

Each of these financial ratios plays a critical role in assessing a company's financial health and stability. Together, they offer a comprehensive picture of a company's profitability, leverage, liquidity, and operational efficiency. In distress prediction models, these ratios are highly valuable because they provide insights into various dimensions of a firm's financial performance, helping to identify early warning signs of financial trouble. By examining these indicators, researchers and analysts can better understand the underlying financial conditions that may lead to distress and create more accurate predictive models.

5. MODIFIED Z''- SCORE

In this research, I consider the Total Debt Ratio (TDR) as an indicator of financial distress rather than equating it with bankruptcy or failure. Financial distress, in my view, refers to a situation where a company faces difficulties in meeting its financial obligations, but this does not necessarily mean that the company is on the verge of insolvency or destined to fail *(Altman E.I., 1968; Ohlson J.A., 1980)*. TDR, as a measure of a company's total debt relative to its assets, allows me to identify firms that may be experiencing financial strain, even though they have not yet reached the point of bankruptcy.

By focusing on TDR as a marker of financial distress, I aim to capture an earlier stage of financial deterioration - one where timely interventions or strategic adjustments may still avert further decline *(Whitaker R.B., 1999)*. Unlike bankruptcy, which signifies a legal state of insolvency, I regard financial distress as a broader condition that includes liquidity challenges, operational inefficiencies, or difficulties in obtaining additional financing *(Wruck K.H., 1990)*. Companies with higher debt ratios may face increased financial pressure, which could make it harder for them to secure funding or service their debt. However, such firms may still operate and potentially recover with the right financial restructuring or management decisions.

This distinction is essential for my study, as it allows me to examine companies that are at risk but have not yet reached a critical failure point. By understanding financial distress through the lens of TDR, I can offer a more nuanced analysis of a firm's stability, providing insights into how debt levels affect long-term financial health without prematurely categorizing the firm as failed *(Gilson S.C., 1989; Andrade G. and Kaplan S.N., 1998).*

For this purpose, I modify the Z"-Score model with the goal of developing an early warning tool that can more effectively predict financial distress at an early stage, particularly with respect to the Total Debt Ratio (TDR). The Z"-Score model, originally formulated by *Altman E.I. (1968),* remains a widely accepted method for assessing the financial health and bankruptcy risk of firms. However, in its traditional form, it may not fully account for the nuanced indicators of financial distress, especially when the aim is to detect early signals of financial strain rather than imminent bankruptcy.

In this modified approach, I exclude the *Working Capital* to *Total Assets* ratio and instead include the *Cash and Cash Equivalents* to *Current Liabilities* ratio *(De Luca F. e Mehmood A., 2023).* This adjustment is based on the premise that liquidity measures, such as cash relative to liabilities, offer a more timely and accurate reflection of a firm's ability to meet its short-term obligations - an essential factor in the early stages of financial distress *(Whitaker R.B., 1999).* By focusing on a company's immediate cash resources in relation to its current liabilities, this ratio provides a clearer indication of its short-term solvency and financial flexibility.

The decision to exclude the *working capital* to *total assets* ratio is grounded in its potential limitations as an early indicator of distress. While working capital remains an important measure of operational efficiency, it may not capture the acute liquidity challenges that firms often face in the early stages of financial distress. On the other hand, the *Cash and Cash Equivalents* to *Current Liabilities* ratio offers a more direct view of liquidity pressure, enabling a more precise assessment of whether a firm can navigate short-term financial challenges without resorting to external financing or risking insolvency *(Wruck K.H., 1990).*

Through this modification, I aim to enhance the predictive accuracy of the Z"-Score model, transforming it into a more effective tool for detecting financial distress before it escalates to bankruptcy. This early detection is critical for firms operating in environments characterized by volatility or high leverage, as it allows management and stakeholders to implement corrective measures in a more proactive and timely manner *(Andrade G. and Kaplan S.N., 1998).*

Ultimately, this approach improves the ability to intervene before financial distress becomes irreversible.

To advance the goal of developing a more precise early warning system for financial distress, I perform the analysis using a modified version of the Z"-Score model, which I refer to as the L-Z"-Score. The model is specified as follows:

$$L - Z''\text{- Score}_{i,t} = \beta_1 \cdot X1_{i,t} + \beta_2 \cdot X2_{i,t} + \beta_3 \cdot X3_{i,t} + \beta_4 \cdot X4_{i,t}$$

In this formulation, β_1, β_2, β_3 and β_4 represent the coefficients associated with each independent variable, while $X1_{i,t}$, $X2_{i,t}$, $X3_{i,t}$ and $X4_{i,t}$ correspond to the selected financial ratios used to predict financial distress. Specifically, $X1_{i,t}$ is the ratio of *Cash and Cash Equivalents* to *Current Liabilities*, $X2_{i,t}$ is *Retained Earnings* to *Total Assets*, $X3_{i,t}$ is *Earnings before Interest and Taxes (EBIT)* to *Total Assets*, and $X4_{i,t}$ is the *Book Value of Equity* to *Total Liabilities*. The outcome, the L - Z" - Score, represents the overall index generated by the model, which serves as a comprehensive measure of financial distress risk.

The inclusion of these particular variables reflects my intent to capture a holistic view of a firm's financial health by focusing on both liquidity and solvency metrics. The ratio of *Cash and Cash Equivalents* to *Current Liabilities* ($X1_{i,t}$) is particularly critical in assessing a firm's immediate ability to meet short-term obligations, a key factor in early-stage financial distress. This measure allows the model to reflect a firm's liquidity and its capacity to avoid short-term solvency issues without relying on additional debt or external financing.

The *Retained Earnings* to *Total Assets* ratio ($X2_{i,t}$) provides insight into the long-term sustainability of a firm by assessing the extent to which profits have been reinvested into the business. This variable highlights the firm's capacity to self-finance its operations and growth, which can serve as a buffer during times of financial strain.

Similarly, *Earnings before Interest and Taxes* to *Total Assets* ($X3_{i,t}$) is a fundamental indicator of operational efficiency, as it measures how effectively a firm is using its assets to generate profits, an essential factor in the firm's overall resilience to financial distress.

Finally, the Book Value of Equity to Total Liabilities ratio ($X4_{i,t}$) offers a view of the firm's leverage and capital structure, providing a gauge of the firm's reliance on debt financing. This ratio is crucial in understanding the firm's vulnerability to financial distress, as a high reliance on debt increases financial risk, especially in times of economic downturns or internal operational challenges.

By combining these four ratios into the L - Z" - Score, I aim to create a more robust and predictive model that can identify firms at risk of financial distress well before they reach a state of bankruptcy. The modification of the original Z" - Score model to include these specific variables reflects a more tailored approach to financial distress prediction, one that emphasizes both liquidity and solvency as key indicators. This modified model provides a more comprehensive early warning tool, allowing firms and stakeholders to take proactive measures to mitigate financial risks before they escalate into more severe financial outcomes.

6. STATISTICAL SIGNIFICANCE in VARIABLE SELECTION

In order to ensure that the variables included in my model are both relevant and impactful, I rely on the statistical measure of the *p-value* for each individual variable. This selection process is crucial as it allows me to assess the statistical significance of each variable in contributing to the model's predictive capacity.

In hypothesis testing, the *p-value* provides a measure of the evidence against the null hypothesis (H_0), which in this context suggests that the variable in question has no meaningful effect on the outcome of interest. For this analysis, I have set the significance level at 95%, a standard threshold in statistical research. This means that I only consider variables with a *p-value* of less than 0.05 for inclusion in the final model. A *p-value* below this threshold indicates that the observed relationship between the variable and the outcome is unlikely to have occurred by chance, allowing me to reject the null hypothesis and infer that the variable likely plays a significant role in predicting the outcome.

On the other hand, variables with a *p-value* greater than 0.05 suggest that there is insufficient evidence to reject the null hypothesis, implying that the variable's contribution to predicting financial distress is not statistically significant. Such variables are excluded from the final model to ensure that the model remains both accurate and parsimonious. By adhering to this strict selection process, I aim to enhance the precision of the model, focusing only on those variables that exhibit a strong statistical relationship with the dependent variable, while eliminating those that may introduce unnecessary complexity or noise into the analysis.

This method of variable selection, rooted in the principles of statistical significance, is foundational to constructing a reliable and valid predictive model. It ensures that the final model is built on statistically robust relationships rather than coincidental or spurious correlations. This is particularly important when predicting financial distress, as the inclusion of irrelevant or weakly correlated variables could compromise the model's predictive accuracy and lead to misleading conclusions. By focusing on variables with a p-value of less than 0.05, I can be confident that the selected variables contribute meaningfully to explaining the outcome, thereby strengthening both the internal validity of the model and its external applicability.

Based on the research questions, the following hypotheses have been formulated to guide the investigation:

- *H_1: firms that effectively monitor and respond to specific financial are less likely to experience financial distress and maintain their going concern status;*
- *H_2: firms that engage in proactive financial restructuring, cost-cutting measures, and strategic realignments are more likely to recover from financial distress and avoid insolvency or liquidation;*
- *H_3: a straightforward index can be created using historical financial data to effectively characterize and predict trends in a firm's financial equilibrium;*
- *H_4: a predictive model can be developed to accurately estimate the probability that a firm will initiate the troubled debt restructuring (TDR) procedure based on key financial indicators and firm-specific characteristics.*

When applying this process to the specific hypotheses guiding my investigation, I find that the rigorous application of *p-value* testing is crucial. For example, in testing *H1*, which posits that firms that effectively monitor and respond to specific financial indicators are less likely to experience financial distress and more likely to maintain their going concern status, I rely on the statistical significance of the financial indicators under examination. Variables that show a *p-value* of less than 0.05 in this context are included, as they provide strong evidence that monitoring and responding to such indicators has a measurable impact on a firm's financial stability.

Similarly, in testing *H2*, which asserts that firms engaging in proactive financial restructuring, cost-cutting measures, and strategic realignments are more likely to recover from financial distress and avoid insolvency or liquidation, the inclusion of only statistically significant variables (those with a *p-value* < 0.05) ensures that the model accurately captures the effect of such actions on a firm's recovery prospects.

For *H3*, which hypothesizes that a straightforward index can be created using historical financial data to effectively characterize and predict trends in a firm's financial equilibrium, the application of the 0.05 significance threshold helps to identify which financial variables are truly predictive of these trends. Only those variables that demonstrate statistical significance are included in the construction of this index, ensuring its practical utility in predicting financial equilibrium.

Finally, for *H4*, which proposes that a predictive model can be developed to estimate the probability that a firm will initiate the troubled debt restructuring (TDR) procedure based on key financial indicators and firm-specific characteristics, the *p-value* threshold of 0.05 ensures that only the most relevant and statistically significant indicators are used. By including only those variables with a *p-value* below 0.05, I enhance the predictive power of the model, making it a more reliable tool for forecasting the likelihood of TDR initiation.

In conclusion, the use of a 0.05 significance level is essential in ensuring that my model focuses on the most statistically relevant variables. This approach not only improves the accuracy of my model but also provides a more robust framework for testing the specific hypotheses that guide this research. Each hypothesis is examined through the lens of statistical significance, allowing me to reject the null hypothesis where appropriate and ensure that the findings are both statistically sound and practically relevant.

7. CALCULATION of the COEFFICIENTS and PREDICTED of CASES

This additional phase in the analysis allows me to further refine the initial set of variables used in the model. While the *p-value* remains a critical factor in determining the statistical significance of each variable, I also consider other important parameters to provide a more comprehensive assessment of the model's overall performance. These additional metrics are essential for evaluating how well the model fits the data, particularly when applied to a test sample.

To this end, I calculate the *coefficients* for the explanatory variables, denoted as $ß_1$, $ß_2$, $ß_3$, and $ß_4$. These coefficients represent the strength and direction of the relationship between each independent variable and the dependent variable, offering insights into the relative importance of each predictor in the model. By interpreting these coefficients, I can better understand the impact of each variable on the outcome, allowing for a more nuanced view of the factors contributing to financial distress.

Beyond the coefficients, I also assess the number of correctly *predicted cases*, a crucial measure of the model’s predictive accuracy. This metric enables me to determine how effectively the model classifies firms based on the likelihood of financial distress, offering a clear indication of its practical utility. By comparing the model's predictions against actual outcomes in the test sample, I can gauge its real-world applicability and reliability.

In addition, I employ a *confusion matrix* to further evaluate the model's classification performance. The confusion matrix provides a detailed breakdown of true positives, true negatives, false positives, and false negatives, offering a more granular perspective on the model's ability to correctly predict cases of financial distress. This analysis allows for a deeper understanding of the model's strengths and potential weaknesses, particularly in terms of misclassifications that may arise in more complex financial scenarios.

Through this comprehensive approach, which includes both statistical significance testing and performance evaluation using metrics like coefficients, predictive accuracy, and the confusion matrix, I ensure that the model is rigorously tested and refined. This methodology not only strengthens the internal validity of the model but also enhances its predictive power and applicability in identifying firms at risk of financial distress.

7.1. Coefficients

In the context of my analysis, the coefficients play an essential and pivotal role, as they reveal the nature of the relationship between each explanatory variable (X_n) and the dependent variable (Y). Understanding these relationships is fundamental not only for interpreting the underlying model but also for deriving meaningful insights and drawing informed conclusions about the financial phenomena being studied. Each coefficient, represented as β_n, quantifies the direction and magnitude of the relationship between an independent variable and the predicted outcome. In essence, these coefficients help determine how changes in each explanatory variable influence the probability of a given financial outcome, such as financial distress or firm stability.

If a coefficient is positive, it indicates a direct relationship between the explanatory variable and the outcome. Specifically, this means that an increase in the value of the corresponding variable (X_n) leads to an increased likelihood of the specific outcome under consideration. For example, in the context of a model designed to predict financial distress, a positive coefficient would suggest that as the value of a financial indicator - such as the debt ratio or interest coverage - increases, so does the probability that the firm will experience financial difficulties or failure. This direct relationship is critical for understanding how certain financial characteristics contribute to the increased vulnerability of a firm. Identifying such risk factors through positive coefficients allows for more targeted analysis and facilitates the development of strategies aimed at mitigating financial risk. This approach is particularly significant in financial modeling, as it aids in pinpointing the variables that most strongly contribute to adverse financial outcomes, aligning with foundational studies in the field *(Altman E.I., 1968; Ohlson J.A., 1980).*

Conversely, if the coefficient is negative, it implies an inverse relationship between the explanatory variable and the predicted outcome. In this case, an increase in the value of the variable (X_n) decreases the likelihood of the specific outcome. For instance, in a model predicting financial distress, a negative coefficient for variables such as liquidity ratio or retained earnings would suggest that as the firm's liquidity improves or retained earnings increase, the probability of financial distress diminishes. This inverse relationship is particularly valuable in identifying protective or stabilizing factors that contribute to a firm's financial resilience. Variables with negative coefficients often represent measures of operational efficiency, sound financial management, or conservative capital structures, all of which serve to buffer a firm against adverse conditions. The ability to quantify these protective factors is essential, as it offers a nuanced view of the dynamics influencing firm performance, highlighting areas where strategic improvements may bolster long-term sustainability.

In my thesis, I employ linear discriminant analysis (LDA) to calculate these coefficients. LDA allows for the estimation of the relationships between each variable and the dependent outcome, offering a robust methodological framework to assess how well the independent variables discriminate between different financial states, such as distress and non-distress. Through this analytical technique, I can assign a weight to each explanatory variable, as represented by the coefficients, which signifies the variable's relative contribution to the overall prediction of financial distress. By interpreting both the sign and the magnitude of these coefficients, I am able to determine the relative impact of each variable on the likelihood of a firm experiencing financial distress. This detailed analysis offers critical insights into firm-specific factors that are most predictive of unfavorable financial outcomes, providing a data-driven basis for decision-making and intervention.

Moreover, variables with statistically significant coefficients are those that demonstrate a meaningful relationship with the dependent variable, thus providing stronger evidence for their inclusion in the final model. This careful selection process enhances the robustness of the model, as only those variables that contribute significantly to the prediction of financial distress are retained. By doing so, I ensure that the model is not only theoretically sound but also empirically validated, thereby increasing its practical utility.

The application of this approach is consistent with established methodologies in both financial and econometric modeling. As noted by *Greene W.H. (2008),* the interpretation of coefficients is a fundamental step in understanding the underlying dynamics of any statistical model. By focusing on both the direction and statistical significance of these coefficients, I ensure that the model reflects the true relationships between the variables and the outcome of interest, rather than capturing spurious correlations or random noise. Furthermore, the insights derived from the coefficients offer practical implications for financial management and risk assessment, as firms can better understand which financial indicators are most indicative of distress and take proactive steps to address those risks.

The calculation and interpretation of coefficients form the cornerstone of my model's analysis, allowing for a deeper understanding of the factors that drive financial outcomes. By leveraging linear discriminant analysis and focusing on statistically significant variables, I aim to create a model that is not only theoretically robust but also practically relevant for predicting financial distress. This method provides a comprehensive view of how various financial metrics contribute to firm performance, highlighting both risk factors and protective measures that can influence a firm's trajectory.

7.2. Correctly predicted cases

In the context of my thesis, the *"number of correctly predicted cases"* serves as a key metric in assessing the predictive power of the model, particularly in binary classification tasks where the outcomes are either success or failure of firms. This metric provides a direct measure of how effectively the model can forecast the outcomes of interest, helping to validate the model's practical utility.

To calculate this metric, the model's predictions are compared with the actual outcomes observed in the dataset. For instance, in predicting whether a company will succeed or fail, I determine how many of the model's predictions accurately match the real-world outcomes in the dataset. This comparison enables me to quantify the number of correct predictions, encompassing both true positives (accurately predicted successes) and true negatives (accurately predicted failures).

However, assessing the number of correctly predicted cases can be approached through two different methodologies. The *original count* reflects the model's performance on the dataset without any further validation, offering a preliminary view of its accuracy. On the other hand, *cross-validation* involves dividing the dataset into several subsets (or "folds") and training the model on different combinations of these subsets. This technique provides a more robust estimate of how well the model generalizes to new data, as it ensures that the model's performance is not overly dependent on any single portion of the dataset *(Hastie T., Tibshirani R., and Friedman J., 2009; Kuhn M. and Johnson K., 2013)*. By comparing both the original count and the cross-validated count, I can gain a more comprehensive understanding of the model's reliability.

Once the number of correctly predicted cases is determined, it is often expressed as a percentage of the total number of observations in the dataset. This is calculated by dividing the number of correct predictions by the total number of observations and multiplying by 100%. This percentage serves as a straightforward measure of the model's overall accuracy, offering a clear indication of its predictive power. A higher percentage indicates a greater degree of accuracy, meaning the model is performing well in aligning its predictions with real outcomes.

Interpreting this percentage, however, requires careful consideration. While a high percentage of correctly predicted cases reflects the model's strength, additional metrics such as precision, recall, and the confusion matrix are necessary to gain a more nuanced understanding, particularly when the dataset may be imbalanced. For instance, in a dataset where successful firms vastly outnumber failing firms, the model might achieve a high accuracy score by disproportionately predicting success. Hence, other evaluation measures ensure a deeper, more reliable understanding of the model's performance in predicting financial distress *(Sokolova M. and Lapalme G., 2009).*

The importance of accurately predicting financial outcomes cannot be overstated. The percentage of correctly predicted cases reflects not just the model's technical performance but also its real-world implications. In financial prediction models like mine, the ability to correctly forecast outcomes such as corporate success or failure has direct consequences for decision-making processes. A model with a high accuracy rate can provide valuable foresight for firms looking to mitigate risks or capitalize on opportunities, whereas a model with lower accuracy might lead to misinformed decisions that could amplify financial risks.

The decision of whether or not to proceed with the current version of the model often hinges on this metric. If the accuracy, as measured by the percentage of correctly predicted cases, is found to be satisfactory - both in the original count and in cross-validation—I can confidently move forward with the analysis and utilize the model's outputs to draw meaningful conclusions. However, if the accuracy is unsatisfactory, it may be necessary to revisit the model, possibly refining the set of variables or adjusting parameters to improve its predictive capacity.

This evaluation process ensures that the model's results are both reliable and meaningful, allowing for well-informed decision-making based on sound statistical principles. By carefully measuring the number of correctly predicted cases and employing cross-validation, I can ensure that the model is sufficiently robust to handle future predictions, mitigating the risk of overfitting or underperforming on unseen data.

7.3.Confusion matrix

The *confusion matrix* provides a more comprehensive analysis than merely counting the cases that were predicted correctly, offering detailed insights into the performance of a classification model. It is a fundamental tool in both machine learning and statistics, especially in binary classification problems where the outcomes can be split into two classes, such as the success or failure of firms. One of the key strengths of the confusion matrix is that it allows us to not only assess how many cases were predicted correctly but also to understand the specific types of errors the model is making. This nuanced understanding is crucial when evaluating a model's overall performance because it identifies where and how the model may be misclassifying data, which is critical for improving accuracy.

In essence, the confusion matrix breaks down the predictions and actual outcomes into four categories: true positives (TP), true negatives (TN), false positives (FP), and false negatives (FN). These categories allow for the precise evaluation of both correct predictions and errors, offering insights into how well the model distinguishes between the two classes. Below is the standard representation of this matrix:

Table 6: Confusion Matrix Representation of Predicted and Actual Outcomes

	Forecast: 0	*Forecast: 1*
Actual: 0	TN (00)	FP (10)
Actual: 1	FN (01)	TP (11)

1. *True Negatives (TN)*: these are cases where the model correctly predicted the negative class. In the context of predicting company success or failure, this would indicate that the model accurately classified a firm as healthy;
2. *True Positives (TP)*: these represent cases where the model correctly predicted the positive class. In this scenario, the model successfully identified a firm as financially distressed;
3. *False Negatives (FN)*: also referred to as Type II errors, these occur when the model incorrectly predicts a firm as healthy (the negative class), when in reality, the firm is financially distressed (the positive class). This type of error can be particularly costly, as it may result in missed opportunities for intervention or restructuring;
4. *False Positives (FP)*: known as Type I errors, these arise when the model predicts that a firm will fail (the positive class), but the firm is actually healthy (the negative class). False

positives may lead to unnecessary actions or costs based on the assumption of impending failure.

The confusion matrix is essential for assessing both the precision and recall of a model, which go beyond mere accuracy *(Hastie T., Tibshirani R.,and Friedman J., 2009).* Precision measures the proportion of true positive predictions among all positive predictions made by the model, indicating how many predicted failures were actually failures. Recall, or sensitivity, measures the proportion of actual failures that were correctly identified by the model, revealing the model's effectiveness in capturing true financial distress cases. These measures are particularly important when evaluating financial prediction models, as both types of errors - false positives and false negatives - carry significant implications for decision-making.

Another crucial aspect of model construction, particularly in my thesis, is the selection of variables. When building predictive models, it is essential to ensure that the variables chosen are not highly correlated with each other, as this can weaken the model's discriminatory power. High correlations between variables introduce *multicollinearity*, which can inflate the variance of the coefficients and reduce the model's reliability. Therefore, an essential step in variable selection is the calculation of a *correlation matrix*, which illustrates the relationships between the different variables under consideration.

The correlation matrix is represented as follows:

Table 7: Correlation Matrix of Selected Variables

	X_1	X_2	X_n
X_1	1	p (X_1, X_2)	p (X_1, X_n)
X_2	p (X_1, X_2)	1	P (X_2, X_n)
X_n	p (X_1, X_n)	p (X_2, X_n)	*1*

Here, $p(X_i,X_j)$ represents the *Pearson correlation coefficient* between variables X_i and X_j, which ranges from -1 to 1. The interpretation of the Pearson coefficient is straightforward:

- a value greater than zero indicates a positive linear correlation, meaning that as one variable increases, the other tends to increase as well;
- a value less than zero signals a negative linear correlation, where an increase in one variable corresponds with a decrease in the other;

- the closer the Pearson coefficient is to 1 (or -1), the stronger the linear relationship between the two variables *(Greene W.H., 2008).*

For practical purposes, a correlation is often considered strong when the coefficient exceeds 0.7 or falls below -0.7, while correlations with coefficients between 0.3 and 0.7 are considered moderate *(Wooldridge J.M., 2012).* When two variables are strongly correlated, including both in the model may lead to redundancy and multicollinearity, reducing the model's predictive performance. Therefore, in my analysis, if two variables exhibit a strong correlation, I make a methodological decision to exclude one to avoid redundancy and ensure the model remains robust.

In addition to examining pairwise correlations, it is important to consider *outlier observations*, as these can distort the correlation matrix. Outliers may artificially inflate or deflate correlations, leading to misleading interpretations. Therefore, before constructing the correlation matrix, I carefully examine the data for outliers and make adjustments where necessary, ensuring that the relationships between variables are accurately captured.

To further assess multicollinearity and its impact on the model, I compute the *Variance Inflation Factor (VIF)* for each variable. The VIF quantifies how much the variance of a regression coefficient is inflated due to multicollinearity among the independent variables. Generally, a VIF between 1 and 10 suggests that multicollinearity is not a serious concern, while a VIF exceeding 10 indicates that multicollinearity may be problematic *(Kutner M.H., Nachtsheim C.Y., and Neter J., 2004).* In my thesis, if the VIF for a given variable is greater than 10, I reconsider its inclusion in the model. Eliminating highly collinear variables helps simplify the model and improves interpretability without sacrificing predictive accuracy.

Furthermore, reducing the number of variables not only mitigates multicollinearity but also enhances the model's generalizability. Models with fewer variables tend to perform better on unseen data, as they are less prone to overfitting, where the model captures noise rather than the underlying pattern in the data. This approach aligns with the principle of parsimony in statistical modeling, which holds that simpler models are often more effective and generalizable than overly complex ones *(Breiman L., 2001).*

In summary, both the confusion matrix and correlation analysis play crucial roles in the construction and evaluation of predictive models in my thesis. The confusion matrix offers detailed insights into the types of errors the model is making, while correlation analysis ensures that the model is free from multicollinearity. By addressing these aspects, I am able to construct a model that is both accurate and robust, offering meaningful predictions regarding financial distress.

CHAPTER 4
EMPIRICAL ANALYSIS & DISCUSSION

1. DESCRIPTIVE STATISTICS of INDEPENDENT VARIABLES

I present the descriptive statistics of the independent variables used in my study for both distressed and non-distressed firms. Descriptive statistics are a fundamental part of my analysis, as they allow me to summarize the key characteristics of the dataset. By examining measures such as the minimum, maximum, mean, and standard deviation, I can better understand the central tendencies and variability of the financial indicators that I am analyzing. This step is essential for gaining an initial understanding of how these variables behave across the two groups of firms - those experiencing distress and those that are not.

Descriptive statistics also play a critical role in helping me identify patterns and differences between the groups, and they can uncover any potential anomalies or outliers that may influence the analysis. Furthermore, they provide a solid foundation for the more complex statistical models that follow, ensuring that the data distribution is suitable for further investigation. Large variations or skewness in the data, for instance, may indicate the need for data transformation or additional cleaning.

Through this preliminary analysis, I gain valuable insights into the variables that may be significantly associated with financial distress. This process helps me build a deeper understanding of the factors driving my research and provides the groundwork for interpreting the final results with greater clarity.

1.1 Descriptive Statistics for the Overall Sample

In analyzing the results from my sample, I observe notable differences in the mean values between the distressed and non-distressed groups across all independent variables considered. These variables include critical financial ratios such as the *retained earnings* to *total assets* ratio, the *earnings before interest and taxes* to *total assets ratio*, the *book value of equity* to *total liabilities ratio*, and the *cash and cash equivalents* to *current liabilities ratio*. These differences, as highlighted in *Table 8*, are particularly significant as they reflect the distinct financial profiles of the two groups. For instance, firms classified as financially distressed tend to exhibit lower retained earnings and weaker liquidity positions, as reflected in these ratios, when compared to their non-distressed counterparts. This divergence in financial health indicators supports the hypothesis that specific financial metrics can serve as reliable predictors of distress, and further underscores the importance of incorporating these ratios into models designed to forecast business failure or troubled debt restructuring.

Table 8: Descriptives - OVERALL SAMPLE

Label2 = Non - Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	642	-0,003204812574906	0,874449060112009	0,343600486238331	0,197919923569674
EBIT_TA	642	-0,065540273920276	0,789118197974498	0,088160310418502	0,083554792088608
BVOE_TL	642	0,010729318620350	8,569293658873830	0,999031007575682	1,121070916949610
CAR	642	0,000015459750173	3,882045001232660	0,259604271787887	0,472367245241036
Valid N (listwise)	642				

a. Label2 = 0

Label2 = Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	642	-5,417748752256140	0,718609756623996	-0,012760954354872	0,365819398701864
EBIT_TA	642	-1,883974690764660	0,268403279292198	-0,055883235540286	0,207872958911123
BVOE_TL	642	-0,724687115601060	9,888045540797260	0,201549189742869	0,527858354526500
CAR	642	0,000027481942245	1,844305759708630	0,066746192555341	0,139630052794710
Valid N (listwise)	642				

a. Label2 = 1

Based on the descriptive statistics presented in *Table 8*, additional observations can be made that further differentiate the financial profiles of distressed and non-distressed firms in the sample. These insights provide a deeper understanding of the financial characteristics that may contribute to the likelihood of financial distress, thereby enriching the broader analysis of corporate failure.

The descriptive analysis of *Table 8* is presented below:

1. *Retained Earnings* to *Total Assets* (RE_TA): the mean value for *retained earnings* to *total assets (RE_TA)* is significantly different between the two groups, with non-distressed firms having a mean of *0.34*, while distressed firms show a negative mean of *-0.013*. This sharp contrast highlights a critical financial disparity: non-distressed firms tend to accumulate positive retained earnings, which suggests more sustainable operations and the ability to reinvest profits. In contrast, distressed firms, with negative or minimal retained earnings, may be operating at a loss or drawing down on reserves, making them more vulnerable to

financial shocks. This negative *RE_TA* ratio in distressed firms reinforces the idea that retained earnings are a key determinant of a firm's long-term stability;

2. *Earnings Before Interest and Taxes* to *Total Assets (EBIT_TA):* the *EBIT* to *total assets ratio* is also markedly lower for distressed firms, with a mean of *-0.056* compared to *0.088* for non-distressed firms. This negative mean for distressed firms suggests that they are not generating sufficient operational profits to cover their assets, which could signal inefficient use of assets or operational losses. Non-distressed firms, on the other hand, demonstrate positive EBIT_TA values, indicating operational profitability. The higher standard deviation in the distressed group also points to greater variability in operational efficiency, suggesting that some distressed firms may be performing significantly worse than others, adding complexity to the assessment of distress;
3. *Book Value of Equity* to *Total Liabilities (BVOE_TL):* another notable difference is observed in the *book value of equity* to *total liabilities (BVOE_TL)* ratio, which averages at *0.999* for non-distressed firms and *0.202* for distressed firms. The higher mean for non-distressed firms indicates that these companies have a stronger equity base relative to their liabilities, providing them with greater financial stability and lower risk of insolvency. In contrast, distressed firms have a much lower *BVOE_TL*, which signals a weaker equity position and higher reliance on debt, thus increasing their risk of default. This observation is further reinforced by the higher standard deviation in both groups, particularly in distressed firms, where some firms may have dangerously low equity relative to their liabilities;
4. *Cash and Cash Equivalents* to *Current Liabilities (CAR):* the *cash and cash equivalents* to *current liabilities (CAR)* ratio, often seen as a measure of liquidity, is notably higher in non-distressed firms, with a mean of *0.260* compared to *0.067* in distressed firms. This considerable gap highlights the liquidity challenges faced by distressed firms, which may struggle to meet short-term obligations due to limited cash reserves. The higher liquidity among non-distressed firms allows them to more easily cover immediate liabilities, ensuring smoother operational continuity. The relatively low standard deviation in the distressed group indicates that liquidity shortages are a widespread issue across the distressed firms in the sample.

The differences in means and the spread of data (as shown by the standard deviations) between distressed and non-distressed firms reveal more than just the financial ratios themselves - they underscore the broader financial health and risk profiles of these companies. Non-distressed firms, on average, demonstrate stronger profitability, better liquidity, and a more robust equity base, which provides them with greater resilience in the face of financial challenges. Conversely, distressed firms tend to exhibit a much higher degree of financial instability, as reflected in their lower or negative earnings, weaker equity positions, and diminished liquidity.

These descriptive statistics serve as a foundation for the inferential analyses that follow, allowing for deeper exploration of the predictive power of these financial ratios in determining the likelihood of financial distress. The stark contrasts in the financial profiles of the two groups underscore the importance of these ratios as key indicators of financial distress, offering valuable insights for both academics and practitioners seeking to understand and predict corporate failure.

In conclusion, the findings from the descriptive analysis bring to light significant areas of financial vulnerability in distressed firms. A prominent issue is the lack of retained earnings, which limits the ability of these firms to build reserves and sustain operations during downturns. This deficiency, coupled with a pronounced liquidity shortfall, exacerbates their financial fragility. Additionally, distressed firms show a higher dependency on debt, which further strains their financial structure, increasing the burden of repayment and heightening the risk of insolvency.

In contrast, non-distressed firms display a healthier financial profile, as evidenced by stronger performance across key financial ratios. These firms manage to generate and retain a higher proportion of their earnings, maintain more robust liquidity, and exhibit more prudent leverage management. Such characteristics enable them to navigate financial challenges more effectively, ensuring smoother operations and long-term stability.

The disparities in the mean values of the financial ratios between the two groups underscore the profound differences in financial health. Distressed firms exhibit weak profitability and inefficient use of assets, while non-distressed firms demonstrate operational efficiency and a more balanced financial structure. This divergence highlights the predictive value of financial ratios in identifying early signs of distress. The ability to accurately measure liquidity, profitability, and leverage serves not only as a diagnostic tool but also as a critical mechanism for anticipating the need for intervention or restructuring.

Thus, the results provide valuable insights into the role of financial ratios as robust indicators of corporate health. They illustrate that firms struggling to maintain earnings, liquidity, and debt management are far more susceptible to financial distress, while those maintaining strength in these areas are better positioned for sustained success. These distinctions offer important implications for developing predictive models of financial distress and inform decision-making processes related to corporate risk management.

1.2. Descriptive Statistics for the Construction Sector Sample

Upon analyzing the results of *Table 9*, which focuses on the construction sector, I observe several significant differences between financially distressed and non-distressed firms across key financial ratios. These differences offer important insights into the factors that contribute to financial health and distress within this sector.

Table 9: Descriptives - 01. Construction

Label2 = Non - Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	48	0,027637789768736	0,562572886114453	0,237644296291562	0,165298853191700
EBIT_TA	48	0,016717688392880	0,136183376464155	0,051372277042785	0,027242019385182
BVOE_TL	48	0,119799870496781	1,511245751068280	0,502919588740976	0,376773117735626
CAR	48	0,000479911592434	3,882045001232660	0,480438196701547	0,889606057165253
Valid N (listwise)	48				

a. Label2 = 0

Label2 = Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	48	-1,145908478593640	0,389145828474159	0,026528309464934	0,254779209992393
EBIT_TA	48	-1,175733977317090	0,095964087688451	-0,035194456078185	0,216097840204839
BVOE_TL	48	-0,456465726623664	0,878114349443189	0,165980214085663	0,262300976802585
CAR	48	0,000070614596378	0,765855611097342	0,068345393603215	0,138580559827709
Valid N (listwise)	48				

a. Label2 = 1

The results of the independent variables under consideration are described below, based on the descriptive statistics:

1. *Retained Earnings* to *Total Assets (RE_TA):* I notice a clear disparity in the *retained earnings* to *total assets (RE_TA)* ratio between the two groups. Non-distressed firms show a mean value of *0.238*, indicating that these companies retain a healthy portion of their earnings, which helps them build a stronger equity base and remain financially stable. The relatively low standard deviation further suggests consistency among non-distressed firms in terms of their ability to reinvest earnings. In contrast, distressed firms exhibit a significantly lower mean of *0.027*, and some even experience substantial negative retained earnings, as indicated by the minimum value of *-1.146*. This lack of retained earnings

suggests that distressed firms struggle to maintain financial reserves, a critical factor in their vulnerability to external shocks and financial instability;

2. *Earnings Before Interest and Taxes* to *Total Assets (EBIT_TA):* the difference in operational profitability is also notable. Non-distressed firms have a mean *EBIT_TA* of *0.051*, reflecting their ability to generate consistent operational profits relative to their asset base. The low variability in this ratio suggests that most non-distressed firms in the construction sector maintain a positive and stable operational performance. On the other hand, distressed firms demonstrate a negative *EBIT_TA* mean of *-0.035*, indicating that, on average, they are incurring operational losses. The large standard deviation within the distressed group points to significant variation in profitability, with some firms experiencing severe financial losses. This negative performance could be a major contributor to their financial distress, as it signals inefficiencies in managing operations relative to the firm's asset base;
3. *Book Value of Equity* to *Total Liabilities (BVOE_TL):* another critical observation comes from the *book value of equity* to *total liabilities (BVOE_TL)* ratio. Non-distressed firms have a mean of *0.503*, suggesting a relatively balanced financial structure where equity provides a reasonable cushion against liabilities. This equity buffer is crucial in reducing the risk of insolvency and enabling firms to absorb financial shocks. However, distressed firms exhibit a much lower mean *BVOE_TL* of *0.166*, indicating that they are more heavily reliant on debt, with a significantly weaker equity position. Such a reliance on debt can make these firms more susceptible to financial distress, as it reduces their ability to manage debt repayments and increases their risk of default;
4. *Cash and Cash Equivalents* to *Current Liabilities (CAR):* liquidity, measured by the *cash and cash equivalents* to *current liabilities (CAR)* ratio, presents a stark contrast between the two groups. Non-distressed firms maintain a mean *CAR* of *0.480*, suggesting they have sufficient liquidity to cover their short-term obligations. However, the high standard deviation suggests some variability in liquidity levels, indicating that while some firms have strong liquidity positions, others might be more constrained. In contrast, distressed firms show a much lower mean *CAR* of *0.068*, underscoring the liquidity issues that these firms face. The lack of available cash to meet current liabilities likely exacerbates their financial struggles, as it impairs their ability to address short-term obligations, which can lead to deeper financial distress.

These results suggest that firms in the construction sector exhibit clear financial indicators that differentiate those that are financially stable from those that are distressed. Non-distressed firms generally maintain positive retained earnings, operational profitability, and sufficient liquidity, which contribute to their resilience in the face of financial challenges. In contrast, distressed firms exhibit weaknesses across all key indicators, including low or negative retained earnings, operational losses, weaker equity positions, and severe liquidity constraints.

The consistency of these patterns across the financial ratios reinforces the importance of these metrics as early warning signs of financial distress. Retained earnings, in particular, play a critical role in building a firm's financial buffer, while operational profitability ensures that the firm can cover its costs and generate value from its assets. The combination of a strong equity base and sufficient liquidity further stabilizes the firm, reducing the likelihood of default. Conversely, firms that exhibit deficits in these areas are more vulnerable to financial failure, particularly in capital-intensive sectors like construction, where maintaining liquidity and managing debt are essential for long-term survival.

The descriptive analysis of firms in the *construction sector* clearly demonstrates the key financial differences between distressed and non-distressed firms. The observed disparities in retained earnings, operational profitability, equity, and liquidity underscore the importance of these financial metrics in assessing a firm's financial health. This analysis serves as an important foundation for the inferential models that follow, offering a deeper understanding of how these variables interact to predict financial distress and inform strategies for crisis management and recovery.

1.3. Descriptive Statistics for the Manufacturing Sector Sample

The descriptive analysis of the manufacturing sector reveals significant differences in the financial ratios between distressed and non-distressed firms. As observed in the construction sector, there is also a substantial distinction between the financial health of distressed and non-distressed firms in the manufacturing industry. Non-distressed firms consistently demonstrate stronger financial performance across key metrics such as retained earnings, profitability, equity base, and liquidity, while distressed firms exhibit weaker financial profiles, with operational losses, lower liquidity, and higher reliance on debt. These differences highlight the common financial challenges faced by distressed firms across both sectors, reinforcing the importance of these financial ratios as indicators of corporate stability and distress.

Table 10: Descriptives - 02. Manufacturing

Label2 = Non - Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	279	-0,003204812574906	0,836605431106616	0,405075464182760	0,202180048621363
EBIT_TA	279	-0,065540273920276	0,789118197974498	0,096955067351575	0,098259477121462
BVOE_TL	279	0,022377106834662	6,818697540935310	1,297799725165320	1,240140625439240
CAR	279	0,000018589207307	3,087521294718910	0,163334839252056	0,315072840266149
Valid N (listwise)	279				

a. Label2 = 0

Label2 = Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	279	-5,417748752256140	0,718609756623996	0,031164901828061	0,412319313192499
EBIT_TA	279	-1,786742164918020	0,268403279292198	-0,062157033715767	0,203334220178474
BVOE_TL	279	-0,724687115601060	9,888045540797260	0,308883004092706	0,694244462262800
CAR	279	0,000027481942245	0,967109424414928	0,049060219795883	0,092274473048441
Valid N (listwise)	279				

a. Label2 = 1

The following observations provide insights into the financial characteristics of both groups, with the results demonstrated in *Table 10***.**

1. *Retained Earnings* to *Total Assets (RE_TA)*: for non-distressed firms, the mean *RE_TA* is *0.405*, indicating that these firms retain a substantial portion of their earnings relative to total assets. This suggests strong operational performance and the ability to reinvest earnings back into the business, contributing to long-term financial stability. The standard deviation of *0.202* shows moderate variability, indicating that most non-distressed firms in the manufacturing sector consistently generate positive retained earnings.
   In contrast, distressed firms exhibit a much lower mean *RE_TA* of *0.031*, indicating minimal retained earnings, with some firms even showing negative retained earnings as reflected by the minimum value of *-5.418*. The higher standard deviation of *0.412* suggests greater variability among distressed firms, with many struggling to retain earnings or experiencing operational losses. This inability to generate and retain earnings puts these firms at higher risk of financial distress, as they lack the internal reserves to buffer against financial shocks;
2. *Earnings Before Interest and Taxes* to *Total Assets (EBIT_TA*): the *EBIT_TA* ratio for non-distressed firms is also significantly higher, with a mean of *0.097*. This indicates that these firms are efficiently generating operational profits relative to their asset base, which contributes to their financial resilience. The standard deviation of *0.098* suggests some

variability in operational profitability, but overall, non-distressed firms are performing well in terms of their ability to convert assets into earnings before interest and taxes.

Conversely, distressed firms show a negative mean *EBIT_TA* of *-0.062*, indicating operational losses. The presence of losses among distressed firms is further emphasized by the minimum value of *-1.787*, indicating severe financial difficulties for certain firms. The higher standard deviation of *0.203* reflects the wide range of operational performance among distressed firms, with some managing to operate close to break-even while others face significant losses. These findings suggest that operational inefficiency and the inability to generate profits from assets are key contributors to financial distress in the manufacturing sector;

3. *Book Value of Equity* to *Total Liabilities (BVOE_TL):* the *BVOE_TL* ratio, which measures the firm's equity base relative to its liabilities, is another critical indicator of financial health. Non-distressed firms have a mean *BVOE_TL* of *1.298*, indicating that they maintain a strong equity position relative to their liabilities. This strong equity buffer provides these firms with greater financial stability and reduces their reliance on debt. The standard deviation of *1.240* suggests some variability, but on average, non-distressed firms in the manufacturing sector are well-capitalized.

   Distressed firms, on the other hand, exhibit a much lower mean *BVOE_TL* of *0.309*, indicating that their equity base is significantly smaller in relation to their liabilities. This weak equity position increases their financial risk, as they have less internal capital to cover liabilities and are more dependent on external financing. The standard deviation of *0.694* suggests that some distressed firms may be more heavily indebted than others, further heightening their vulnerability to financial distress;

4. *Cash and Cash Equivalents* to *Current Liabilities (CAR)*: the liquidity measure, *cash and cash equivalents* to *current liabilities (CAR)*, shows a substantial difference between the two groups. Non-distressed firms in the manufacturing sector have a mean *CAR* of *0.163*, indicating that they maintain reasonable liquidity levels to cover a portion of their short-term liabilities. While the standard deviation of *0.315* suggests some firms maintain significantly higher liquidity levels, overall, the non-distressed group demonstrates the ability to manage liquidity effectively.

   Distressed firms, however, exhibit a much lower mean *CAR* of *0.049*, reflecting serious liquidity constraints. This limited liquidity suggests that distressed firms in the manufacturing sector struggle to meet their short-term obligations, heightening the risk of financial distress. The relatively low standard deviation of *0.092* indicates that liquidity shortages are a consistent issue across distressed firms, with limited variability in their ability to manage current liabilities.

The analysis of the manufacturing sector reveals clear differences between distressed and non-distressed firms across all key financial ratios. Non-distressed firms maintain stronger retained earnings, operational profitability, equity positions, and liquidity, which together contribute to their financial resilience. In contrast, distressed firms struggle in each of these areas, with lower retained earnings, operational losses, weaker equity bases, and insufficient liquidity to cover their liabilities.

These findings suggest that firms in the manufacturing sector that are able to retain earnings, generate profits, maintain a solid equity base, and manage liquidity effectively are better positioned to avoid financial distress. Conversely, firms that exhibit weaknesses in these areas are more likely to experience financial difficulties and may require restructuring or other intervention to regain financial stability.

The descriptive statistics presented in *Table 10* demonstrate that the financial health of firms in the manufacturing sector is closely tied to key financial ratios such as retained earnings, profitability, equity, and liquidity. Non-distressed firms exhibit stronger financial positions across all these metrics, while distressed firms show clear signs of financial instability. These results provide valuable insights into the financial dynamics of the manufacturing sector and reinforce the importance of these ratios as indicators of financial distress.

1.4. Descriptive Statistics for the Transportation, Communications, Electric, Gas, and Sanitary Services Sector Sample

The descriptive analysis of the *Transportation, Communications, Electric, Gas, and Sanitary Services* sector provides a detailed picture of the financial health of firms in this diverse and capital-intensive sector. This sector, characterized by high operational costs, infrastructure investments, and regulatory requirements, presents unique financial challenges. As shown in *Table 11*, the analysis highlights stark differences between non-distressed firms, which demonstrate financial resilience and stability, and distressed firms, which exhibit significant financial vulnerabilities. Understanding these differences is crucial for identifying the key factors that contribute to financial distress and for formulating effective strategies for mitigating risk.

Table 11: Descriptives - 03. Transportation, Communications, Electric, Gas, and Sanitary Services

Label2 = Non - Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	102	0,004094195891964	0,758833495004058	0,279990697877897	0,167675797668968
EBIT_TA	102	-0,034745131736932	0,242055367644667	0,072592526814329	0,050568035174522
BVOE_TL	102	0,010729318620350	4,084513908370770	0,705328375766168	0,744216865355709
CAR	102	0,000066473948409	3,800526388913550	0,364187598946791	0,634542422740176
Valid N (listwise)	102				

a. Label2 = 0

Label2 = Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	102	-1,732193447852300	0,453628491753627	-0,102025915601815	0,387232079651143
EBIT_TA	102	-1,243486940018020	0,215418437353786	-0,054779853305415	0,231069549378011
BVOE_TL	102	-0,619615018143976	0,862240783202366	0,052498972649502	0,243842983635051
CAR	102	0,000036170587962	0,988738787873142	0,058956970710822	0,143641727453962
Valid N (listwise)	102				

a. Label2 = 1

Below is a detailed description of the independent variables under consideration, drawn from the descriptive statistical analysis as shown in *Table 11:*

1. *Retained Earnings* to *Total Assets (RE_TA):* in non-distressed firms, the *retained earnings* to *total assets (RE_TA)* ratio reveals a mean value of *0.280*, indicating that these companies are effectively generating and retaining a substantial portion of their earnings. This ability to accumulate retained earnings not only strengthens their balance sheets but also provides a financial cushion that can be used for reinvestment, debt repayment, or absorbing financial shocks. The low standard deviation ***(0.168)*** across non-distressed firms suggests that this is a common characteristic within the group, reflecting overall sound financial management.

   By contrast, the distressed firms present a significantly different picture, with a mean *RE_TA* of *-0.102*, indicating either negative or very limited retained earnings. This points to operational difficulties, where companies may be incurring losses or depleting their equity reserves to meet obligations. The wide variation in the distressed group, indicated by the high standard deviation ***(****0.387*), shows that while some firms may be operating close to breakeven, others are in a more precarious financial position with substantial negative retained earnings. Such disparities highlight the financial instability that characterizes distressed firms in this sector, where retained earnings are a critical measure of long-term financial health;

2. *Earnings Before Interest and Taxes* to *Total Assets (EBIT_TA):* the *EBIT* to *total assets (EBIT_TA)* ratio for non-distressed firms averages *0.073*, signaling effective operational profitability relative to their asset base. This positive *EBIT_TA* suggests that these firms are not only covering their operational costs but are also generating sufficient returns on their assets, which is essential for long-term viability in capital-heavy industries such as transportation and utilities. The relatively low standard deviation ***(0.051)*** suggests that most non-distressed firms manage to maintain steady operational performance.

   By contrast, the distressed firms exhibit a negative mean *EBIT_TA* of *-0.055*, highlighting a widespread inability to generate sufficient operational profits. The broader range of performance in this group, as indicated by the higher standard deviation ***(0.231),*** reflects varying degrees of operational inefficiency. Some firms are experiencing substantial losses, with *EBIT_TA* ratios as low as *-1.243*. These operational challenges underline the significant financial struggles faced by distressed firms, whose inability to convert assets into profits is a key factor contributing to their financial distress;

3. *Book Value of Equity* to *Total Liabilities (BVOE_TL):* the *book value of equity* to *total liabilities (BVOE_TL)* ratio further differentiates the financial stability of non-distressed firms from distressed ones. Non-distressed firms demonstrate a mean *BVOE_TL* of *0.705*, indicating that they have a solid equity base relative to their liabilities. This equity position not only reduces their reliance on external debt but also provides a buffer against financial risk, allowing them to manage liabilities more effectively. The standard deviation ***(0.744)*** shows some variability, but overall, non-distressed firms maintain a healthy financial structure.

   On the other hand, distressed firms show a significantly lower mean *BVOE_TL* of *0.052*, reflecting a very weak equity base. This low ratio suggests that distressed firms are highly leveraged, with a heavy reliance on debt financing and insufficient equity to cover liabilities. The minimal standard deviation ***(0.244)*** within the distressed group indicates that this is a consistent problem across these firms. The weakest firms, as shown by a minimum *BVOE_TL* of *-0.620*, are in particularly dire situations where liabilities exceed equity, placing them at serious risk of insolvency;

4. *Cash and Cash Equivalents* to *Current Liabilities (CAR):* liquidity, as measured by the *cash and cash equivalents* to *current liabilities (CAR)* ratio, is another key area where the financial health of non-distressed and distressed firms diverges. Non-distressed firms, with

a mean *CAR* of *0.364*, maintain relatively strong liquidity positions, allowing them to cover a significant portion of their short-term liabilities with available cash. This liquidity is critical for day-to-day operations and helps ensure that these firms can meet their obligations without relying heavily on external financing. However, the high standard deviation ***(0.635)*** indicates that while some non-distressed firms maintain substantial liquidity, others may still face occasional liquidity challenges.

Distressed firms, however, face much more severe liquidity constraints, with a mean *CAR* of only *0.059*. This limited liquidity suggests that these firms struggle to cover even a small portion of their current liabilities, making them vulnerable to cash flow issues and increasing the likelihood of default. The relatively low standard deviation ***(0.144)*** suggests that liquidity shortages are a widespread issue among distressed firms, which exacerbates their financial instability and heightens the risk of failure.

The differences observed in key financial ratios between distressed and non-distressed firms within the *Transportation, Communications, Electric, Gas, and Sanitary Services* sector underscore the challenges faced by firms operating in capital-intensive industries. Non-distressed firms demonstrate better financial management, with stronger profitability, liquidity, and equity structures. These firms are better positioned to withstand financial shocks and maintain long-term operational stability.

In contrast, distressed firms are characterized by operational losses, low liquidity, and weak equity positions, all of which contribute to their financial vulnerability. The negative retained earnings and operational inefficiencies observed in distressed firms suggest that they are not only struggling to manage their day-to-day operations but are also heavily reliant on debt financing, which further increases their risk of failure. These financial weaknesses suggest that distressed firms are in urgent need of restructuring or external financial support to avoid insolvency.

The descriptive analysis of the *Transportation, Communications, Electric, Gas, and Sanitary Services* sector, as presented in *Table 11*, highlights the significant financial discrepancies between distressed and non-distressed firms. Non-distressed firms exhibit relatively strong financial health across all key metrics, allowing them to manage operations and liabilities effectively. Conversely, distressed firms face persistent financial challenges, including negative retained earnings, operational losses, and liquidity constraints. These findings suggest that the ability to manage profitability, liquidity, and debt is essential for firms in this sector to avoid financial distress. The insights gained from this analysis provide a foundation for further exploration into the factors driving financial distress and offer potential strategies for mitigating risk in this sector.

1.5. Descriptive Statistics for the Wholesale Trade Sector Sample

The descriptive analysis of the *Wholesale Trade* sector highlights important differences between financially distressed and non-distressed firms, similar to the other sectors analyzed. The financial ratios provide key insights into the stability and financial health of these companies, which are crucial in understanding the dynamics of this capital-intensive industry.

Table 12: Descriptives - Wholesale Trade

Label2 = 0

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	42	0,005953611804084	0,571775928618340	0,250678292923153	0,155614066037552
EBIT_TA	42	0,007286730812241	0,253719777860633	0,074324442525857	0,064438382162234
BVOE_TL	42	0,055768950021556	1,539095810030420	0,518159277815716	0,388232047407738
CAR	42	0,000925721587859	1,345384796896800	0,255622825750332	0,365436093218957
Valid N (listwise)	42				

a. Label2 = 0

Label2 = 1

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	42	-0,772108412724535	0,378150713121730	0,064866286643160	0,242228199029174
EBIT_TA	42	-0,868839223577778	0,052875052920372	-0,041065107475502	0,181301797523534
BVOE_TL	42	-0,425427604081502	0,627568713440031	0,188071365002348	0,240840818406710
CAR	42	0,000970955777346	0,238321146266642	0,056558994729703	0,052617145940846
Valid N (listwise)	42				

a. Label2 = 1

As demonstrated by the results in *Table 12*, these differences reveal significant variations in the ability of firms to manage their earnings, profitability, equity, and liquidity.

1. *Retained Earnings* to *Total Assets (RE_TA):* the *retained earnings to total assets (RE_TA)* ratio plays a critical role in evaluating how effectively a firm is reinvesting its profits to support its asset base. For non-distressed firms in the wholesale sector, the mean *RE_TA* is *0.251*, indicating a solid capacity to generate and retain earnings relative to their total assets. This suggests that non-distressed firms have been able to maintain profitability over time, allowing them to strengthen their balance sheets and reinvest profits to drive future growth. The relatively low standard deviation ***(0.156)*** indicates that there is not much variation in

the retained earnings across these firms, suggesting consistent financial management practices within this group.

In stark contrast, distressed firms show a much lower mean *RE_TA* of *0.065*, indicating that they retain far less of their earnings, with some firms even experiencing negative retained earnings, as reflected by the minimum value of *-0.772*. This indicates that many distressed firms are either struggling to generate profits or are using their earnings to cover immediate financial obligations rather than reinvesting them. The higher standard deviation ***(0.242)*** points to greater variability within the distressed group, reflecting the challenges some of these firms face in maintaining profitability and reinvestment strategies;

2. *Earnings Before Interest and Taxes* to *Total Assets (EBIT_TA):* the *EBIT to total assets (EBIT_TA)* ratio is another key metric used to evaluate the operational efficiency and profitability of firms. Non-distressed firms in the wholesale sector exhibit a mean *EBIT_TA* of *0.074*, which suggests that these firms are managing to generate reasonable operational profits relative to their assets. This operational performance reflects the ability of non-distressed firms to efficiently utilize their assets to drive revenue. The standard deviation of *0.064* indicates some variability in operational efficiency, but overall, the non-distressed firms are performing well in terms of profitability. On the other hand, distressed firms show a negative mean *EBIT_TA* of *-0.041*, indicating operational losses. This negative ratio suggests that many distressed firms are not able to generate sufficient revenue from their operations to cover costs, which puts them at a significant disadvantage. The wide range of performance in this group is reflected by the minimum value of *-0.869* and the higher standard deviation of *0.181*, indicating substantial variability in the operational performance of distressed firms. This operational inefficiency could be a major factor contributing to their financial difficulties, as it limits their ability to generate the profits needed to sustain business operations and reduce financial risk;
3. *Book Value of Equity* to *Total Liabilities (BVOE_TL):* the *book value of equity* to *total liabilities (BVOE_TL)* ratio provides insight into a firm's ability to cover its liabilities with equity. For non-distressed firms, the mean *BVOE_TL* is *0.518*, indicating that these companies have a relatively balanced financial structure, where equity plays a significant role in managing liabilities. A higher ratio suggests a more favorable position in terms of financial stability, as firms with greater equity relative to their liabilities are less reliant on debt and are better positioned to manage financial risks. The standard deviation ***(0.388)*** shows some variability, but overall, non-distressed firms demonstrate a healthy balance between equity and liabilities.

Distressed firms, however, exhibit a much lower mean *BVOE_TL* of *0.188*, reflecting a weaker equity position relative to their liabilities. This suggests that distressed firms rely heavily on debt financing, which increases their financial risk and limits their ability to manage their liabilities effectively. The lower standard deviation ***(0.241)*** indicates that many distressed firms in the wholesale sector face this challenge, and the minimum value of *-0.425* reveals that some firms even have negative equity, meaning their liabilities exceed their equity base, putting them at significant risk of insolvency.

4. *Cash and Cash Equivalents* to *Current Liabilities (CAR):* liquidity is a critical factor in determining a firm's ability to meet short-term financial obligations. The *cash and cash equivalents* to *current liabilities (CAR)* ratio for non-distressed firms in the wholesale sector is *0.256*, indicating that these companies have sufficient liquidity to cover a portion of their current liabilities. While this ratio does not indicate full coverage, it suggests that non-distressed firms are maintaining a reasonable level of liquidity to manage their short-term obligations. However, the relatively high standard deviation ***(0.365)*** suggests some variation in liquidity management, with certain firms maintaining higher levels of liquidity than others.

   Distressed firms, on the other hand, exhibit much more constrained liquidity, with a mean *CAR* of only *0.057*, which indicates significant liquidity shortages. This low level of cash relative to current liabilities suggests that distressed firms may struggle to meet short-term financial obligations, heightening the risk of default or further financial strain. The relatively low standard deviation ***(****0.053*) indicates that this lack of liquidity is a consistent issue across distressed firms, and the minimum value of *0.00097* shows that some firms have almost no liquidity at all, further exacerbating their financial difficulties.

The results presented in *Table 12* underscore the significant financial challenges faced by distressed firms in the wholesale sector. Non-distressed firms exhibit healthier financial positions, with stronger retained earnings, better operational profitability, more favorable equity-to-liabilities ratios, and higher liquidity levels. These firms are better equipped to manage both short-term and long-term financial obligations, which provides them with a competitive advantage and financial resilience.

In contrast, distressed firms demonstrate clear signs of financial vulnerability, including negative operational profitability, lower retained earnings, weaker equity positions, and insufficient liquidity. These factors suggest that distressed firms in the wholesale sector are likely struggling to manage their financial obligations, which could lead to heightened financial distress or even insolvency if corrective actions are not taken. The variability in financial performance across distressed firms, as reflected by the higher standard deviations, further suggests that some firms are facing more severe financial challenges than others, pointing to the need for targeted interventions to stabilize their financial health.

The descriptive statistics in *Table 12* highlight the key differences in financial performance between distressed and non-distressed firms in the *Wholesale Trade* sector. Non-distressed firms exhibit stronger financial health, reflected in their ability to retain earnings, generate operational profits, maintain a stable equity base, and manage liquidity effectively. In contrast, distressed firms face significant financial pressures, with operational losses, limited retained earnings, lower equity, and severe liquidity shortages. These findings are consistent with the patterns observed in the previously analyzed sectors, such as *Construction* and *Manufacturing*, where similar trends in financial performance were noted. Across all sectors, non-distressed firms tend to demonstrate stronger financial positions, while distressed firms exhibit common vulnerabilities that contribute to their financial instability. This consistency reinforces the importance of effective financial management across sectors as a critical factor in avoiding financial distress.

1.6. Descriptive Statistics for the Retail Trade Sector Sample

The descriptive analysis of the *Retail Trade* sector highlights significant differences between distressed and non-distressed firms, reflecting a trend that has been consistently observed across other sectors previously analyzed, such as *construction, transportation, communications, electric, gas, and sanitary services, as well as wholesale* and *manufacturing.* The *retail trade* sector, driven by intense competition, consumer demand, and often unpredictable operational costs, faces unique financial challenges that can rapidly lead to financial distress if not managed effectively.

As demonstrated in *Table 13*, the findings provide valuable insights into how key financial metrics behave across both distressed and non-distressed firms, further reinforcing the patterns seen in other industries. These differences in financial performance reveal crucial indicators that underpin both financial health and operational success. The analysis confirms that managing profitability, retained earnings, equity, and liquidity is just as critical in retail as it is in other sectors, making these financial variables essential tools in assessing and safeguarding the financial stability of firms operating in this competitive environment.

Table 13: Descriptives - 05. Retail Trade

Label2 = Non - Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	36	0,149482909889994	0,874449060112009	0,490380958256435	0,208367708126085
EBIT_TA	36	-0,007820885586491	0,239909329554744	0,110993683214208	0,059872912884272
BVOE_TL	36	0,232216687524644	8,569293658873830	2,044639596740000	2,050011704770150
CAR	36	0,011239903127011	2,702582774010320	0,543018993551950	0,643217130253192
Valid N (listwise)	36				

a. Label2 = 0

Label2 = Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	36	-1,427206228643360	0,268130950712876	-0,136972403547375	0,421066301643644
EBIT_TA	36	-0,727592375044201	0,067214360180083	-0,099597886075009	0,197554884304592
BVOE_TL	36	-0,538696606615606	0,525741171450536	0,054559640225521	0,248280096631941
CAR	36	0,002268312024588	0,280375767040199	0,070873392428201	0,074237994296522
Valid N (listwise)	36				

a. Label2 = 1

Below is the analysis of the key financial ratios presented in *Table 13*, which provides insights into the financial performance of firms in the *Retail Trade* sector**:**

1. *Retained Earnings* to *Total Assets (RE_TA):* for non-distressed firms in the retail trade sector, the *retained earnings* to *total assets (RE_TA)* ratio displays a mean value of *0.490*, indicating a strong ability to generate and retain earnings relative to the asset base. This high retention suggests that non-distressed firms are effectively managing their profitability, allowing them to reinvest profits and strengthen their financial stability. The relatively moderate standard deviation of *0.208* indicates some variability, but overall, the firms demonstrate consistent financial health in terms of retained earnings.

   In contrast, distressed firms exhibit a mean RE_TA of -*0.137*, indicating negative or very limited retained earnings. This figure highlights that many distressed firms are operating at a loss or are unable to retain a sufficient portion of their earnings to support growth or cover liabilities. The wide range of performance, indicated by the standard deviation of *0.421* and the minimum value of -*1.427*, shows that certain distressed firms face substantial financial difficulties, as they are depleting their retained earnings or struggling to generate profits altogether. This stark contrast with non-distressed firms underscores the importance of retained earnings as a key indicator of financial health in the retail sector;

2. *Earnings Before Interest and Taxes* to *Total Assets (EBIT_TA):* the *EBIT to total assets (EBIT_TA)* ratio serves as a crucial measure of operational profitability. Non-distressed retail firms show a mean *EBIT_TA* of *0.111*, reflecting a strong ability to generate profits from their asset base. This operational efficiency is vital in the retail sector, where firms must manage both inventory and overhead costs to remain competitive. The standard deviation of *0.060* indicates moderate variation in profitability, but overall, non-distressed firms demonstrate effective management of their assets to yield returns.

   On the other hand, distressed firms have a negative mean *EBIT_TA* of *-0.100*, signaling operational losses. This suggests that distressed firms are not generating enough revenue to cover their operational expenses, which may be due to inefficiencies in managing inventory, controlling costs, or maintaining sales volumes. The broader range in operational performance, highlighted by a standard deviation of *0.198* and a minimum value of *-0.728*, points to significant variability within the distressed group. Some firms may be facing severe operational challenges, contributing to their overall financial distress;

3. *Book Value of Equity* to *Total Liabilities (BVOE_TL):* the *book value of equity to total liabilities (BVOE_TL)* ratio is an important measure of a firm's financial structure, particularly its ability to cover liabilities with equity. Non-distressed firms in the retail trade sector have a mean *BVOE_TL* of *2.045*, suggesting a strong equity position relative to their liabilities. This ratio indicates that non-distressed firms rely less on debt and have built a solid equity base, which provides financial stability and lowers their risk of insolvency. The standard deviation of *2.050* shows that while there is some variability in how these firms manage their equity and liabilities, they are generally in a strong financial position.

   In contrast, distressed firms show a significantly lower mean *BVOE_TL* of *0.055*, indicating a weak equity base relative to their liabilities. This low ratio suggests that distressed firms are heavily reliant on debt, which increases their financial risk and limits their ability to manage liabilities effectively. The relatively low standard deviation *(0.248*) indicates that many distressed firms face similar challenges with equity management, and the minimum value of *-0.539* reflects the extent of financial distress for some firms, where liabilities exceed their equity base. Such firms are at serious risk of insolvency if they cannot improve their financial position;

4. *Cash and Cash Equivalents* to *Current Liabilities (CAR):* liquidity, measured by the *cash and cash equivalents* to *current liabilities (CAR)* ratio, is a key factor in determining a firm's

ability to meet short-term obligations. Non-distressed firms in the retail trade sector have a mean *CAR* of *0.543*, indicating that these firms maintain sufficient liquidity to cover a substantial portion of their current liabilities. This high level of liquidity suggests that non-distressed firms are well-positioned to manage their short-term financial commitments, ensuring smoother day-to-day operations. However, the standard deviation of *0.643* points to variability in liquidity management across firms, with some maintaining very high liquidity levels while others operate with tighter cash reserves.

Distressed firms, however, face significant liquidity challenges, as evidenced by a mean *CAR* of *0.071*. This low ratio indicates that distressed firms are struggling to meet their short-term liabilities with available cash, heightening their vulnerability to cash flow problems and increasing the risk of default. The standard deviation of *0.074* suggests that liquidity shortages are widespread among distressed firms in the retail sector, further exacerbating their financial distress. The minimum value of *0.002* indicates that some firms have almost no liquidity available, which could lead to operational disruptions or failure if not addressed promptly.

The results from the *Retail Trade* sector analysis, as shown in *Table 13*, indicate that financial health in this industry is closely tied to a firm's ability to manage retained earnings, operational profitability, equity, and liquidity. Non-distressed firms display strong financial management across all key metrics, which enables them to remain resilient in the face of competition and market fluctuations. These firms' ability to generate and retain profits, maintain a solid equity base, and ensure adequate liquidity positions them for long-term success.

In contrast, distressed firms exhibit clear financial vulnerabilities, particularly in terms of negative operational profitability, weak equity positions, and severe liquidity constraints. These issues suggest that distressed firms in the retail sector may struggle to manage their financial obligations, which could lead to further distress or failure if corrective actions are not implemented. The significant differences in financial performance between distressed and non-distressed firms emphasize the importance of sound financial management practices in this competitive and fast-paced industry.

The descriptive statistics in *Table 13* illustrate the significant financial differences between distressed and non-distressed firms in the *Retail Trade sector*, echoing the trends observed in other sectors such as wholesale and manufacturing. Non-distressed firms demonstrate strong financial performance across all key metrics - higher retained earnings, operational profitability, equity, and liquidity - enabling them to navigate the dynamic retail environment effectively. On the other hand, distressed firms face considerable financial challenges, including operational losses, lower retained earnings, weak equity structures, and liquidity shortages, which contribute to their financial instability. As seen in the previously analyzed sectors, sound financial management remains a crucial determinant of a firm's success or failure in the retail sector. These findings highlight the importance of addressing financial vulnerabilities early to avoid the escalation of distress and ensure long-term stability.

1.7. Descriptive Statistics for the Finance, Insurance, and Real Estate Sector Sample

The descriptive analysis of the *Finance, Insurance,* and *Real Estate* sector offers valuable insights into the financial health and performance of firms in this heavily regulated and capital-intensive industry. As illustrated in *Table 14*, the findings closely mirror the patterns observed in other sectors, such as construction, transportation, wholesale, manufacturing, and retail trade. A clear distinction emerges between distressed and non-distressed firms, with key financial ratios shedding light on the strengths and weaknesses that define each group.

Table 14: Descriptives - 06. Finance, Insurance, and Real Estate

Label2 = Non - Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	39	0,097945894800224	0,626420191256422	0,354711112354597	0,137334185424684
EBIT_TA	39	-0,001734319065434	0,400923340952201	0,078882400815921	0,086327192004880
BVOE_TL	39	0,183641787408023	5,391791715909230	0,871496475753634	0,837433803913856
CAR	39	0,000900888712390	0,581796386658823	0,185415013799264	0,157693243248775
Valid N (listwise)	39				

a. Label2 = 0

Label2 = Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	39	-1,026385216233420	0,215954615158473	-0,045520148114874	0,225674781736980
EBIT_TA	39	-0,716656825462745	0,086896761369265	-0,047298956920570	0,168387438359014
BVOE_TL	39	-0,474468789283201	1,724892753062670	0,153528518507038	0,462968042730537
CAR	39	0,000645472004440	1,072700505073290	0,081139665083753	0,182609419055109
Valid N (listwise)	39				

a. Label2 = 1

The analysis of *Table 14* is detailed for each individual financial variable under consideration, providing a comprehensive understanding of how factors like retained earnings, operational profitability, equity, and liquidity impact the financial stability of firms in this sector. This sector-specific breakdown offers a clearer picture of the common challenges and opportunities faced by firms depending on their financial condition.

Below is the analysis of each individual variable under consideration:

1. *Retained Earnings* to *Total Assets (RE_TA):* in the non-distressed group, the *RE_TA* ratio reflects a solid financial foundation, with a mean value of *0.355*. This indicates that these firms are efficiently retaining their earnings, allowing them to reinvest in their operations and maintain a robust financial position. A relatively low standard deviation of *0.137* suggests that most non-distressed firms in this sector have a steady ability to retain earnings, reflecting their overall profitability and stable performance.

   Conversely, distressed firms show a very different financial picture, with a negative mean *RE_TA* of *-0.046*. This highlights significant struggles in retaining earnings, with some firms even operating at a loss, as indicated by the minimum value of *-1.026*. The higher variability, reflected in the standard deviation of *0.226*, shows that while some distressed firms are performing marginally better, many are facing substantial challenges. This trend of weak retained earnings among distressed firms is consistent with the findings across other sectors, underscoring the importance of profitability and reinvestment capacity as indicators of financial health.

2. *Earnings Before Interest and Taxes* to *Total Assets (EBIT_TA):* the *EBIT_TA* ratio is a critical measure of a firm's operational efficiency, and the results again highlight the divide between distressed and non-distressed firms. For non-distressed firms, the mean *EBIT_TA* of *0.079* signals that these companies are managing to generate operational profits relative to their assets, contributing to their financial stability. The relatively moderate standard deviation of *0.086* suggests that operational efficiency is fairly consistent across this group. However, distressed firms present a less favorable picture, with a negative mean *EBIT_TA* of *-0.047*, indicating that many of these companies are experiencing operational losses. The broader variability, as evidenced by a standard deviation of *0.168*, highlights that some firms in this group are struggling more than others, with some experiencing severe operational inefficiencies. This operational instability is a common issue among distressed firms in all sectors analyzed, emphasizing that profitability is crucial for maintaining financial health.

3. *Book Value of Equity* to *Total Liabilities (BVOE_TL):* the *BVOE_TL* ratio provides insight into how firms manage their capital structure, particularly the balance between equity and liabilities. Non-distressed firms in the finance sector demonstrate a relatively healthy equity position, with a mean *BVOE_TL* of *0.871*, indicating that these firms are less reliant on debt and maintain a solid equity base. The variability in equity management, with a standard deviation of *0.837*, reflects differences in how these firms structure their finances, but overall, the sector shows stability in this regard.

   On the other hand, distressed firms are facing a much more precarious situation, with a significantly lower mean *BVOE_TL* of *0.154*, suggesting a heavy reliance on debt financing. This imbalance between equity and liabilities places these firms at greater financial risk, especially in an industry where capital requirements are high. The standard deviation of *0.463* suggests that some distressed firms are in an even worse position, with equity levels falling short of what is needed to manage their liabilities effectively. This pattern is consistent with what has been observed in other sectors, where distressed firms tend to struggle with high debt levels and insufficient equity buffers.
4. *Cash and Cash Equivalents* to *Current Liabilities (CAR):* liquidity is another critical factor, particularly in the finance sector, where firms must maintain the ability to meet short-term obligations. Non-distressed firms show a mean *CAR* of *0.185*, indicating that they maintain adequate liquidity to cover a portion of their current liabilities. The standard deviation of *0.158* suggests that liquidity levels vary somewhat across firms, but overall, these companies appear capable of managing their immediate financial commitments.

   In contrast, distressed firms face significant liquidity challenges, with a much lower mean *CAR* of *0.081*. This low liquidity ratio reflects their inability to cover short-term obligations effectively, increasing their financial vulnerability. The higher standard deviation of *0.183* points to widespread liquidity issues among distressed firms in this sector, a trend that is also mirrored in other sectors where distressed companies frequently struggle to maintain sufficient cash reserves to meet their liabilities. Without adequate liquidity, these firms are at greater risk of financial distress and operational disruption.

The findings from this analysis are consistent with those observed in other sectors, where non-distressed firms exhibit stronger financial health across all key metrics. In the *Finance, Insurance, and Real Estate* sector**,** non-distressed firms demonstrate higher profitability, better equity management, and stronger liquidity, which are critical for maintaining financial stability in an industry with high regulatory and capital demands. These firms are well-positioned to manage both short-term and long-term financial obligations, giving them a significant advantage over distressed firms.

Distressed firms, on the other hand, exhibit financial weaknesses across all areas, including profitability, equity, and liquidity. Their reliance on debt, coupled with operational inefficiencies and liquidity shortages, makes them more vulnerable to financial shocks and increases their risk of insolvency. This pattern of financial instability among distressed firms is consistent with the trends observed in previously analyzed sectors, reinforcing the importance of robust financial management practices to ensure stability and reduce the risk of distress.

In conclusion, the descriptive analysis of the *Finance, Insurance,* and *Real Estate sector***,** as presented in *Table 14*, reveals substantial differences between distressed and non-distressed firms, in line with the patterns seen across other sectors. Non-distressed firms in this sector display greater financial stability, characterized by higher retained earnings, operational profitability, and stronger equity positions, which allow them to navigate the complexities of the industry more effectively. Distressed firms, however, face significant challenges in managing their finances, particularly in terms of profitability, equity, and liquidity. These findings emphasize the critical role of sound financial management in maintaining stability and reducing the likelihood of financial distress across all sectors, particularly in an industry as capital-intensive as finance and real estate.

1.8. Descriptive Statistics for the Services Sector Sample

The descriptive analysis of the *Services* sector offers valuable insights into the financial performance of firms in this highly diverse and dynamic industry. Characterized by a wide range of activities, from professional services to consumer-oriented businesses, the services sector presents unique financial challenges that can significantly impact a firm's ability to remain financially stable. As demonstrated in *Table 15***,** the analysis reveals a clear divide between distressed and non-distressed firms, following the same trend observed in other sectors *(construction, manufacturing, transportation, communications, electric, gas and sanitary services, wholesale trade, retail trade, finance, insurance, and real estate).* The differences in key financial ratios between these two groups offer critical insights into their operational efficiency, equity management, and liquidity positions.

Table 15: Descriptives - 07. Services

Label2 = Non - Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	99	0,020410578713044	0,734345999590015	0,271163234564935	0,171692858509605
EBIT_TA	99	-0,012524721680600	0,529695447676121	0,099040582882032	0,089335849238927
BVOE_TL	99	0,023844535793856	2,910164153841460	0,591215845312247	0,597247608677538
CAR	99	0,000015459750173	1,217381623996680	0,249561199735617	0,295936441650227
Valid N (listwise)	99				

a. Label2 = 0

Label2 = Distressed

Descriptive Statistics[a]

	N	Minimum	Maximum	Mean	Std. Deviation
RE_TA	99	-1,900354455143480	0,558096387345822	-0,034640273490683	0,287534675713465
EBIT_TA	99	-1,883974690764660	0,189196507297255	-0,045888703793072	0,220946547187139
BVOE_TL	99	-0,653177245931216	3,380823767724560	0,153925329392808	0,431001072493158
CAR	99	0,000056955492242	1,844305759708630	0,119800879625957	0,229923214671493
Valid N (listwise)	99				

a. Label2 = 1

Below is the detailed analysis of the variables used in *Table 15*, which provides insights into the financial performance of firms in the *Services* sector:

1. *Retained Earnings* to *Total Assets (RE_TA):* for non-distressed firms in the services sector, the *Retained Earnings* to *Total Assets (RE_TA)* ratio demonstrates a mean value of *0.271***,** indicating a relatively strong ability to retain earnings. This ability is essential for firms in the services industry, where reinvestment in human capital, technology, and infrastructure is critical to maintaining competitiveness. The standard deviation of *0.172* suggests moderate variability among non-distressed firms, but overall, these firms demonstrate a stable capacity to generate and retain profits, which supports their long-term financial health.

   In contrast, distressed firms exhibit a significantly lower mean *RE_TA* of *-0.035*, reflecting a struggle to retain earnings, with some firms even operating at a loss. The wide range of performance, highlighted by the minimum value of *-1.900* and the standard deviation of *0.288*, suggests that distressed firms in the services sector are facing substantial financial difficulties. These firms may lack the internal financial strength to reinvest in their operations, further exacerbating their challenges. This pattern is consistent with the struggles seen in distressed firms across other sectors, where the inability to retain earnings is a key indicator of financial distress;

2. *Earnings Before Interest and Taxes* to *Total Assets (EBIT_TA):* the *EBIT* to *Total Assets (EBIT_TA)* ratio serves as a critical measure of operational profitability. Non-distressed firms in the services sector exhibit a mean *EBIT_TA* of *0.099*, indicating that these companies are generally able to generate operational profits from their asset base. The standard deviation of *0.089* suggests some variability, but overall, non-distressed firms demonstrate effective management of their operational costs and assets, enabling them to maintain profitability in a competitive environment.

   On the other hand, distressed firms show a negative mean *EBIT_TA* of *-0.046*, signaling operational losses. This negative ratio reflects inefficiencies in managing operational costs or generating sufficient revenue to cover expenses. The broader variability in operational performance, as indicated by the standard deviation of *0.221*, highlights that some firms within the distressed group are experiencing significant difficulties, with some reporting severe operational losses, as evidenced by the minimum value of *-1.884*. This operational inefficiency mirrors the challenges seen in distressed firms across other sectors, underscoring the importance of maintaining profitability to ensure financial stability;

3. *Book Value of Equity* to *Total Liabilities (BVOE_TL):* the *Book Value of Equity* to *Total Liabilities (BVOE_TL)* ratio is a crucial indicator of a firm's financial structure, revealing how much equity a firm holds relative to its liabilities. Non-distressed firms in the services sector have a mean *BVOE_TL* of *0.591*, indicating a balanced financial structure with a moderate reliance on equity to manage liabilities. This equity position provides these firms with greater financial resilience, reducing their dependence on debt and enabling them to better manage financial risks. The standard deviation of *0.597* suggests variability in how non-distressed firms manage their equity, but overall, these firms are in a relatively strong financial position.

   In contrast, distressed firms display a much lower mean *BVOE_TL* of *0.154*, indicating a weaker equity base relative to their liabilities. This suggests that these firms are more heavily reliant on debt financing, which increases their financial vulnerability and limits their ability to absorb financial shocks. The variability within the distressed group, as indicated by the standard deviation of *0.431*, further underscores the challenges these firms face in maintaining a healthy balance between equity and liabilities. This trend of weak equity positions among distressed firms is consistent with findings in other sectors, where high levels of debt and insufficient equity are common challenges for struggling firms;

4. *Cash and Cash Equivalents* to *Current Liabilities (CAR):* liquidity is a critical factor in determining a firm's ability to meet short-term obligations, particularly in the services sector, where firms often face fluctuating demand and operational costs. Non-distressed firms in this sector maintain a mean *CAR* of *0.250*, indicating that these companies generally have sufficient liquidity to cover a portion of their current liabilities. This level of liquidity enables firms to manage day-to-day financial commitments without relying too heavily on external financing. However, the standard deviation of *0.296* suggests that liquidity levels vary significantly across non-distressed firms, with some maintaining stronger liquidity positions than others.

   Distressed firms, however, show a much lower mean *CAR* of *0.120*, reflecting severe liquidity constraints. This low level of liquidity suggests that many distressed firms are struggling to meet their short-term obligations, increasing their risk of default or operational disruption. The standard deviation of *0.230* indicates that liquidity shortages are widespread among distressed firms in the services sector, further highlighting their financial vulnerability. This pattern is consistent with liquidity challenges observed in distressed firms across other sectors, where limited cash reserves and an inability to manage short-term liabilities are common issues.

The results from the analysis of the *Services* sector**,** as presented in *Table 15*, align with the broader trends seen in other sectors. Non-distressed firms consistently demonstrate stronger financial health across all key metrics, including profitability, equity management, and liquidity. These firms are better positioned to manage both short-term and long-term financial obligations, which is critical in the services sector, where firms must balance operational costs with fluctuating demand.

In contrast, distressed firms exhibit clear financial weaknesses, including negative retained earnings, operational losses, weak equity positions, and liquidity shortages. These challenges make it difficult for distressed firms to navigate the competitive services industry, increasing their risk of financial distress and failure. The similarities in financial performance between distressed and non-distressed firms in the services sector and those in other industries emphasize the importance of robust financial management practices in maintaining stability and reducing the risk of distress.

The descriptive analysis of the *Services* sector***,*** as demonstrated in *Table 15*, highlights significant financial differences between distressed and non-distressed firms, consistent with the findings from other sectors such as construction, finance, and retail. Non-distressed firms exhibit stronger financial performance, characterized by higher retained earnings, operational profitability, a more stable equity base, and better liquidity management. These attributes allow non-distressed firms to maintain financial stability in a highly competitive and dynamic industry. Distressed firms, on the other hand, face significant challenges in all these areas, which contribute to their financial instability and increase their risk of failure. These findings underscore the critical role of sound financial management in determining a firm's ability to succeed in the services sector, and by extension, across all sectors.

The in-depth analysis of key financial ratios across multiple sectors - *construction, manufacturing, transportation, communications, electric, gas and sanitary services, wholesale trade, retail trade, finance, insurance, and real estate,* and *services* - reveals important insights into the financial dynamics of distressed and non-distressed firms. While each sector presents its own unique challenges and operational characteristics, the overarching trends observed suggest that financial stability hinges on the effective management of profitability, liquidity, and capital structure. Across all sectors, there is a consistent pattern: non-distressed firms display superior financial health, while distressed firms face common vulnerabilities, often characterized by insufficient profitability, poor liquidity, and excessive reliance on debt.

In sectors such as *construction* and *manufacturing***,** the ability to retain earnings and generate operational profits is critical. Non-distressed firms in these capital-intensive industries tend to exhibit higher retained earnings-to-assets ratios, reflecting their capacity to reinvest profits into growth and manage their financial obligations effectively. Distressed firms in these sectors, however, often struggle with negative retained earnings, operational inefficiencies, and liquidity shortages. This leaves them vulnerable to economic downturns and sector-specific risks, such as fluctuating demand in construction and global supply chain disruptions in manufacturing. The high variability in performance among distressed firms further underscores the financial instability that plagues many companies in these industries.

Similarly, the analysis of the *transportation***,** *communications***,** *electric, gas, and sanitary services* sectors reinforces the importance of maintaining operational profitability and a stable capital structure. Non-distressed firms in these infrastructure-driven sectors typically manage to balance their assets and liabilities effectively, often displaying stronger equity positions relative to their liabilities. These firms are better equipped to absorb shocks and manage long-term financial commitments, which are especially critical in sectors where large investments in infrastructure and services are required. In contrast, distressed firms often exhibit weak equity positions, a heavier reliance on debt, and substantial liquidity constraints. This pattern of financial strain, coupled with operational losses, increases the risk of failure, particularly in sectors where capital expenditure and maintenance costs are high.

The analysis of *wholesale* and *retail trade* further highlights the significance of liquidity and cash flow management. Retail firms, in particular, are highly dependent on consumer demand, and the ability to quickly convert inventory into cash is crucial. Non-distressed firms in both sectors maintain stronger liquidity positions, as indicated by higher cash-to-liabilities ratios, which allows them to manage day-to-day operations and respond to market fluctuations more effectively. However, distressed firms in wholesale and retail often face severe liquidity challenges, compounded by low retained earnings and operational inefficiencies. These financial weaknesses make it difficult for distressed firms to navigate the competitive pressures of the market, leading to a higher likelihood of financial distress or restructuring.

In the *finance, insurance, and real estate* sector, financial health is closely tied to a firm's ability to manage equity and debt. Non-distressed firms in this sector typically display stronger equity positions relative to their liabilities, providing them with the flexibility to meet both regulatory requirements and operational needs. The analysis shows that these firms are able to maintain a balance between profitability and risk management, allowing them to operate in a highly regulated environment with less reliance on external financing. Distressed firms, however, are characterized by weak equity positions and heavy reliance on debt, leaving them exposed to financial risks that could lead to insolvency. Their inability to generate sufficient operational profits and retain earnings further exacerbates these risks, reflecting a trend that is seen across all sectors.

Finally, the *services* sector presents a diverse range of activities, but the financial trends observed are consistent with those found in other industries. Non-distressed firms in services typically exhibit stronger profitability and liquidity, enabling them to manage both operational costs and demand fluctuations effectively. The ability to retain earnings and reinvest in service offerings is critical for maintaining a competitive edge in this sector. Distressed firms, on the other hand, often struggle with operational losses, liquidity shortages, and weak equity positions. These challenges make it difficult for them to manage their financial obligations, increasing the risk of failure in an industry where service delivery and operational efficiency are key to success.

The general trend across all sectors is one of clear financial disparity between distressed and non-distressed firms. Non-distressed firms, regardless of the industry, consistently show higher retained earnings, stronger operational profitability, better liquidity management, and a more balanced capital structure with lower reliance on debt. These firms have the financial resilience to weather industry-specific challenges and broader economic downturns. Their ability to reinvest earnings into growth and manage liabilities effectively allows them to maintain competitive advantages and long-term financial stability.

Distressed firms, on the other hand, exhibit similar financial vulnerabilities across all sectors. They are typically characterized by low or negative retained earnings, operational inefficiencies, liquidity shortages, and weak equity positions. The reliance on debt to finance operations and the inability to generate sustainable profits leaves these firms more exposed to external shocks and financial risks. Whether in construction, retail, or services, the lack of financial stability is a key determinant of their distress, often leading to restructuring or even insolvency if these issues are not addressed.

The cross-sector analysis underscores the critical importance of sound financial management in determining a firm's ability to succeed or fail. Across all sectors analyzed - *construction, manufacturing, transportation, communications, electric, gas and sanitary services, wholesale trade, retail trade, finance, insurance, real estate, and services* - the same financial patterns emerge. Non-distressed firms demonstrate stronger financial health through higher profitability, better liquidity, and a more balanced approach to managing debt and equity. In contrast, distressed firms are plagued by common financial weaknesses, which severely hinder their ability to operate efficiently and meet their financial obligations.

This comprehensive analysis emphasizes that, regardless of the industry, effective management of financial resources is essential for maintaining a company's operational viability. Whether navigating capital-intensive industries like construction and manufacturing or consumer-driven sectors like retail and services, the ability to manage retained earnings, generate operational profits, and maintain sufficient liquidity is crucial for avoiding financial distress. The findings from this study provide a foundation for further investigation into sector-specific strategies that can help firms enhance their financial resilience and avoid the pitfalls of distress.

2. *CALCULATION of LDA CANONICAL DISCRIMINANT COEFFICIENTS*

In this analysis, I decided not to split the data into training and testing sets, a common approach in larger datasets, where it is crucial to assess the model's predictive performance on unseen data. However, due to the relatively small size of my sample, splitting the data would have significantly reduced the amount of information available for both training and testing. This could have weakened the model's robustness, as each subset would have contained less data, leading to potentially less reliable results.

By using the entire dataset for the *Linear Discriminant Analysis (LDA)***,** I ensure that the model benefits from the full breadth of the data, maximizing the insights derived from the available sample. In this context, utilizing the entire sample is particularly important for achieving more accurate and generalizable results. Given the limited size of the dataset, splitting it would have risked introducing noise or bias, while using the full sample provides a more comprehensive and reliable assessment of the factors that distinguish distressed from non-distressed firms.

Moreover, since my goal is to examine the financial characteristics of distressed and non-distressed firms within a specific time frame, avoiding the training and testing split allows me to retain the full complexity of the relationships between the financial variables. This decision enhances the validity of the analysis, as I can focus on drawing meaningful conclusions from the complete dataset, providing a clearer and more accurate representation of the patterns and trends in financial distress without compromising the integrity of the findings.

For each group, I consider data from three years. For distressed firms, I examine financial data from the three years leading up to their filing for TDR, which provides insight into the financial deterioration preceding their distress. In contrast, for non-distressed firms, I use financial data from their best-performing years between 2011 and 2019, allowing for a comparison between the most financially stable companies and those on the brink of distress. This approach ensures that the analysis reflects the full breadth of the available data, providing more reliable insights into the factors influencing financial distress.

Below are the coefficients obtained for the entire sample using the modified L-Z'' Score formula:

$$L - Z''\text{-Score}_{i,t} = 0.818 \cdot X_{1i,t} + 1.795 \cdot X_{2i,t} + 2.102 \cdot X_{3i,t} + 0.346 \cdot X_{4i,t}$$

Table 16 presents the coefficients derived from the study model for the overall sample, as well as for each individual industry sector analyzed. These coefficients provide valuable insights into how the different financial variables contribute to distinguishing between distressed and non-distressed firms across the various sectors. By examining the coefficients for the entire sample and for each sector, I gain a deeper understanding of the sector-specific financial dynamics that influence the risk of financial distress.

Table 16: Canonical Discriminant Function Coefficients

	Overall	01. Construction	02. Manufacturing	03. Transportation, Communications, Electric, Gas, and Sanitary Services	04. Whalesale Trade	05. Retail Trade	06. Finance, Insurance, and Real Estate	07. Services
	Function 1	Function 1	Function 1	Function 1	Function 1	Function 1	Function 1	Function 1
RE_TA	1,795	1,962	1,330	1,645	-2,787	1,505	5,354	3,958
EBIT_TA	2,102	0,134	2,849	0,671	5,060	2,685	-0,762	0,037
BVOE_TL	0,346	1,431	0,388	0,838	2,462	0,262	0,092	0,028
CAR	0,818	0,521	0,726	0,769	1,764	0,307	2,071	0,606
(Constant)	-0,674	-0,882	-0,736	-0,633	-0,789	-0,619	-1,139	0,588

Unstandardized coefficients

a. Overall Sample

In the overall sample, the *Canonical Discriminant Function 1* coefficients reveal the critical financial indicators that differentiate distressed from non-distressed firms. Notably, the *Retained Earnings to Total Assets (RE_TA)* ratio, with a coefficient of *1.795*, indicates a strong role for retained earnings in maintaining financial stability. Firms that retain earnings are better positioned to reinvest in their operations and sustain growth, allowing them to avoid distress. Additionally, the *Earnings Before Interest and Taxes to Total Assets (EBIT_TA)* ratio, with a coefficient of *2.102*, highlights the importance of operational profitability. Firms that can generate higher profits relative to their assets are better able to weather financial challenges. Meanwhile, the *Book Value of Equity to Total Liabilities (BVOE_TL)* ratio (coefficient: *0.346*) and the *Cash and Cash Equivalents to Current Liabilities (CAR)* ratio (coefficient: *0.818*) play supporting roles, indicating that equity and liquidity management contribute to financial stability, albeit to a lesser extent than profitability and retained earnings. These findings suggest that a combination of profitability, retained earnings, and liquidity are key to distinguishing financially healthy firms from those at risk of distress.

Sector-Specific Analysis

b. *Construction Sector*

When examining the *Construction* sector, the importance of *RE_TA* becomes even more pronounced, with a coefficient of *1.962*. This underscores the vital role that retained earnings play in this capital-intensive industry, where firms must continuously reinvest in projects and infrastructure. Construction firms that accumulate retained earnings are better equipped to navigate the cyclical nature of the sector. In contrast, *EBIT_TA* (coefficient: *0.134*) plays a much smaller role in construction compared to the overall sample. This reflects the long project cycles and upfront costs that often delay profitability. Instead, the *BVOE_TL* ratio (coefficient: *1.431*) assumes greater importance, as firms with stronger equity positions are better able to absorb financial risks associated with large-scale projects and economic fluctuations. Liquidity, reflected by *CAR* (coefficient: *0.521*), remains relevant but is less critical than in other sectors, indicating that construction firms are more focused on long-term capital investment than short-term liquidity needs.

c. *Manufacturing Sector*

In Manufacturing, the dynamics shift, with *EBIT_TA* taking on a much larger role. The coefficient of *2.849* emphasizes the importance of operational profitability in this sector. Manufacturing firms must efficiently use their assets to generate profits, particularly in an industry characterized by tight margins and global competition. Unlike construction, where equity and long-term capital are prioritized, profitability is paramount in manufacturing, driving firms' ability to remain competitive. *RE_TA*, with a coefficient of *1.330*, still plays a significant role but is slightly less critical than in construction. This may be because manufacturing firms have more opportunities to access external financing, making retained earnings less central to their growth strategies. The *BVOE_TL* ratio (coefficient: *0.388*) is also less important in manufacturing compared to construction, indicating that equity management, while relevant, is not as vital in distinguishing distressed firms. *CAR* (coefficient: *0.726*) shows moderate importance, as liquidity is necessary for managing inventory and operational costs, but it is not the dominant factor in this asset-driven industry.

d. Transportation, Communications, Electric, Gas, and Sanitary Services Sector

In the *Transportation, Communications, Electric, Gas, and Sanitary Services* sectors, we see a balanced importance of *RE_TA* (coefficient: *1.645*) and *BVOE_TL* (coefficient: *0.838*). Retained earnings remain crucial for these industries, as firms must reinvest in infrastructure and services to meet long-term demand. Similarly, equity plays a significant role, as firms with stronger equity positions are better equipped to manage their extensive liabilities, which often arise from large-scale infrastructure investments. Interestingly, *EBIT_TA* (coefficient: *0.671*) is less significant in this sector than in manufacturing or the overall sample. This may reflect the fact that firms in these sectors often operate under long-term contracts or regulated environments, which provide stable revenue streams, making profitability less volatile. Liquidity, represented by *CAR* (coefficient: *0.769*), is still important, as these firms need to maintain sufficient cash flow to cover operational and maintenance costs. This balanced financial profile reflects the long-term, capital-heavy nature of the industry, where both equity and retained earnings help firms manage large projects and ongoing operational needs.

e. Wholesale Trade Sector

In the Wholesale Trade sector, the coefficients present a unique perspective. The *RE_TA* coefficient is notably negative *(-2.787*), which contrasts sharply with other sectors. This suggests that retained earnings may not function as a strong indicator of financial health in this industry. One possible explanation is that wholesale firms reinvest heavily in inventory rather than retaining cash, making high retained earnings less reflective of financial stability. In contrast, *EBIT_TA* (coefficient: *5.060*) emerges as the most significant factor across all sectors, underscoring the critical importance of operational profitability in wholesale. Efficient asset management and profit generation are vital for success in this competitive, margin-sensitive industry. The *BVOE_TL* coefficient of *2.462* also highlights the importance of a strong equity base in wholesale trade, where firms that can balance liabilities with equity are more resilient in volatile market conditions. *CAR* (coefficient: *1.764*) emphasizes the need for liquidity, as wholesale firms must manage large inventories and complex supply chains while maintaining operational flexibility. The emphasis on liquidity and profitability reflects the fast-paced, high-volume nature of wholesale trade.

f. Retail Trade Sector

In the *Retail Trade* sector, the *RE_TA* coefficient of *1.505* indicates that retained earnings remain an important indicator of financial health, though not as critical as in construction or finance. Retailers that can retain earnings are better positioned to reinvest in inventory, marketing, and store expansions, which are key to maintaining competitiveness in a highly dynamic market. *EBIT_TA* (coefficient: *2.685*) remains a strong factor, as profitability is essential for retail firms, where margins are often narrow and competition is fierce. However, the *BVOE_TL* coefficient of *0.262* is the lowest among all sectors, suggesting that equity relative to liabilities is less of a concern in retail compared to industries like wholesale or construction. This may be because retail firms often rely on short-term financing and revolving credit lines rather than long-term equity financing. Similarly, *CAR* (coefficient: *0.307*) is also relatively low, indicating that liquidity, while important, plays a lesser role compared to profitability in the retail sector. Retailers must focus on generating consistent profits to sustain their operations, with liquidity and equity playing supporting roles.

g. Finance, Insurance, and Real Estate Sector

In the *Finance, Insurance, and Real Estate* sector, *RE_TA* stands out as the most influential variable, with a coefficient of 5.354. This highlights the critical role of retained earnings in this sector, where firms must maintain strong reserves to meet regulatory capital requirements and absorb financial shocks. In contrast, *EBIT_TA* has a negative coefficient of *-0.762*, suggesting that profitability, while still important, may not always reflect financial health in finance and insurance. This could be due to external factors, such as interest rate fluctuations, that affect profitability in these sectors. *BVOE_TL* (coefficient: *0.092*) plays a minimal role, indicating that equity management is less of a distinguishing factor, possibly because financial firms typically operate with high leverage. However, *CAR* (coefficient: *2.071*) is highly significant, reflecting the importance of liquidity. Firms in finance, insurance, and real estate must maintain strong cash positions to meet short-term obligations, making liquidity management essential for financial stability.

h. Services Sector

In the *Services* sector, *RE_TA* has a coefficient of *3.958*, underscoring the importance of retained earnings for services firms, which often need to reinvest in personnel, technology, and service offerings. This reinvestment is crucial for maintaining competitiveness in a sector driven by human capital and innovation. Interestingly, *EBIT_TA* (coefficient: *0.037*) plays a minimal role in this sector, which may reflect the diverse nature of the services industry. Profitability can vary significantly depending on the specific type of service offered, making it a less reliable indicator of financial health across the entire sector. *BVOE_TL* (coefficient: *0.028)* is similarly low, indicating that equity is less critical for services firms compared to capital-intensive industries like construction or manufacturing. *CAR* (coefficient: *0.606*) remains relevant, as liquidity is necessary for managing short-term cash flows in a sector where operational efficiency is key.

The analysis of the *Canonical Discriminant Function Coefficients* across the overall sample and individual sectors provides crucial insights into the financial variables that play significant roles in distinguishing distressed firms from non-distressed ones. The coefficients reflect the varying importance of *retained earnings*, *operational profitability*, *equity*, and *liquidity* across industries, highlighting the financial dynamics that drive firm performance within each sector.

The results underscore several strengths. For instance, *Retained Earnings* to *Total Assets (RE_TA)* consistently emerges as a critical variable, particularly in capital-intensive industries such as *construction* and *finance*, where the ability to retain and reinvest earnings is key to maintaining long-term financial stability. Similarly, the *Earnings Before Interest and Taxes* to *Total Assets (EBIT_TA)* ratio demonstrates its relevance in sectors like *wholesale trade* and *manufacturing*, where operational profitability and efficient asset management are essential for firm success. Across most sectors, *liquidity*, as measured by *Cash and Cash Equivalents* to *Current Liabilities (CAR)*, also plays a significant role, indicating the importance of short-term cash flow management, particularly in industries that face frequent fluctuations in demand or operational costs.

However, the analysis also reveals some challenges, particularly in sectors where certain financial ratios show negative coefficients. For example, the negative coefficient for *EBIT_TA* in the *finance, insurance, and real estate* sector suggests that profitability alone may not always be a reliable indicator of financial health, possibly due to the external factors, such as interest rate fluctuations or regulatory changes, that can significantly affect profitability in these industries.

Similarly, the negative *RE_TA* coefficient in the *wholesale trade* sector is noteworthy, indicating that high retained earnings might not always correlate with stronger financial health in this industry. This could be due to the sector's reliance on inventory turnover and external financing rather than retained earnings.

Despite these sector-specific anomalies, the coefficients, when considered across the entire sample, are largely reliable. They offer valuable predictive power in identifying distressed firms, particularly when taken in the context of each sector's unique financial environment. The coefficients reflect the critical financial variables that contribute to firm stability or distress, but it is important to interpret them in light of sector-specific dynamics.

The negative coefficients found in certain sectors do not necessarily undermine the overall reliability of the model but instead point to the complexity of financial performance in those industries. For example, in finance and insurance, profitability is often influenced by external macroeconomic factors rather than internal operational efficiency alone. In wholesale trade, liquidity and inventory turnover may be more critical than retained earnings. These nuances highlight the need for a tailored approach when analyzing financial health within different sectors.

In conclusion, the coefficients derived from the *modified L-Z'' score* are robust and provide reliable indicators for distinguishing between distressed and non-distressed firms across various industries. While there are sector-specific anomalies - such as the negative coefficients in certain industries - these reflect the diverse financial landscapes within each sector rather than weaknesses in the model itself. Ultimately, the analysis offers valuable insights into the key financial factors that drive firm performance and underscores the importance of interpreting financial ratios within the context of each industry's unique characteristics.

3. CLASSIFICATION PERFORMANCE and ACCURACY

In the context of discriminant analysis, understanding the model's accuracy in classifying firms as distressed or non-distressed is paramount for validating the model's predictive power. To ensure the reliability of the results, it is essential to evaluate the model's performance through two complementary methods: the classification of *original grouped cases* and the process of *cross-validation*.

The classification of *original grouped cases* assesses how well the model is able to distinguish between distressed and non-distressed firms using the dataset on which the discriminant function was originally built. This provides an immediate understanding of the model's accuracy within the dataset and allows for the identification of patterns in the financial variables that contribute to the classification. However, the use of the original dataset alone could lead to an overestimation of the model's effectiveness if it is overly fitted to the specific data. Therefore, while the results from original grouped cases give a snapshot of the model's performance, they must be interpreted with caution, especially when applied to new or unseen data.

To address this potential limitation, *cross-validation* is used as a more stringent test of the model's generalizability. In cross-validation, each case in the dataset is classified by a discriminant function derived from all other cases, excluding the case being classified. This process ensures that the model is not overfitted to specific cases and provides a more accurate reflection of its performance when applied to new data. The cross-validation approach assesses the stability of the model and whether it can maintain its accuracy across a broader range of scenarios, reinforcing the model's predictive power.

Together, the classification of *original grouped cases* and the *cross-validation* process offer a comprehensive evaluation of the model's accuracy. High accuracy in both classifications would suggest that the model is not only reliable for the data used to create it but also robust enough to predict outcomes in different contexts. The consistency between these two metrics is a key indicator of the model's strength and validity, providing confidence in its application to financial decision-making.

a. Analysis of Classification Results for the Overall Sample

The classification results for the *discriminant analysis* applied to the overall sample provide significant insights into the model's effectiveness in distinguishing between distressed and non-distressed firms as shown in *Table 17*. These results are particularly useful for understanding the model's accuracy and reliability when applied to both the original dataset and through the process of cross-validation.

Table 17: Overall Sample

Classification Results[a,c]

Label2			Predicted Group Membership 0	1	Total
Original	Count	0	511	131	642
		1	82	560	642
	%	0	79,6	20,4	100,0
		1	12,7	87,3	100,0
Cross-validated[b]	Count	0	510	132	642
		1	81	558	642
	%	0	79,4	20,6	100,0
		1	12,7	87,3	100,0

a. 83,5% of original grouped cases correctly classified.

b. Cross validation is done only for those cases in the analysis. In cross validation, each case is classified by the functions derived from all cases other than that case.

c. 83,4% of cross-validated grouped cases correctly classified.

In the original analysis, the model correctly classified *83.5%* of the grouped cases. This means that, out of the total 1,284 firms (split equally between 642 non-distressed and 642 distressed firms), a large majority were accurately categorized by the discriminant function. Specifically, the model correctly classified *79.6%* of the non-distressed firms (511 out of 642), while *87.3%* of the distressed firms (560 out of 642) were also correctly identified. These high percentages demonstrate the model's strong predictive power, particularly in identifying distressed firms. The fact that the model performs slightly better at classifying distressed firms compared to non-distressed ones suggests that the financial characteristics of distressed firms, as captured by the discriminant function, are more distinct and easier to detect.

However, there are still some misclassifications. *20.4%* of non-distressed firms were incorrectly classified as distressed, while *12.7%* of distressed firms were misclassified as non-distressed. These misclassifications may arise due to certain financial variables in the model that do not entirely capture the nuanced financial conditions of some firms, or due to the presence of outliers that skew the results.

Cross-validation provides a more rigorous test of the model's reliability. In this process, each case is classified by the discriminant functions derived from all the other cases except the one being classified, ensuring that the model's performance is not overly dependent on the specific data used to derive the discriminant functions. Cross-validation helps assess whether the model is robust and generalizable to new data.

The results from the cross-validated classification show a very similar level of accuracy, with *83.4%* of the grouped cases correctly classified. This slight reduction in accuracy from the original classification (*83.5%* to *83.4%)* indicates that the model is highly stable and consistent, as the slight difference suggests minimal overfitting. Specifically, *79.4%* of non-distressed firms were correctly classified in the cross-validation (compared to *79.6%* in the original classification), and *87.3%* of distressed firms were correctly identified (identical to the original classification). The minimal change between the original and cross-validated classifications reinforces the robustness of the model.

The key takeaway from these results is that the model is highly effective in distinguishing between distressed and non-distressed firms, with an overall classification accuracy of over *83%*. The model demonstrates particularly strong performance in identifying distressed firms, with *87.3%* accuracy in both the original and cross-validated cases. This suggests that the financial variables used in the model - such as retained earnings, operational profitability, and liquidity—are particularly well-suited for detecting financial distress.

The slight decrease in classification accuracy during cross-validation is expected and normal, as cross-validation provides a more conservative estimate of model performance. The near-identical results between the original classification and cross-validated classification (*83.5%* vs. *83.4%)* highlight that the model does not suffer from significant overfitting, meaning it is likely to perform similarly well on new or unseen data.

The model does, however, struggle slightly more with classifying non-distressed firms, with around *20%* misclassified as distressed. This could be due to non-distressed firms that exhibit some financial indicators similar to distressed firms, such as low profitability or weak liquidity, but are still managing to avoid financial distress. The model may also misclassify firms on the borderline between financial health and distress, capturing only part of the complexity of their financial situation. Similarly, the *12.7%* of distressed firms that were misclassified as non-distressed could reflect firms that have not yet fully displayed the financial characteristics typical of distress or those that have undertaken significant restructuring efforts to avoid distress, temporarily improving their financial ratios.

In brief, the discriminant analysis model demonstrates strong predictive accuracy, correctly classifying over *83%* of firms. The slightly better performance in identifying distressed firms highlights the model's strength in detecting signs of financial trouble, while the modest misclassification rate in non-distressed firms reflects the complexity of financial performance in firms that may share some characteristics of distress but remain financially healthy. The small difference between the original grouped cases and cross-validated grouped cases further emphasizes the robustness and reliability of the model, confirming its suitability for predicting financial distress across different sectors and scenarios.

b. Analysis of Classification Results for the Construction Sector

The classification results for the *Construction* sector, as presented in *Table 18*, reveal both strengths and limitations in the model's ability to accurately distinguish between distressed and non-distressed firms within this industry. The overall classification accuracy and cross-validation results provide insights into the model’s effectiveness, as well as areas where it may face challenges. Below is a detailed analysis of the strengths and weaknesses associated with the classification results for this sector.

Table 18: 01. Construction Sector

Classification Results[a,c]

Label2			Predicted Group Membership 0	1	Total
Original	Count	0	24	24	48
		1	10	38	48
	%	0	50,0	50,0	100,0
		1	20,8	79,2	100,0
Cross-validated[b]	Count	0	22	26	48
		1	10	38	48
	%	0	45,8	54,2	100,0
		1	20,8	79,2	100,0

a. 64,6% of original grouped cases correctly classified.

b. Cross validation is done only for those cases in the analysis. In cross validation, each case is classified by the functions derived from all cases other than that case.

c. 62,5% of cross-validated grouped cases correctly classified.

One of the key strengths observed in the classification results is the model’s ability to correctly classify a significant portion of the *distressed firms*. In the original classification, the model accurately identified *79.2%* of distressed firms (38 out of 48). This high level of accuracy indicates that the financial variables used in the model are particularly effective at capturing the indicators of financial distress within the construction sector. Given the capital-intensive nature of construction, firms that exhibit signs of financial distress - such as insufficient liquidity or poor profitability - tend to be more distinct, making them easier for the model to classify.

Furthermore, the results remain stable under *cross-validation*, with *79.2%* of distressed firms again correctly classified. This consistency suggests that the model is not overly reliant on the specific dataset used for training and that it performs well when tested on unseen cases. The ability of the model to maintain similar levels of accuracy through cross-validation is a positive indicator of its robustness and reliability when applied to different construction firms within the dataset.

Another strength is that, for *non-distressed firms*, the model achieves a *50%* classification accuracy in the original results, suggesting that it can identify non-distressed firms half of the time. Although not as high as the classification rate for distressed firms, this still indicates some degree of distinction between financially healthy and distressed firms in this sector.

While the model shows strengths in classifying distressed firms, it faces notable challenges in accurately identifying *non-distressed firms*. In the original classification, only *50%* of non-distressed firms were correctly classified, meaning that the model misclassified half of the non-distressed firms as distressed. This misclassification rate suggests that the financial characteristics of some non-distressed construction firms may overlap with those of distressed firms, making it difficult for the model to accurately differentiate between the two groups. This could be due to the cyclical nature of the construction industry, where firms may experience temporary financial strains without necessarily being in distress. The model may misinterpret these short-term fluctuations as indicators of distress, leading to incorrect classifications.

This challenge is further highlighted in the *cross-validation* results, where the classification accuracy for non-distressed firms drops slightly to *45.8%*. This indicates that the model struggles more when classifying non-distressed firms based on data from other cases. The fact that the accuracy decreases in cross-validation suggests that the financial variables used in the model may not fully capture the nuances of non-distressed firms in the construction sector, especially those that operate in a highly variable market with fluctuating costs and demand.

Another point of concern is the *overall classification accuracy*. For the original grouped cases, *64.6%* of firms were correctly classified. While this is a reasonable level of accuracy, it highlights that the model does not perfectly capture the complexity of financial performance in the construction sector. In cross-validation, the overall accuracy drops slightly to *62.5%*, further suggesting that while the model performs moderately well, there are limitations in its ability to generalize across different cases. This reduction in accuracy indicates some level of overfitting to the original dataset, where the model may have learned specific patterns that do not hold as strongly when applied to new data.

The construction sector presents unique financial challenges, including long project timelines, significant upfront capital investments, and vulnerability to cyclical demand. These characteristics can make it difficult for a discriminant function model to accurately classify firms, as firms may exhibit periods of financial strain without necessarily being in distress. For example, a construction firm might temporarily face liquidity shortages due to the time lag between project expenses and revenue realization, which could lead the model to incorrectly classify it as distressed.

Additionally, the construction sector is highly sensitive to external factors such as interest rates, government regulations, and economic downturns, all of which can affect a firm's financial performance. The model may not fully account for these external variables, which could contribute to the misclassification of firms, particularly those on the borderline between distress and financial health.

The classification results for the construction sector reveal both strengths and challenges. The model demonstrates strong performance in identifying distressed firms, with a high classification accuracy of *79.2%* for both original and cross-validated cases. This consistency suggests that the financial variables used—such as retained earnings, liquidity, and profitability - are effective at distinguishing firms that are in financial distress. However, the model struggles to accurately classify non-distressed firms, with only *50%* correctly identified in the original dataset and *45.8%* in cross-validation. This misclassification rate reflects the complexity of the construction industry, where financial performance can fluctuate significantly, even for firms that are not in distress.

Overall, while the model provides valuable insights into the financial health of construction firms, its moderate overall accuracy (*64.6%* for original grouped cases and *62.5%* for cross-validated cases) indicates that there is room for improvement. Refining the model to better capture the specific financial dynamics of non-distressed construction firms—perhaps by incorporating additional variables related to project timelines or economic cycles—could enhance its accuracy and provide a more nuanced understanding of financial performance in this sector.

c. Analysis of Classification Results for the Manufacturing Sector

The classification results for the *Manufacturing* sector, as displayed in *Table 19*, provide a detailed understanding of the model's effectiveness in distinguishing between distressed and non-distressed firms within this industry. The analysis of the original classification and cross-validation results sheds light on the model's strengths and potential areas for improvement when applied to the financial data of manufacturing firms.

One of the most notable strengths of the model in the manufacturing sector is its high *overall classification accuracy*, with *85.0%* of the original grouped cases correctly classified. This high level of accuracy suggests that the financial ratios and variables used in the discriminant analysis - such as retained earnings, operational profitability, and liquidity—are particularly well-suited for identifying financial distress within manufacturing firms. The consistency of this result is further reinforced through *cross-validation*, where the model maintains an accuracy of ***85.0%,*** indicating that the model is stable and performs well even when tested on unseen data. This suggests that the discriminant function has captured robust patterns that are applicable to different manufacturing firms within the sample.

Table 19: 02. Manufacturing Sector

Classification Results[a,c]

Label2			Predicted Group Membership 0	1	Total
Original	Count	0	228	51	279
		1	35	244	279
	%	0	81,7	18,3	100,0
		1	11,7	88,3	100,0
Cross-validated[b]	Count	0	228	51	279
		1	35	244	279
	%	0	81,7	18,3	100,0
		1	11,7	88,3	100,0

a. 85,0% of original grouped cases correctly classified.

b. Cross validation is done only for those cases in the analysis. In cross validation, each case is classified by the functions derived from all cases other than that case.

c. 85,0% of cross-validated grouped cases correctly classified.

When focusing specifically on *distressed firms*, the model demonstrates strong predictive power. It correctly classifies *88.3%* of distressed firms (244 out of 279), highlighting its ability to accurately detect signs of financial distress in this sector. Given that the manufacturing industry often involves high capital expenditures, narrow profit margins, and sensitivity to market demand, firms facing financial difficulties tend to exhibit distinct financial characteristics, such as negative retained earnings or declining profitability. The model's high accuracy in identifying distressed firms suggests that these financial characteristics are effectively captured in the analysis, allowing for reliable classification.

The model also performs well in classifying *non-distressed firms*, correctly identifying *81.7%* of them (228 out of 279). This demonstrates that the model can accurately recognize financially stable firms, which often display strong operational profitability, positive retained earnings, and sufficient liquidity to manage production costs and inventory. This level of accuracy is crucial in the manufacturing sector, where firms need to demonstrate financial stability to withstand fluctuations in raw material prices, labor costs, and global competition.

Despite the model's overall success, there are some limitations that merit attention. One such limitation is the *misclassification rate* for both distressed and non-distressed firms. While the misclassification rates are relatively low, they indicate areas where the model struggles to differentiate between the two groups. For *non-distressed firms*, the model misclassifies *18.3%* as distressed, suggesting that some financially stable firms share financial characteristics with distressed firms, making it difficult for the model to distinguish between the two. This could be attributed to the cyclical nature of the manufacturing industry, where firms may experience temporary downturns or periods of low profitability without necessarily being in financial distress. For example, a firm might face short-term losses due to an investment in new production technologies or a temporary dip in market demand, leading to misclassification by the model.

Similarly, *11.7%* of distressed firms are misclassified as non-distressed. This misclassification might occur in cases where distressed firms exhibit certain financial ratios - such as liquidity levels or equity positions - that are more typical of financially stable firms. It could also reflect the presence of firms that are on the borderline of distress, where early signs of financial trouble have not yet fully manifested in the financial ratios used in the model. These borderline cases pose a challenge to any classification model, as they represent firms that may still have some financial resilience despite underlying vulnerabilities.

The high overall classification accuracy of the model in the *Manufacturing* sector can be attributed to the clear financial signals that distinguish distressed from non-distressed firms. In this sector, operational efficiency and profitability are crucial, as firms need to manage production costs while maintaining competitive pricing. The model's ability to detect these patterns highlights the importance of variables like *Earnings Before Interest and Taxes* to *Total Assets (EBIT_TA)***,** which measures a firm's ability to generate profits from its asset base. Similarly, *Retained Earnings* to *Total Assets (RE_TA)* serves as an indicator of a firm's ability to reinvest earnings into production capabilities, a key factor in maintaining long-term stability in manufacturing.

However, the model's misclassifications suggest that financial ratios alone may not fully capture the complexity of the manufacturing industry. External factors, such as changes in global supply chains, tariffs, and technological advancements, can have significant impacts on manufacturing firms' financial performance. These factors may not be directly reflected in the financial ratios used in the model, leading to misclassifications in cases where firms are adapting to external pressures or undergoing restructuring.

Overall, the *discriminant analysis model* performs strongly in the manufacturing sector, achieving a high classification accuracy of *85.0%* in both the original and cross-validated results. This consistency indicates that the model is reliable and well-suited for identifying financial distress in manufacturing firms, leveraging the financial variables that are most relevant to this capital-intensive industry. The model's ability to accurately classify *88.3%* of distressed firms is particularly noteworthy, as it underscores the clarity with which financially troubled firms can be identified based on their declining profitability and liquidity challenges. However, the model is not without its challenges, as reflected in the misclassification of *18.3%* of non-distressed firms and *11.7%* of distressed firms. These misclassifications suggest that while the model is robust, it could benefit from the inclusion of additional variables or external factors that capture the broader economic environment in which manufacturing firms operate. Addressing these limitations could further enhance the model's accuracy and provide a more nuanced understanding of financial health within the manufacturing sector.

In conclusion, while the discriminant analysis model offers a powerful tool for identifying distressed firms in the manufacturing industry, its effectiveness could be improved with a more comprehensive approach that considers both financial ratios and external economic factors. This would ensure that the model remains a reliable predictor of financial distress, capable of adapting to the evolving challenges faced by manufacturing firms.

d. Analysis of Classification Results for the Transportation, Communications, Electric, Gas, and Sanitary Services Sector

The classification results for the *Transportation, Communications, Electric, Gas, and Sanitary Services* sector presented in *Table 20* provide valuable insights into the effectiveness of the discriminant analysis model for this sector. The analysis of both the original grouped cases and the cross-validated cases highlights the model's strengths in distinguishing between distressed and non-distressed firms, as well as areas where further refinement may be necessary.

The model demonstrates a high degree of accuracy in classifying firms within this sector, with *87.7%* of the original grouped cases correctly classified. This indicates that the discriminant analysis is highly effective at distinguishing between distressed and non-distressed firms based on the financial variables considered in the study. The consistency of the results is further confirmed by the *cross-validated accuracy*, which is *87.3%*, showing that the model remains stable and reliable even when tested on new, unseen data.

A closer look at the classification of *distressed firms* reveals the model's robustness in identifying these entities. Specifically, *92.2%* of distressed firms (94 out of 102) are correctly classified in the original analysis, and this accuracy is maintained during cross-validation. This high success rate suggests that the financial variables used in the model - such as retained earnings, profitability, and liquidity - are effective at capturing the early signs of financial distress within this sector. Given the capital-intensive nature of industries like *transportation, electric utilities, and gas services*, where firms often face significant fixed costs and regulatory pressures, the model's ability to accurately classify distressed firms is particularly valuable. It suggests that the financial strains leading to distress in these industries are relatively distinct and can be effectively identified through discriminant analysis.

Table 20: 03. Transportation, Communications, Electric, Gas, and Sanitary Services Sector

Classification Results[a,c]

Label2			Predicted Group Membership 0	1	Total
Original	Count	0	85	17	102
		1	8	94	102
	%	0	83,3	16,7	100,0
		1	7,8	92,2	100,0
Cross-validated[b]	Count	0	84	18	102
		1	8	94	102
	%	0	82,4	17,6	100,0
		1	7,8	92,2	100,0

a. 87,7% of original grouped cases correctly classified.

b. Cross validation is done only for those cases in the analysis. In cross validation, each case is classified by the functions derived from all cases other than that case.

c. 87,3% of cross-validated grouped cases correctly classified.

The model also shows a good performance in classifying *non-distressed firms*, with *83.3%* of such firms (85 out of 102) accurately identified in the original classification, and *82.4%* correctly classified in the cross-validation. This high accuracy rate is indicative of the model's ability to recognize firms that maintain strong financial health and stability in an industry where consistent cash flow and long-term contracts often contribute to financial resilience. The ability of the model to consistently identify these firms is crucial for sectors like communications and utilities, where long-term investments and stability are key indicators of financial success.

Despite its overall strengths, the model does face some challenges, particularly in terms of *misclassification* rates. In the classification of *non-distressed firms*, *16.7%* of firms were incorrectly classified as distressed in the original analysis, with a slightly higher rate of *17.6%* during cross-validation. These misclassifications may arise due to certain financial ratios of non-distressed firms resembling those of distressed firms. For instance, even firms that are not in distress might experience periods of reduced profitability or temporary liquidity constraints, especially in capital-intensive sectors where fluctuations in demand or changes in regulatory frameworks can impact financial performance. This overlap can cause the model to mistakenly categorize some non-distressed firms as distressed.

Similarly, *7.8%* of distressed firms were classified as non-distressed, both in the original and cross-validated analyses. This indicates that some firms experiencing financial distress may display financial characteristics that resemble those of healthier firms, such as maintaining sufficient liquidity or temporarily improving profitability through cost-cutting measures. These cases often involve firms that are in the early stages of distress or are actively trying to recover, making it challenging for the model to accurately classify them as distressed.

Another potential limitation is the sensitivity of the sector to *external factors* such as economic cycles, regulatory changes, and shifts in energy markets or infrastructure investments. While the model relies on financial ratios to predict distress, it may not fully account for the impact of such external factors, which can significantly influence the financial health of firms in *transportation, communications, and utility services*. This can result in misclassifications, particularly for firms that might appear stable in their financial ratios but are vulnerable to external market shocks.

The overall high classification accuracy of the model in this sector highlights its strength in identifying firms with clear financial distress signals, which is especially valuable in industries that require significant capital investment and have long-term operational commitments. The model's ability to classify *92.2%* of distressed firms correctly suggests that financial distress in these industries often manifests through clear indicators such as declining retained earnings, reduced profitability, and liquidity issues.

However, the model's limitations in misclassifying certain firms underscore the complexity of the financial dynamics in these industries. The *misclassification of non-distressed firms* could reflect the inherent volatility in capital-intensive sectors, where firms may temporarily struggle with cash flow or profitability without facing long-term distress. Similarly, the *misclassification of distressed firms as non-distressed* suggests that some companies in early-stage distress or recovery phases can present a financial profile that temporarily masks deeper issues.

In conclusion, the *discriminant analysis model* applied to the *Transportation, Communications, Electric, Gas, and Sanitary Services* sector shows strong overall performance, with *87.7%* accuracy in original cases and *87.3%* in cross-validation. These high levels of accuracy indicate that the financial variables used are effective in identifying distressed firms in this sector, reflecting the distinct financial characteristics associated with firms in distress. The model's ability to maintain consistent performance in cross-validation underscores its robustness and reliability.

Nonetheless, the model does face challenges, particularly in accurately distinguishing non-distressed firms that may temporarily exhibit distress-like characteristics. Additionally, the complex external factors that influence this sector, such as regulatory changes and economic cycles, suggest that further refinement of the model could involve incorporating additional indicators that capture these external dynamics. Addressing these limitations could improve the model's precision, offering a more nuanced understanding of financial health within this diverse and capital-intensive sector. Ultimately, while the current model provides a solid foundation for predicting financial distress, a more comprehensive approach could further enhance its utility for stakeholders within these industries.

e. Analysis of Classification Results for the Whalesale Trade Sector

The classification results for the *Wholesale Trade* sector in *Table 21* offer a comprehensive view of how well the discriminant analysis model functions in this industry. The analysis covers the performance of the model in classifying both original grouped cases and cross-validated cases, revealing its key strengths and the areas where it encounters challenges in differentiating between distressed and non-distressed firms.

The primary strength of the model lies in its ability to accurately identify *distressed firms* within the wholesale trade sector. In the original classification, *95.2%* of distressed firms (40 out of 42) were correctly classified, a strong indication that the financial ratios used are effective in highlighting distress signals in this industry. This high rate of accuracy points to the model's ability to capture critical financial patterns that often accompany distress, such as declining liquidity, profitability issues, or deteriorating equity positions.

The stability of the model is further supported by its *cross-validation performance*, which maintains a high classification accuracy of *92.9%* for distressed firms. Cross-validation, a process where each firm is classified using a discriminant function derived from the remaining firms, tests the model's ability to generalize its predictions beyond the initial sample. The minimal drop in accuracy between the original and cross-validated results indicates that the model is robust, as it performs reliably even when faced with new data. This consistency is especially valuable in the wholesale trade sector, where accurate identification of distressed firms can assist in early intervention and support strategies.

Table 21: 04. Wholesale Trade Sector

Classification Results[a,c]

Label2			Predicted Group Membership 0	1	Total
Original	Count	0	26	16	42
		1	2	40	42
	%	0	61,9	38,1	100,0
		1	4,8	95,2	100,0
Cross-validated[b]	Count	0	25	17	42
		1	3	39	42
	%	0	59,5	40,5	100,0
		1	7,1	92,9	100,0

a. 78,6% of original grouped cases correctly classified.

b. Cross validation is done only for those cases in the analysis. In cross validation, each case is classified by the functions derived from all cases other than that case.

c. 76,2% of cross-validated grouped cases correctly classified.

Despite these strengths, the model exhibits some limitations, particularly in its classification of *non-distressed firms*. The original classification results show that only *61.9%* of non-distressed firms (26 out of 42) were correctly identified, with *38.1%* being misclassified as distressed. This higher rate of misclassification suggests that the financial profiles of some non-distressed firms overlap with those of distressed firms, potentially due to the sector's inherent financial fluctuations.

The challenge persists in the *cross-validation analysis*, where the classification accuracy for non-distressed firms decreases slightly to *59.5%***.** This indicates that when the model is tested with new cases, it struggles to maintain a clear distinction between stable and at-risk firms. Such misclassification may occur because wholesale trade firms, even when fundamentally sound, can experience periods of reduced liquidity or temporary profitability dips due to seasonal demand changes or shifts in inventory management. The model might interpret these short-term fluctuations as signs of financial distress, thus leading to incorrect classifications.

Another aspect worth noting is the model's *overall classification accuracy* of *78.6%* for the original dataset and *76.2%* for the cross-validated dataset. While this level of accuracy demonstrates the model's general effectiveness, the slight reduction in accuracy during cross-validation suggests a sensitivity to the specifics of the training data. It points to the need for further refinement, such as incorporating additional indicators that might better capture the sector's unique market dynamics, like seasonal sales patterns or changes in supplier relationships that impact inventory turnover and cash flow.

The high accuracy rate in identifying distressed firms emphasizes the model's proficiency in detecting clear financial distress signals, which is particularly beneficial for the wholesale trade industry. In this sector, firms struggling with managing inventory or maintaining adequate cash flow often show distinct financial patterns, making them easier to classify accurately. Variables such as *Cash and Cash Equivalents* to *Current Liabilities (CAR)* and *Earnings Before Interest and Taxes* to *Total Assets (EBIT_TA)* play a critical role in distinguishing firms that face liquidity crises or profitability challenges.

However, the difficulty in correctly classifying *non-distressed firms* points to the complexity of financial stability in the wholesale trade industry. Firms that are fundamentally sound may still experience fluctuations that do not necessarily indicate long-term distress, such as temporary dips in cash reserves due to seasonal sales cycles or large inventory purchases. The model's focus on financial ratios may not fully account for these temporary but normal variations, leading to a higher incidence of misclassifying stable firms as distressed.

In summary, the discriminant analysis model demonstrates a strong capacity for identifying financially distressed firms in the *Wholesale Trade* sector, with 95.2% accuracy in the original dataset and *92.9%* accuracy in cross-validation. These results suggest that the model is effective in recognizing key distress indicators, such as declining liquidity and profitability, which are crucial for predicting financial trouble in this industry. The model's consistent performance across original and cross-validated data highlights its reliability for practical application in this sector.

However, the model's limitations in distinguishing *non-distressed firms* - as seen in the *38.1%* misclassification rate for the original dataset and *40.5%* in cross-validation - reveal the challenges of interpreting financial data in a sector where short-term variability is common. These results indicate that while the model is generally effective, a more nuanced approach could improve its classification of stable firms. Integrating additional sector-specific variables, such as inventory turnover rates or measures of seasonal cash flow, could refine the model and enhance its ability to differentiate between temporary financial pressures and genuine distress.

Overall, the discriminant analysis model provides a strong foundation for evaluating financial health in the *Wholesale Trade* sector, particularly in identifying firms at risk of distress. With further refinements to address the complexity of non-distressed firms, the model has the potential to offer even more precise insights into the financial dynamics of this critical industry.

f. Analysis of Classification Results for the Retail Trade Sector

The classification results for the *Retail Trade* sector, as illustrated in *Table 22*, provide a comprehensive view of the model's effectiveness in distinguishing between distressed and non-distressed firms. The findings from both the original and cross-validated cases highlight the model's key strengths in identifying distressed entities, as well as some areas where its performance can be critiqued.

Table 22: 05. Retail Trade sector

Classification Results[a,c]

Label2			Predicted Group Membership 0	1	Total
Original	Count	0	29	5	34
		1	0	36	36
	%	0	85,3	14,7	100,0
		1	0,0	100,0	100,0
Cross-validated[b]	Count	0	29	5	34
		1	0	36	36
	%	0	85,3	14,7	100,0
		1	0,0	100,0	100,0

a. 92,9% of original grouped cases correctly classified.

b. Cross validation is done only for those cases in the analysis. In cross validation, each case is classified by the functions derived from all cases other than that case.

c. 92,9% of cross-validated grouped cases correctly classified.

The most striking strength of the model is its perfect performance in identifying *distressed firms***.** In both the original and cross-validated analyses, the model correctly classified *100%* of distressed firms (36 out of 36). This indicates that the discriminant function used in the analysis is highly effective at identifying the financial characteristics that are typical of distressed firms within the retail trade sector. The retail industry often faces significant financial pressures, such as fluctuations in consumer demand, seasonal sales cycles, and intense competition. These pressures can lead to clear indicators of distress, such as declining profitability, reduced retained earnings, or cash flow challenges, which the model has successfully captured. This high level of accuracy suggests that the financial variables included in the model, such as profitability ratios and liquidity measures, are well-suited for capturing signs of distress in this sector.

Additionally, the model's *overall classification accuracy* is *92.9%* for both the original grouped cases and the cross-validated cases, indicating that the model is robust and maintains its predictive power when applied to new, unseen data. Cross-validation ensures that the model's accuracy is not limited to the specific dataset used for training, providing confidence in its broader applicability. The consistency between the original results and the cross-validated results reinforces the reliability of the model in predicting financial distress across different retail firms.

The model also performs relatively well in identifying *non-distressed firms*, with *85.3%* of such firms (29 out of 34) correctly classified in both the original and cross-validated analyses. This suggests that the model is capable of recognizing the financial stability and operational efficiency that typically characterize healthy retail firms. Strong financial health in this sector is often reflected in positive retained earnings, solid profitability, and adequate liquidity to manage inventory and operating costs, all of which are captured by the model's financial variables.

Despite its high accuracy in identifying distressed firms, the model exhibits certain limitations, particularly in the classification of *non-distressed firms*. In both the original and cross-validated analyses, *14.7%* of non-distressed firms (5 out of 34) were misclassified as distressed. This suggests that some firms, which are fundamentally financially stable, exhibit characteristics that the model mistakenly interprets as signs of distress. This could be due to the inherent volatility in the retail sector, where even healthy firms may experience periods of financial strain, such as temporary drops in profitability during off-peak seasons or increased expenditure for inventory buildup before major sales events.

These temporary fluctuations can result in financial ratios that resemble those of distressed firms, leading to misclassifications. For example, a non-distressed firm may show a short-term decline in profitability or a decrease in liquidity due to strategic investments, such as opening new stores or investing in e-commerce platforms. The model may interpret these temporary financial changes as indicative of distress, resulting in incorrect classifications. This indicates that while the model captures long-term financial distress effectively, it may need refinement to better differentiate between short-term strategic decisions and true signs of financial trouble.

Another potential limitation is the model's rigidity in classifying *distressed firms*. While *100%* accuracy is desirable, it may also suggest that the model's criteria for identifying distress are highly stringent. This could mean that the model classifies any firm that slightly deviates from ideal financial health as distressed, potentially overlooking firms that are in the process of recovery or stabilization. This rigid classification could limit the model's flexibility and reduce its ability to detect firms that are on the edge of distress but are taking corrective measures.

In summary, the discriminant analysis model demonstrates strong performance in the *Retail Trade* sector, with an overall classification accuracy of *92.9%* and perfect identification of distressed firms. These results highlight the model's effectiveness in recognizing the financial distress signals that are critical in the retail industry, such as declining profitability and liquidity constraints. The model's consistency across both original and cross-validated cases further underscores its robustness, making it a valuable tool for stakeholders looking to assess financial health in this competitive sector.

However, the model's tendency to misclassify a portion of non-distressed firms suggests that it could benefit from further refinement. Integrating additional variables that capture the sector's cyclical nature or adjusting the sensitivity of the discriminant function to temporary financial changes could improve its ability to accurately differentiate between true distress and temporary challenges. Such enhancements would make the model more versatile, offering a more nuanced understanding of the financial dynamics that shape the performance of firms in the *Retail Trade* sector. Ultimately, while the model is a powerful predictor of financial distress, a more tailored approach could enhance its accuracy and provide deeper insights into the financial health of retail firms.

g. *Analysis of Classification Results for the Finance, Insurance, and Real Estate Sector*

The findings from *Table 23* regarding the *Finance, Insurance, and Real Estate sector* provide a comprehensive evaluation of the model's performance in this intricate industry. Characterized by its sensitivity to external economic factors and regulatory shifts, this sector presents unique challenges for financial analysis. The discriminant analysis model, evaluated through both original classification and cross-validation methods, demonstrates notable predictive strength while also pointing out specific aspects that could be improved for greater accuracy and reliability.

The model achieves a high level of *overall classification accuracy*, correctly classifying *93.6%* of cases in the original analysis and *91.0%* in cross-validation. This high accuracy suggests that the financial indicators utilized in the discriminant analysis are well-suited for capturing the dynamics of distress and stability within the finance, insurance, and real estate industries. Given that firms in this sector often have complex financial structures, the model's ability to maintain a high classification rate is a notable achievement, demonstrating its robustness in distinguishing between firms based on their financial profiles.

Table 23: 06. Finance, Insurance, and Real Estate Sector

Classification Results[a,c]

Label2			Predicted Group Membership 0	1	Total
Original	Count	0	37	2	39
		1	3	36	39
	%	0	94,9	5,1	100,0
		1	7,7	92,3	100,0
Cross-validated[b]	Count	0	36	3	39
		1	4	35	39
	%	0	92,3	7,7	100,0
		1	10,3	89,7	100,0

a. 93,6% of original grouped cases correctly classified.

b. Cross validation is done only for those cases in the analysis. In cross validation, each case is classified by the functions derived from all cases other than that case.

c. 91,0% of cross-validated grouped cases correctly classified.

The model is particularly effective in classifying *non-distressed firms* within the sector. In the original classification, *94.9%* of non-distressed firms (37 out of 39) were accurately identified, with a slight decrease to *92.3%* in cross-validation. This suggests that the financial characteristics of stable firms in the finance, insurance, and real estate industries are distinct enough for the model to capture effectively. The stability of these industries often depends on strong liquidity management, consistent earnings, and prudent leverage, which are likely reflected in the financial ratios used in the analysis. This high rate of correct classification suggests that the model can reliably identify firms that maintain strong balance sheets and stable cash flows, characteristics that are crucial for surviving market volatility in these industries.

The model's performance remains relatively consistent during cross-validation, which reinforces its reliability. Cross-validation ensures that the model's accuracy is not merely a result of overfitting to the specific training data. The ability of the model to maintain over *91%* accuracy in cross-validation indicates that it has generalized well beyond the initial dataset, suggesting its applicability to new data within the finance, insurance, and real estate sectors.

While the model demonstrates notable strengths, it faces some challenges in correctly identifying *distressed firms*. In the original classification, *92.3%* of distressed firms (36 out of

39) were correctly classified, but *7.7%* (3 out of 39) were misclassified as non-distressed. In the cross-validation process, this rate of misclassification increases slightly, with *10.3%* of distressed firms incorrectly identified as non-distressed.

This slight increase in misclassification during cross-validation suggests that some distressed firms in this sector have financial profiles that resemble those of stable firms, at least temporarily. The finance, insurance, and real estate industries are highly sensitive to changes in interest rates, market cycles, and regulatory shifts, which can lead to short-term improvements in profitability or liquidity even in firms that are fundamentally in distress. For instance, a distressed firm may temporarily show improved liquidity due to asset sales or short-term adjustments in debt management, which could lead the model to classify it as stable. Such scenarios highlight the complexity of interpreting financial distress within these industries and suggest that the model may benefit from incorporating additional variables that account for external economic conditions or sector-specific risks.

The misclassification of *non-distressed firms* is relatively low, with *5.1%* in the original analysis and *7.7%* in cross-validation. However, even these small rates of misclassification warrant attention. For example, the misclassification of a non-distressed firm as distressed could have significant implications for stakeholders, such as investors or regulators, potentially leading to unwarranted concerns about a firm's stability. This issue underscores the importance of fine-tuning the model to better distinguish between firms undergoing temporary adjustments and those experiencing deeper financial troubles.

Conversely, the misclassification of *distressed firms* as non-distressed, while less common, could pose a risk if not addressed. Distressed firms that appear stable due to temporary financial boosts may continue to face underlying challenges that could lead to failure if not managed properly. This scenario highlights the need for a more nuanced approach that can differentiate between temporary financial improvements and sustainable stability. Addressing this limitation could involve adding variables that capture longer-term financial trends or incorporating indicators that track changes in market conditions more closely.

The classification results for the *Finance, Insurance, and Real Estate* sector reveal a model that is highly effective at distinguishing between financially healthy and distressed firms, with an overall accuracy of over *90%* in both the original and cross-validated analyses. The model's strength in accurately identifying non-distressed firms demonstrates its ability to recognize stable financial patterns, which is crucial in an industry that relies heavily on strong cash flow management and stable earnings. The consistency between the original results and the cross-validation outcomes reinforces the model's robustness, suggesting its utility in real-world applications.

However, the model's slightly higher misclassification rate for distressed firms indicates that further refinement is needed. The presence of *7.7%* to *10.3%* misclassification of distressed firms suggests that the model could benefit from additional parameters that capture the nuances of economic cycles or temporary financial adjustments in these industries. By incorporating a broader range of variables or adjusting the sensitivity of the discriminant function, the model could more accurately differentiate between firms experiencing short-term improvements and those that are genuinely recovering from distress.

Overall, the model provides a reliable framework for assessing financial health in the *Finance, Insurance, and Real Estate* sector, offering valuable insights into the factors that contribute to stability or distress. With further adjustments, it has the potential to become an even more effective tool for stakeholders seeking to navigate the complexities of financial performance in this dynamic sector.

h. Analysis of Classification Results for the Service Sector

The analysis of the *Services* sector, presented in *Table 24,* offers valuable insights into the discriminant analysis model's performance within a diverse and dynamic industry. This sector encompasses a wide range of activities, including professional services, hospitality, healthcare, and more, each with unique financial characteristics. As such, the results of the model's classification provide a deeper understanding of how well it captures these nuances when distinguishing between distressed and non-distressed firms. Both the original classification results and the cross-validation outcomes provide a balanced view of the model's strengths and its areas of improvement.

One of the key strengths of the model lies in its *overall classification accuracy* of *83.2%* for the original grouped cases and *81.7%* in cross-validation. These figures indicate that the discriminant analysis model is generally reliable when applied to the financial data of firms in the services sector. The relatively small decrease in accuracy from the original classification to cross-validation suggests that the model is fairly robust and maintains its ability to generalize to new cases within this industry. This stability is particularly important given the varied nature of the services sector, where firms range from those offering essential services to those whose performance is closely tied to economic cycles.

Table 24: 07. Services Sector

Classification Results[a,c]

Label2			Predicted Group Membership 0	1	Total
Original	Count	0	77	21	98
		1	12	87	99
	%	0	78,6	21,4	100,0
		1	12,1	87,9	100,0
Cross-validated[b]	Count	0	76	22	98
		1	14	85	99
	%	0	77,6	22,4	100,0
		1	14,1	85,9	100,0

a. 83,2% of original grouped cases correctly classified.

b. Cross validation is done only for those cases in the analysis. In cross validation, each case is classified by the functions derived from all cases other than that case.

c. 81,7% of cross-validated grouped cases correctly classified.

The model also performs well in identifying *distressed firms*, with *87.9%* of distressed firms (87 out of 99) correctly classified in the original analysis, and *85.9%* in cross-validation. This strong performance suggests that the financial indicators used - such as liquidity ratios, profitability metrics, and retained earnings - are effective in capturing the early signs of financial distress in service-oriented businesses. The ability to consistently identify firms that are struggling financially is crucial in the services sector, where challenges like fluctuating demand, high labor costs, and dependency on consumer spending can quickly affect a firm's stability.

Despite the model's strengths, it faces notable challenges in accurately classifying *non-distressed firms* within the services sector. In the original classification, *21.4%* of non-distressed firms (21 out of 98) were misclassified as distressed, and this misclassification rate increases slightly to *22.4%* in cross-validation. This relatively high rate of misclassification suggests that the financial profiles of some stable firms in the services sector resemble those of distressed firms, making it difficult for the model to distinguish between them accurately.

This overlap may be due to the diverse nature of service-based businesses, where even fundamentally sound firms can experience periods of financial strain due to factors like seasonality or economic downturns. For example, a non-distressed firm in the hospitality industry may show temporary declines in profitability during off-peak seasons, which the model might interpret as distress. Similarly, a professional services firm might experience fluctuations in revenue that do not necessarily indicate long-term distress but rather reflect the cyclical nature of client demand. These temporary financial shifts can lead to incorrect classifications, highlighting a limitation of the model when dealing with firms whose financial health is sensitive to short-term market conditions.

The model's ability to classify *distressed firms* with relatively high accuracy remains a significant advantage, as it can help stakeholders identify those firms most in need of intervention. However, the *12.1%* misclassification rate for distressed firms in the original analysis and the slightly increased *14.1%* in cross-validation suggest that some firms exhibiting early signs of distress may be incorrectly classified as non-distressed. This misclassification can be critical, as it may delay necessary restructuring efforts or prevent timely support that could mitigate financial instability.

Moreover, the *misclassification of non-distressed firms* can have practical implications for stakeholders such as investors, creditors, and management. Overestimating the risk of distress in stable firms could lead to overly conservative lending practices or unnecessary caution in business partnerships. This issue underlines the need for a more refined approach that could better capture the complexities of the services sector. For instance, integrating indicators that track longer-term trends or adjust for seasonality could help reduce the model's tendency to misclassify firms during temporary downturns.

The services sector, with its broad range of activities, presents unique challenges for financial modeling. Firms in this industry often face fluctuating operating conditions, such as shifts in consumer demand or changes in regulatory environments that can impact financial performance. The model's ability to maintain an accuracy rate above *80%* indicates that it has captured some of these dynamics well, particularly in identifying firms with clear signs of distress.

However, the challenges in classifying non-distressed firms suggest that the model could benefit from adjustments tailored to the specific characteristics of service-based businesses. For example, developing a deeper understanding of how different types of services firms react to economic cycles or adjusting the model's sensitivity to short-term financial changes could enhance its accuracy. Such refinements could help the model distinguish between firms experiencing temporary difficulties and those with more serious underlying issues.

In conclusion, the *discriminant analysis model* applied to the *Services* sector demonstrates a solid ability to classify firms, with *83.2%* accuracy for original cases and *81.7%* in cross-validation. The model's effectiveness in identifying distressed firms suggests that it is well-equipped to detect significant financial stress within this diverse industry. This capability is particularly valuable for stakeholders seeking to identify firms at risk and implement timely support measures.

However, the higher misclassification rate among non-distressed firms points to the need for further refinement. The tendency to misinterpret temporary financial challenges as indicators of distress highlights the limitations of the current approach. By incorporating additional sector-specific factors, such as adjustments for seasonal trends or more nuanced measures of revenue stability, the model could better account for the complexities of the services sector.

Overall, the model provides a strong framework for analyzing financial health in the *Services* sector, but with targeted improvements, it could become even more adept at distinguishing between firms experiencing short-term challenges and those facing deeper financial distress. This would enhance its utility for guiding investment decisions, lending practices, and strategic planning in a sector that plays a crucial role in the broader economy.

4. VALIDATION of the Z''-Score ADJUSTMENT: ALIGNMENT with HYPOTHESES

The modification of the Z"-Score model in this study plays a crucial role in providing empirical support for the assumptions outlined in the research hypotheses. The adjustment, which involves the replacement of a specific ratio with one better suited to capturing early signs of financial distress, has been validated through the results of the analysis. This adjustment is integral to a more accurate assessment of a firm's financial health, directly tying back to the hypotheses formulated at the beginning of this study. Below, I elaborate on how this modification aligns with and substantiates each hypothesis.

a. *Alignment with Hypothesis H_1: Effective Financial Monitoring and Going Concern Status*

H_1: firms that effectively monitor and respond to specific financial are less likely to experience financial distress and maintain their going concern status.

Hypothesis *H1* posits that firms capable of effectively monitoring and responding to specific financial metrics are less likely to experience financial distress and can maintain their going concern status. The modification of the Z"-Score model, particularly through the inclusion of a new ratio that better reflects a firm's liquidity and cash management capabilities, enhances the model's ability to identify firms that engage in such proactive monitoring. This is critical, as liquidity is often a leading indicator of a firm's ability to weather short-term financial pressures and avoid distress.

The empirical results demonstrate that the modified Z"-Score is more sensitive to changes in liquidity, a key factor for firms striving to remain solvent. By replacing a less responsive ratio with one that directly measures cash reserves relative to current liabilities, the modified model allows for a more precise identification of firms that are able to manage their cash flow effectively. This capability aligns with H_1 by showing that those firms that exhibit stronger liquidity management through the modified ratio are indeed less likely to fall into financial distress, thus maintaining their operational continuity. In essence, the replacement improves the model's predictive power in distinguishing between firms that are merely at risk and those that have the capacity to adapt and stabilize.

b. Supporting Hypothesis H_2: The Role of Financial Restructuring in Recovery

H_2: firms that engage in proactive financial restructuring, cost-cutting measures, and strategic realignments are more likely to recover from financial distress and avoid insolvency or liquidation

Hypothesis H_2 suggests that firms engaging in proactive financial restructuring, cost-cutting, and strategic realignments are more likely to recover from financial distress and avoid insolvency. The modification of the Z"-Score, specifically the inclusion of a cash-based ratio, provides a clearer lens through which to observe these efforts. The adjusted model captures the effects of restructuring activities that often manifest through improved cash positions and reductions in short-term liabilities, directly influencing the financial ratios used in the modified score.

The results indicate that firms that have implemented effective restructuring measures often display better performance on the modified Z"-Score, as the new ratio highlights improvements in liquidity and short-term financial management. This supports H_2 by showing that firms actively managing their cash flows and liabilities, key components of restructuring strategies, tend to exhibit stronger scores in the modified model, reflecting their increased resilience against financial distress. The enhanced sensitivity of the modified Z"-Score thus allows for a more nuanced understanding of the impact of these strategic adjustments, reinforcing the hypothesis that such measures contribute to a firm's ability to recover and sustain operations.

c. *Validation of Hypothesis H_3: Constructing a Predictive Index*

H_3: a straightforward index can be created using historical financial data to effectively characterize and predict trends in a firm's financial equilibrium.

Hypothesis *H_3* asserts that a straightforward index, constructed using historical financial data, can effectively characterize and predict trends in a firm’s financial equilibrium. The adjustment to the Z"-Score model strengthens this claim by introducing a ratio that better captures liquidity trends over time, thereby improving the model’s ability to predict shifts in a firm’s financial stability. The modified index is more reflective of ongoing financial realities, as it integrates a measure that directly correlates with a firm's ability to meet its obligations without relying on short-term borrowing or asset liquidation.

The empirical validation of the modified Z"-Score across various sectors demonstrates its robustness as a predictive tool. By incorporating a ratio that is more aligned with the actual cash management practices of firms, the model provides a more accurate and timely indicator of financial stability. This directly supports *H_3*, as it shows that a more tailored index—constructed with the right mix of financial ratios—can indeed offer a clearer picture of a firm's financial health. The modification thus proves to be a critical step in refining the model’s predictive accuracy, allowing for a more precise anticipation of financial trends and potential distress.

d. *Conclusion: Justifying the Z"-Score Modification*

Overall, the modification of the Z"-Score model, through the replacement of a ratio, is justified by the empirical results and their alignment with the first three hypotheses of this study. The adjustment improves the model's ability to differentiate between distressed and non-distressed firms, offering a more accurate representation of a firm’s financial health. The enhanced sensitivity to liquidity and cash flow management provides critical insights into a firm's operational stability, its potential for recovery, and its likelihood of initiating restructuring measures.

This refined approach not only supports the hypotheses but also enhances the practical applicability of the Z"-Score as a predictive tool for financial analysts and stakeholders. By offering a more precise measure of financial risk, the modified Z"-Score contributes to a better understanding of the factors that drive financial distress and recovery, thus providing a valuable framework for assessing firm stability in various economic conditions.

The findings from my study further substantiate the distinction between predicting firm failure and predicting financial distress, a difference that has been highlighted in prior research *(Astebro T. and Winter J.K., 2012; Gupta J. et al., 2018; De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023).* While these concepts are often conflated in the literature, my results demonstrate that they represent distinct phenomena, each requiring a tailored approach for accurate assessment. This distinction is particularly important in the context of my research, as I focus on developing a model specifically designed to predict financial distress, rather than outright failure or bankruptcy.

Financial distress, as analyzed in my study, refers to a condition where a firm faces significant financial challenges, such as declining liquidity, deteriorating profitability, or an inability to meet its short-term obligations. These difficulties often precede failure, but they do not necessarily lead to bankruptcy if managed effectively through interventions like restructuring or strategic realignments. On the other hand, failure typically implies a point of no return, where the firm is no longer able to sustain its operations and ultimately ceases to function as a viable entity. This difference is not merely semantic; it has profound implications for how we design predictive models and interpret their results.

In developing my model for predicting financial distress, I have sought to capture early warning signs—those indicators that reveal a firm's increasing vulnerability well before it reaches the brink of failure. The results demonstrate that this focus on distress prediction yields higher predictive accuracy, suggesting that the model is particularly sensitive to the financial characteristics that precede failure, rather than those that signal its immediate onset. By emphasizing variables that reflect liquidity management, profitability trends, and leverage ratios, my model effectively identifies firms that are struggling but may still have the opportunity to recover if appropriate measures are taken.

This approach aligns with the insights of those who argue *(Astebro T. and Winter J.K., 2012; Gupta J. et al., 2018; De Luca F. and Meschieri E., 2017; De Luca F. and Mehmood A., 2023)* that financial distress and failure should be treated as separate stages within the financial trajectory of a firm.

My findings suggest that models designed for distress prediction offer a more nuanced understanding of a firm’s financial health, focusing on a broader spectrum of risk factors. These models are not only about identifying firms that are likely to collapse but also about highlighting those that are on the edge, thereby providing stakeholders with the information necessary for timely intervention.

The higher prediction accuracy achieved by my model reinforces the validity of focusing on distress rather than failure. By doing so, the model is better equipped to identify firms that exhibit signs of financial strain, even when those signs are not yet severe enough to predict an imminent collapse. This distinction is crucial for managers, investors, and creditors, as it allows them to adopt proactive strategies to address emerging financial challenges, potentially averting a downward spiral that could lead to insolvency.

My research highlights that the process of financial distress prediction is inherently different from failure prediction, both in terms of its objectives and its methodological requirements. By developing a model that targets financial distress, I am able to provide a more precise and forward-looking assessment of a firm's risk profile. This approach not only aligns with existing theoretical perspectives but also contributes to the broader understanding of how firms can be supported before they reach a critical point of no return. The results clearly indicate that focusing on distress prediction offers a more accurate and actionable tool for stakeholders, emphasizing the importance of treating these two concepts as distinct, yet interconnected, elements of corporate financial analysis.

5. DEVELOPING the TDR PROBABILITY MODEL

In the subsequent phase of my research, I focus on developing a *Troubled Debt Restructuring (TDR) probability* model, designed to predict the likelihood that a firm will undergo restructuring due to financial distress. This model is rooted in the application of our modified *Z"-Score* framework, which serves as a critical tool for assessing the financial health of firms. The Z"-Score model, which I have adapted to include more relevant ratios, provides a comprehensive assessment of the financial conditions that precede the need for TDR.

The development of this probability model involves the estimation of coefficients that quantify the impact of various financial ratios on the likelihood of a firm entering TDR. By focusing on both distressed and non-distressed firms, I aim to create a model that not only identifies firms that are at risk but also differentiates them from those that maintain a stable financial outlook. This distinction is crucial, as it allows stakeholders - such as managers, creditors, and investors - to understand the specific financial challenges a firm faces and to adopt appropriate measures before a restructuring becomes necessary.

The modified Z"-Score model forms the foundation for this probability estimation. It incorporates key financial ratios, such as *liquidity measures, profitability ratios, and leverage* indicators**,** that have been adjusted to reflect the nuances of firms facing potential restructuring. By using these adjusted coefficients, I am able to calculate the probability of distress for each firm in the sample, providing a more precise measure of the risk that a firm will initiate TDR procedures. The integration of these coefficients allows the model to capture subtle shifts in financial health, offering a predictive edge over traditional models that may overlook early signs of distress.

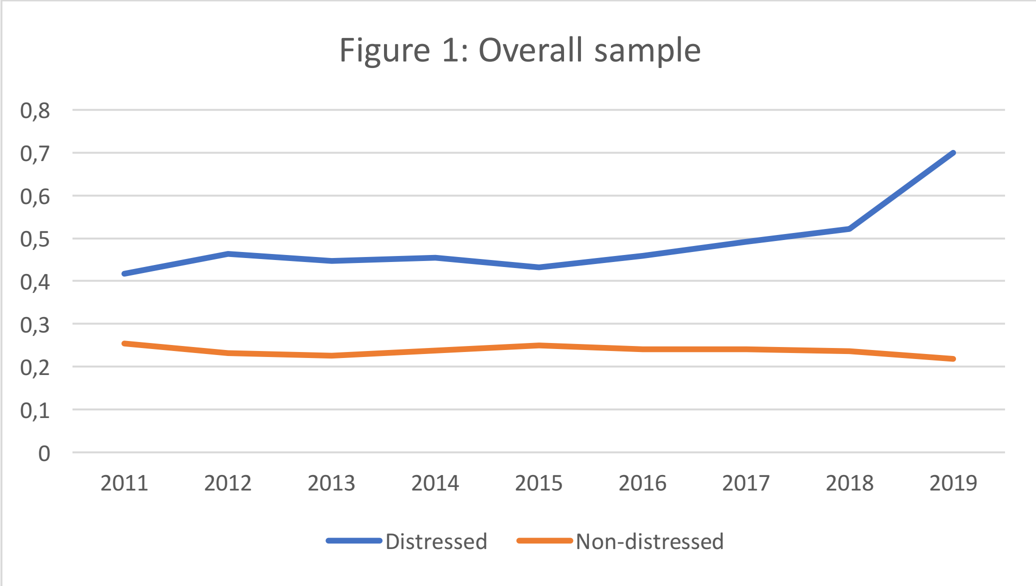
This modeling approach serves two primary objectives: first, to provide a more accurate forecast of financial distress, and second, to offer a practical tool for evaluating the effectiveness of early interventions. By estimating probabilities that quantify a firm's risk of entering TDR, the model offers a clearer view of how financial conditions evolve over time, and which firms are likely to benefit from preemptive measures, such as restructuring or strategic realignment. This enables stakeholders to make more informed decisions, targeting firms with the greatest potential for recovery before their financial challenges become irreversible.

The TDR probability model aims to bridge the gap between theoretical assessments of financial health and real-world decision-making. It translates the financial indicators embedded in the modified Z"-Score into actionable probabilities, which can be used to prioritize firms for intervention. This methodology not only enriches the academic understanding of financial distress but also provides practical insights for mitigating risks within firms that are on the brink of restructuring. The following analysis explores the calculation process, the results derived from the model, and its implications for managing financial distress in a targeted and strategic manner.

a. TDR Probability Model for the Overall Sample

To gain a deeper understanding of the dynamics of *TDR probabilities* over time, I calculate the *year-wise average probabilities* for firms potentially entering *Troubled Debt Restructuring (TDR)***.** This analysis includes the entire sample, covering both *distressed* and *non-distressed firms***.** By averaging the TDR probabilities for each year, my goal is to identify any overarching trends or patterns that emerge throughout the observed period. This method allows me to capture the fluctuations in the likelihood of firms entering TDR year by year, offering a clearer picture of how the risk of restructuring evolves over time.

The line chart presented in *Figure 1* illustrates the year-wise trends of *TDR probabilities* for both *distressed* and *non-distressed firms* in the overall sample, covering the period from *2011 to 2019***.** The graph highlights the differences in the probability of entering *Troubled Debt Restructuring (TDR)* between these two groups over time, offering insights into the divergent financial trajectories they follow.



The *distressed firms*, represented by the *blue line*, exhibit a markedly upward trend in their *TDR probabilities* over the period analyzed. Initially, from *2011* to *2014*, there is a slight increase in the probability, indicating that the financial distress among these firms is gradually becoming more pronounced. During this phase, the probability remains relatively stable but consistently above the levels seen for non-distressed firms, suggesting a persistent risk of restructuring even in the earlier years.

From *2015* onwards, the upward trend becomes more pronounced, with a noticeable acceleration in *2017*, leading to a significant rise through *2018* and *2019*. By *2019***,** the TDR probability for distressed firms reaches a peak of approximately *0.7***.** This sharp increase towards the end of the period indicates a growing likelihood that these firms would need to engage in restructuring efforts to manage their financial instability. The steep rise could reflect various external factors, such as economic downturns, tightening credit conditions, or structural issues within certain industries that disproportionately affect financially weaker firms.

The trend observed for distressed firms suggests that their financial challenges tend to compound over time, with risks escalating notably in the later years of the analysis. The rising TDR probability underscores the inability of these firms to stabilize their financial positions, potentially due to factors like declining revenues, increasing debt burdens, or unsuccessful restructuring attempts. This trend is indicative of a deteriorating financial environment for distressed firms, where the need for TDR becomes increasingly critical as time progresses.

In contrast, the *non-distressed firms***,** represented by the *red line*, display a *stable and relatively flat* trend throughout the same period. The TDR probability for these firms starts at a lower level compared to distressed firms, with values around *0.2***,** and remains within a narrow range throughout the years from *2011* to *2019*. This stability indicates that non-distressed firms maintain a relatively low risk of requiring debt restructuring over the observed period.

The minor fluctuations in the TDR probability for non-distressed firms suggest that while there may be occasional shifts in financial conditions - such as market volatility or sector-specific challenges - these firms are generally able to manage their financial obligations effectively. Their stable TDR probabilities imply that they have stronger cash flow management, profitability, and a more balanced debt structure, enabling them to avoid the financial pressures that might lead to restructuring.

The difference in trends between distressed and non-distressed firms emphasizes the resilience of non-distressed entities in maintaining financial stability even during periods where distressed firms show heightened risks. This stability might be attributed to better risk management practices, stronger market positions, or more diversified revenue streams that allow these firms to weather economic challenges without resorting to TDR.

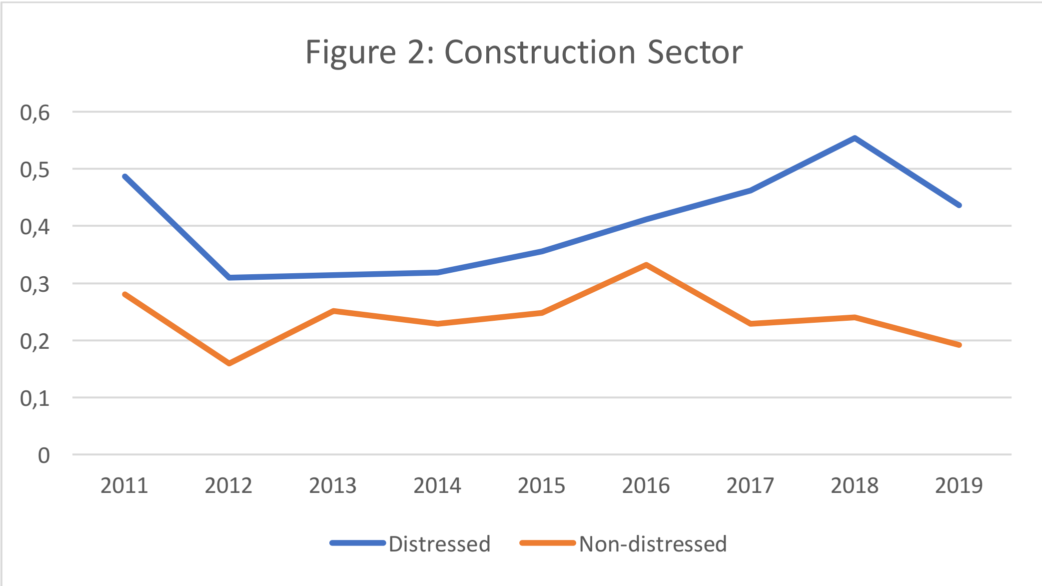
The contrasting trends between the two groups reveal important insights into the dynamics of financial distress and stability. While distressed firms show a trajectory of increasing risk, especially in the latter years, non-distressed firms maintain a steady outlook. This divergence suggests that the factors driving financial distress and the subsequent need for restructuring are not uniformly affecting all firms but are more concentrated among those already exhibiting weaknesses in their financial structure.

The sharp rise in the TDR probabilities of distressed firms towards *2018* and *2019* may indicate that economic conditions during these years had a more pronounced impact on vulnerable firms, pushing them closer to the need for restructuring. Conversely, the steadiness of the non-distressed firms implies that their financial fundamentals allowed them to remain largely unaffected by such conditions, avoiding the escalation of risk that characterized the distressed group.

Overall, *Figure 1* highlights the critical role of financial resilience in determining the likelihood of a firm entering TDR. For distressed firms, the inability to stabilize their finances over time results in a sharp increase in restructuring risk, suggesting a deepening of financial challenges. In contrast, the consistently lower and stable TDR probabilities of non-distressed firms underscore the importance of maintaining strong financial fundamentals to avoid distress scenarios. This comparative analysis serves to illustrate how different financial strategies and market conditions impact firms' restructuring probabilities, offering valuable insights into the effectiveness of early intervention and risk management practices.

b. *TDR Probability Model for the Construction Sector*

Figure 2 presents the year-wise trends of *Troubled Debt Restructuring (TDR) probabilities* for *distressed* and *non-distressed firms* within the *construction* sector from *2011* to *2019*. The graph allows for a comparative analysis of how the likelihood of requiring restructuring evolves over time for firms in varying financial conditions. It also highlights the disparities between firms that are facing financial distress and those that maintain a more stable outlook.



In the initial years, *2011* and *2012*, the TDR probability for distressed firms is relatively high, starting close to *0.5* in *2011***.** However, it drops sharply in *2012***,** settling around *0.3***.** For non-distressed firms, the probabilities begin at approximately *0.3* and decrease slightly in *2012*. It is important to note that the results for these early years are less significant due to the *small sample size* during this period, meaning that the observed fluctuations may not fully represent broader trends in the construction sector.

The limited number of companies in the sample for *2011* and *2012* can amplify the impact of outliers or extreme cases, leading to potentially skewed results. Thus, while the trends provide an initial snapshot of the sector, they should be interpreted with caution, as they might not capture the complete picture of financial distress during these years.

From *2013* onwards, the *distressed firms* (represented by the *blue line*) show a gradual increase in *TDR probabilities*, reaching a peak around *0.55* in *2018* before slightly declining in *2019***.** This upward trajectory suggests that financial vulnerabilities among distressed firms in the construction sector become more pronounced over time. The rise in probabilities indicates that these firms increasingly face liquidity constraints, declining profitability, or higher leverage ratios, pushing them closer to the need for restructuring.

The peak in *2018* could be attributed to various factors, such as a slowdown in construction activity, tighter lending conditions, or increased costs that could have exacerbated financial strain. The slight dip in *2019* may indicate that some distressed firms were able to stabilize temporarily, possibly through internal restructuring efforts or improvements in market conditions. Nevertheless, the overall trend points to a sector where distressed firms struggle to regain financial stability over the medium term, highlighting the persistent challenges that these companies face in maintaining viable operations.

The *non-distressed firms* (represented by the *red line*) display a more stable trend over the same period, with *TDR probabilities* consistently lower than those of distressed firms. Their probabilities hover around *0.2* to *0.3*, showing minor fluctuations but without any significant upward or downward shifts. This relative stability suggests that non-distressed firms in the construction sector generally manage to maintain their financial health, avoiding the pressures that could necessitate a restructuring.

However, it is crucial to emphasize that, despite their better overall financial standing, non-distressed firms still exhibit a baseline probability of entering TDR. While these firms are not as vulnerable as their distressed counterparts, the presence of a *0.2* to *0.3 probability* indicates that they are not entirely immune to the risks inherent in the construction industry, such as market downturns, project delays, or cost overruns. This underlying risk underscores the importance of proactive financial management, even for firms that appear stable, as they remain susceptible to sudden shifts in the economic environment that could increase their restructuring needs.

The contrast between the rising trend for distressed firms and the stable trend for non-distressed firms highlights the divergent financial paths within the construction sector. While distressed firms continue to experience growing pressures over time, potentially due to an inability to recover from initial financial setbacks, non-distressed firms maintain a relatively steady outlook. This difference suggests that the ability to manage financial resources effectively can significantly impact a firm's likelihood of entering TDR, even within a sector as volatile as construction.

One of the key strengths of this trend analysis lies in its ability to differentiate between firms that are at a higher risk of restructuring and those that remain relatively stable. For policymakers, investors, and lenders, this distinction is critical, as it allows them to allocate resources and support where it is most needed. The analysis also emphasizes that while non-distressed firms are in a better position, they still require vigilant management to prevent their baseline risk from escalating into a more serious threat of distress.

Figure 2 illustrates the distinct challenges faced by firms in the construction sector, with *distressed firms* displaying a clear trend towards increased vulnerability over time, peaking in *2018* before a slight improvement in *2019*. Meanwhile, *non-distressed firms* maintain a lower probability of restructuring, yet remain subject to sectoral risks that require careful management. The analysis underscores the need for tailored financial strategies that address the unique dynamics of each group, while also highlighting the broader challenges that construction firms face in maintaining financial resilience. Understanding these trends is essential for identifying opportunities for early intervention and support, ultimately helping to mitigate the risk of distress across the sector.

The trend observed in *Figure 2* for *non-distressed firms* in the *construction* sector shows a notable peak around *2014-2015*, followed by a gradual decline from *2016* onwards. This peak in the *TDR probability* among firms that were otherwise considered financially stable can be largely attributed to the economic and industry-specific challenges that impacted the *construction* sector in Italy during this period.

During the years around ***2014-2015*,** the Italian construction industry faced significant economic pressures. This period coincided with the aftermath of the *Eurozone debt crisis*, which had severe repercussions on the construction market. The construction industry, already weakened by the financial crisis of *2008*, struggled with reduced investment levels, tight credit conditions, and a slowdown in public and private sector projects. These factors exerted pressure even on firms that had previously managed to maintain financial stability, thereby slightly raising their *TDR probabilities* despite being classified as non-distressed.

This rise in TDR probabilities reflects how broader economic downturns can impact even healthier firms. Many construction companies, despite having strong fundamentals, faced challenges such as delayed payments, reduced access to financing, and a shrinking market for new projects. As a result, these firms became more vulnerable to cash flow issues, leading to a temporary increase in the likelihood that they might require restructuring measures to maintain stability.

However, starting from *2016***,** the sector began to show signs of recovery, which is reflected in the *gradual decline in TDR probabilities* for non-distressed firms. The Italian economy, along with the broader Eurozone, started to stabilize, leading to improved credit conditions and a renewed focus on infrastructure projects and urban development. This period saw an increase in both public and private investments in construction, which helped firms regain their financial footing.

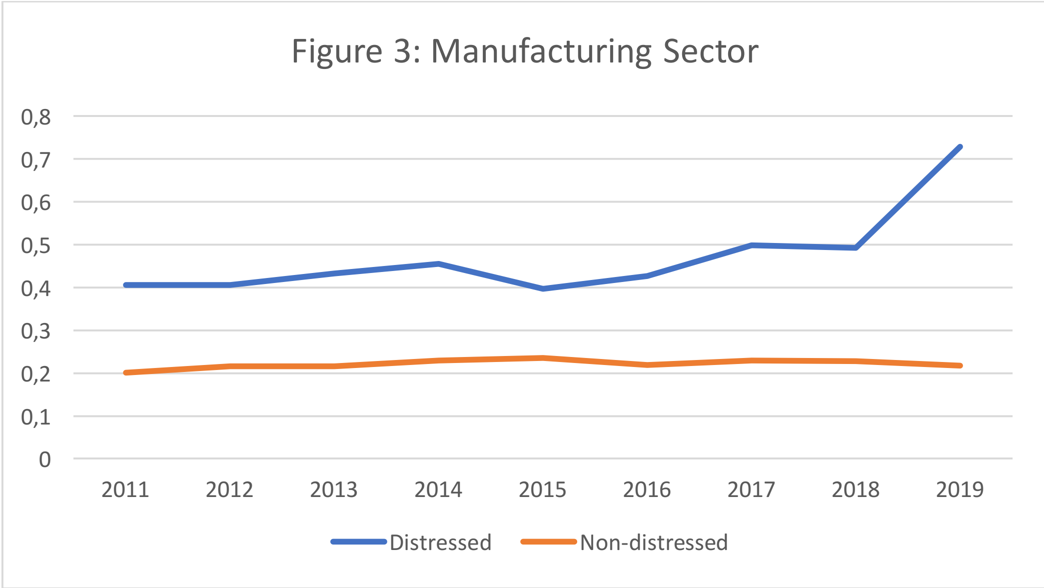
With the easing of economic pressures, non-distressed construction firms were able to strengthen their liquidity positions, manage their debt more effectively, and capitalize on the resurgence of market demand. As a result, their TDR probabilities decreased, moving back towards the baseline risk levels observed earlier in the decade. This improvement highlights the resilience of the sector's healthier firms and their ability to adapt to changing economic conditions when external pressures begin to subside.

The peak in *TDR probabilities* for non-distressed construction firms during *2014-2015* reflects the challenging environment faced by the sector due to economic stagnation and financial uncertainty in Italy. The subsequent decline in probabilities from *2016* onwards demonstrates the sector's recovery, as firms adapted to better market conditions, revitalized investment flows, and improved credit availability. This trend underscores the sensitivity of the construction sector to macroeconomic shifts and the importance of a stable economic environment in supporting the long-term financial health of even the most resilient firms.

c. TDR Probability Model for the Manufacturing Sector

Figure 3 illustrates the year-by-year trends of *Troubled Debt Restructuring (TDR) probabilities* for both *distressed* and *non-distressed firms* in the *manufacturing* sector over the period from *2011* to ***2019*.** This graph provides a comparative view of how the likelihood of entering restructuring evolves for firms facing financial distress and those maintaining a more stable financial position. It highlights the significant differences in TDR probabilities between the two groups and reflects broader economic and industry-specific factors that shape these trends.

The *distressed firms* in the manufacturing sector, represented by the *blue line*, show a trend that is characterized by a relatively stable probability of *around 0.4* from *2011* to *2014*. During this period, the TDR probabilities for distressed firms do not exhibit significant fluctuations, suggesting that many of these firms were consistently experiencing financial challenges, such as declining profitability and strained liquidity, but without a sharp increase in the likelihood of restructuring.



From *2015* onwards, however, there is a noticeable increase in the TDR probabilities for distressed firms. This rise continues with minor fluctuations until *2018***,** followed by a steep increase that peaks in *2019* at a probability of around *0.75*. The significant escalation in *2019* suggests that the distressed firms in the manufacturing sector faced increasing financial pressures that made restructuring more likely. This upward trend could be attributed to various external factors, such as global trade tensions, shifts in manufacturing demand, and challenges in adapting to new technologies or production processes.

The sharp rise towards the end of the period implies that despite initial resilience, distressed manufacturing firms struggled to manage their financial challenges as conditions worsened. This might reflect structural issues within the sector, such as a decline in traditional manufacturing outputs or increased competition from international markets, which could have further strained the financial stability of these firms.

In contrast, the *non-distressed firms*, represented by the *red line*, display a notably *stable trend* over the entire period. Their TDR probabilities remain consistently low, fluctuating only slightly around *0.2* throughout *2011* to *2019*. This stability suggests that non-distressed manufacturing firms have been able to maintain their financial health despite broader economic challenges and sector-specific shifts.

The low and stable TDR probabilities for non-distressed firms indicate that these companies have been better positioned to manage risks related to cash flow, debt obligations, and profitability. Their ability to maintain lower probabilities of restructuring suggests that they have either developed more adaptive strategies to cope with changes in the manufacturing landscape or possess stronger financial fundamentals, such as better capital reserves or more diversified revenue streams.

While the probabilities for non-distressed firms remain lower than those for distressed firms, it is important to note that the baseline risk of *0.2* reflects the inherent challenges of the manufacturing sector. Factors such as fluctuations in demand, supply chain disruptions, and shifts in global trade dynamics could affect even financially stable firms, albeit to a lesser extent. Thus, while non-distressed firms remain more resilient, they are not completely insulated from the broader economic environment.

The gap between the distressed and non-distressed groups becomes particularly pronounced towards the later years of the analysis. The sharp divergence seen around *2018-2019*, where distressed firms' probabilities rise dramatically, while non-distressed firms maintain their lower risk levels, underscores the differences in financial resilience between these groups.

The rising trend for distressed firms suggests that as economic conditions became more challenging, particularly in the face of global uncertainties like trade wars or shifts in manufacturing supply chains, those firms already struggling with financial instability were less capable of adapting. This inability to recover is reflected in the increasing likelihood of needing restructuring measures.

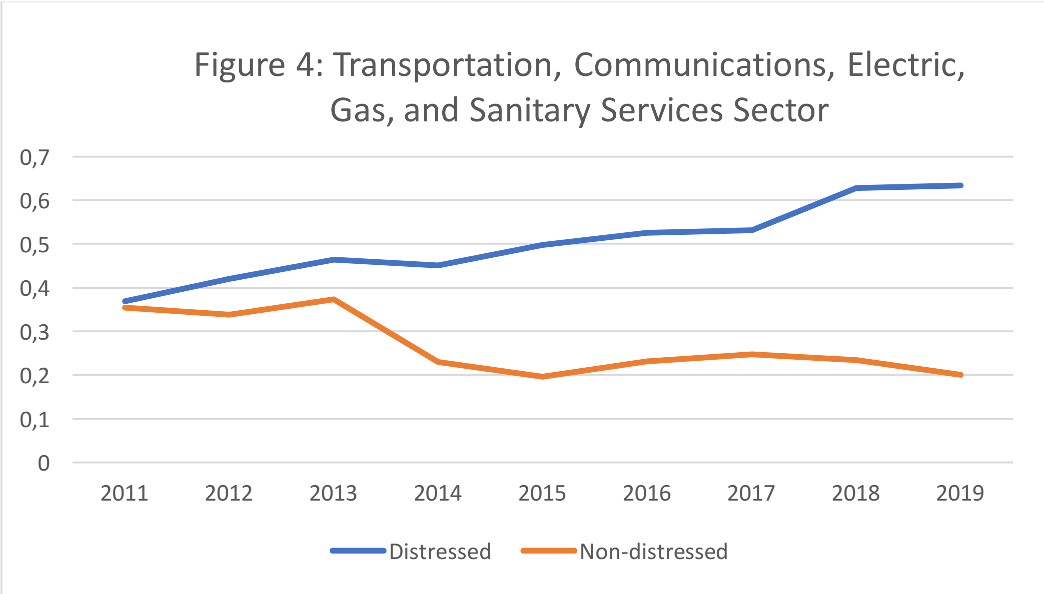
Non-distressed firms, meanwhile, appear to have managed these pressures more effectively, with their TDR probabilities showing no significant increase. This resilience may indicate that such firms were able to leverage efficiencies, maintain profitability, or adjust their production strategies to better withstand market fluctuations.

Figure 3 provides a detailed view of how *TDR probabilities* evolve differently for *distressed* and *non-distressed firms* within the *manufacturing sector*. The steady rise in probabilities for distressed firms highlights the growing financial pressures that these firms faced, particularly towards the end of the period, while the stability of non-distressed firms underscores their ability to navigate the challenges inherent in the manufacturing industry.

This analysis emphasizes the importance of maintaining strong financial health in a sector that is sensitive to both domestic and international economic conditions. For distressed firms, the increasing probabilities suggest an urgent need for strategic adjustments or restructuring to address deepening financial issues. In contrast, non-distressed firms demonstrate that with robust financial management and adaptive strategies, it is possible to maintain stability even in a sector facing significant external challenges. Understanding these trends is crucial for stakeholders looking to identify risks and opportunities within the *manufacturing* sector and to develop strategies that support long-term resilience.

d. TDR Probability Model for the Transportation, Communications, Electric, Gas, and Sanitary Services Sector

Figure 4 presents an in-depth view of the year-by-year *Troubled Debt Restructuring (TDR) probabilities* for *distressed* and *non-distressed* firms within the *transportation, communications, electric, gas, and sanitary services* sector between *2011* and *2019*. This analysis allows us to explore how firms in these industries, which are often sensitive to economic fluctuations and regulatory changes, experience different levels of restructuring risk. The trends observed offer crucial insights into the underlying financial stability of these firms, as well as the external factors influencing their restructuring probabilities over time.



The *distressed firms* in this sector, represented by the *blue line*, exhibit a clear pattern of increasing *TDR probabilities* throughout the period. Beginning in *2011* with a TDR probability close to *0.36*, these firms demonstrate an already significant risk of requiring restructuring measures due to pre-existing financial vulnerabilities. The initial rise through *2012* suggests that some of these firms were struggling with challenges such as high debt levels or reduced demand for services, leading to an elevated need for financial adjustments.

From *2013* to *2015***,** the probability remains relatively stable, fluctuating between *0.46* and *0.49*. This period indicates that, while distressed firms continued to face financial pressures, there was no substantial change in their risk profile. This steadiness could be attributed to a period of adjustment, where firms might have sought internal restructuring or temporary measures to stabilize their finances, albeit without achieving significant long-term improvements.

However, starting from *2016*, the TDR probabilities for distressed firms begin to climb more sharply, reaching *0.62* by *2018*, where they remain stable through *2019*. This trend suggests an accumulation of financial pressures during the later years, potentially driven by rising operational costs, changes in industry regulations, or increased competition in the market. The peak and subsequent stabilization indicate that many of these firms reached a critical point where their financial distress necessitated consideration of TDR measures, yet external conditions did not improve sufficiently to lower their restructuring risk.

The plateau from *2018* to *2019* could signify that distressed firms have exhausted typical methods of internal financial adjustment, such as debt renegotiation or asset sales, making external restructuring the next viable step. This stagnation at a high probability level highlights the depth of the challenges these firms face, suggesting a sustained struggle to recover financial stability amidst evolving market conditions.

In contrast, *non-distressed firms* in this sector, represented by the *red line*, display a more stable trajectory over the same period, albeit with some notable variations. Starting at around *0.35* in *2011*, these firms initially show a moderate probability of requiring TDR, reflecting the inherent risks of operating in a capital-intensive and regulated industry. This probability level suggests that, even among firms considered financially stable, there is a baseline level of risk related to external shocks or sectoral volatility.

By *2013*, the TDR probability for non-distressed firms reaches a local peak before experiencing a significant decline during *2014-2015*, dropping below *0.2*. This dip indicates a temporary period of improved financial conditions, possibly due to a more favorable economic environment or successful internal adjustments that enhanced liquidity and profitability. During this time, non-distressed firms likely benefited from stable demand for essential services, cost reductions, or favorable credit conditions, which helped to reduce their likelihood of requiring restructuring.

After *2015*, the trend shifts slightly upwards, with the TDR probability rising back to around *0.25* by *2017*. This increase could reflect emerging challenges such as regulatory changes, increased competition, or fluctuations in fuel and energy costs, which affected the sector's overall stability. Despite this rise, the probability remains substantially lower than that of distressed firms, indicating that non-distressed firms, while facing similar market conditions, managed to maintain a stronger financial position.

By ***2019*,** the TDR probability for non-distressed firms begins to decline again, suggesting that these firms have adapted to the new market dynamics and restored their financial resilience. This downward trend points to effective risk management strategies and a capacity to absorb market shocks without significant disruption to their operational stability.

The contrasting trends observed between *distressed* and *non-distressed firms* underscore the varying levels of financial resilience within the sector. The *distressed firms* show a consistent increase in TDR probabilities, indicating that their financial challenges deepen over time, ultimately reaching a point where the likelihood of restructuring becomes almost inevitable. This continuous rise, particularly from *2016* onward, suggests that distressed firms in this sector struggle to cope with high operational costs, regulatory complexities, and market uncertainties.

On the other hand, *non-distressed firms* demonstrate a different pattern of adjustment, with a notable dip in probabilities during *2014-2015*, followed by a controlled rise and subsequent stabilization. This ability to maintain lower probabilities of TDR indicates stronger internal financial controls, more adaptive management practices, and potentially better access to financing or diversified income streams. The resilience of these firms is further evidenced by their capacity to quickly adapt to changing economic conditions, thus avoiding the steep rise in TDR probability observed in distressed firms.

Despite their more stable outlook, it is important to note that *non-distressed firms* consistently show a baseline TDR probability of around *0.2* to *0.3*. This persistent, albeit lower, risk level reflects the inherent challenges of operating in capital-heavy industries like transportation, energy, and utilities, where even financially stable firms must navigate fluctuations in demand, regulatory shifts, and infrastructure maintenance costs.

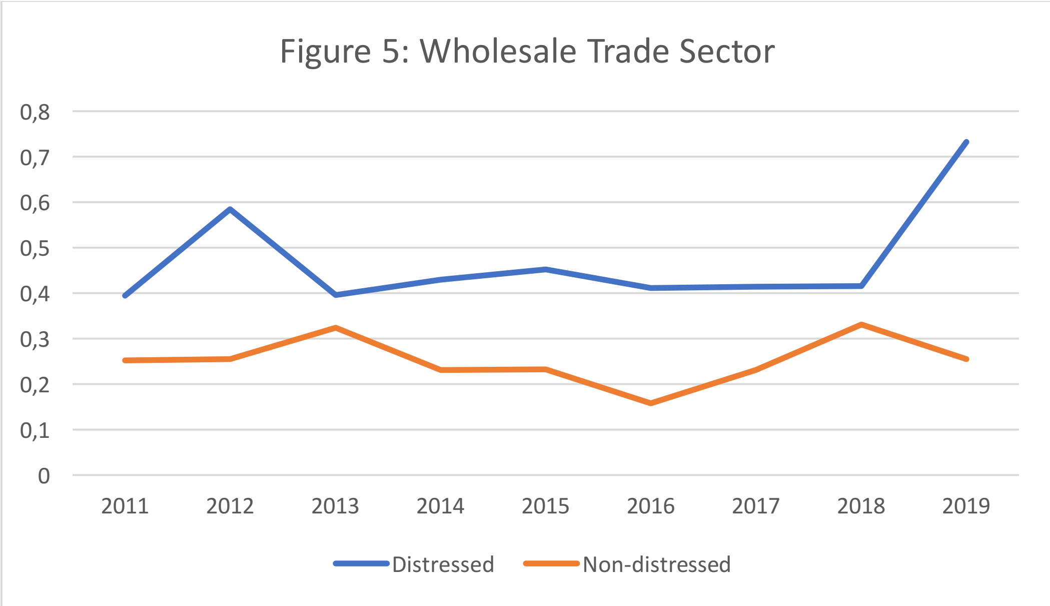
The trends illustrated in *Figure 4* emphasize the importance of financial resilience and adaptability in the *transportation, communications, electric, gas, and sanitary services* sector**.** For *distressed firms*, the steady increase in TDR probabilities points to the need for more effective intervention strategies, whether through targeted financial restructuring or policy support, to prevent further decline. The stabilization at high probability levels in *2018-2019* suggests that many of these firms may be at a tipping point where strategic support could help avert more severe outcomes.

For *non-distressed firms*, the ability to maintain lower TDR probabilities through internal adjustments highlights the effectiveness of their financial management practices. Their capacity to recover from mid-period challenges and stabilize their restructuring risk offers valuable lessons in risk mitigation and strategic flexibility.

The analysis of *Figure 4* thus provides critical insights into the sector's financial dynamics, showing how firms with differing risk profiles respond to the complexities of a highly regulated and capital-intensive environment. It also highlights areas where strategic focus and tailored interventions could significantly impact the overall financial stability of the sector, guiding stakeholders in making more informed decisions about resource allocation, investment, and support for struggling firms.

e. TDR Probability Model for the Wholesale Trade Sector

Figure 5 provides a nuanced representation of the *Troubled Debt Restructuring (TDR) probabilities* over time for both *distressed* and *non-distressed firms* within the *wholesale trade* sector, covering the period from *2011* to *2019*. The graph offers insights into how these firms, operating within a sector sensitive to market fluctuations and inventory dynamics, experience varying levels of financial pressure that may lead to restructuring.



The *blue line* represents the TDR probabilities for *distressed firms*, displaying a pattern that combines periods of volatility with a late-stage surge. Starting in *2011*, these firms show a probability around *0.4*, indicating an already substantial risk of needing to restructure due to existing financial struggles. This likelihood then spikes sharply in *2012*, exceeding *0.55*, before dropping back to *0.4* in *2013*. This early volatility suggests that distressed firms were initially grappling with challenges like fluctuating demand or rising costs, which temporarily heightened their restructuring risk. The drop in *2013* might reflect short-term adjustments or stabilization efforts, yet the subsequent probabilities remain elevated, indicating that these firms continued to face underlying financial pressures.

From *2014* through *2017*, the probabilities for distressed firms stabilize at around *0.4*, suggesting that while these companies managed to avoid a further increase in distress, they were unable to significantly improve their financial standing. This period of steadiness likely reflects ongoing efforts to control costs or renegotiate debts, yet the persistence of a high risk level indicates that deeper structural challenges remained unaddressed. These challenges could include issues like low profit margins, dependence on certain market segments, or difficulties in adapting to shifts in wholesale trade practices.

A shift occurs starting in *2018*, where the TDR probabilities for distressed firms rise markedly, reaching *0.73* by *2019*. This sudden increase points to a worsening environment for these firms, potentially driven by broader economic changes or intensified competition. It suggests that earlier efforts to stabilize were no longer sufficient in the face of new pressures, possibly including the rise of digital trade and changing consumer behaviors, which may have further strained traditional wholesale models. The high probability in *2019* indicates that many distressed firms were at a critical juncture, where external restructuring or intervention became increasingly necessary.

The *red line**,*** depicting the TDR probabilities for *non-distressed firms*, reveals a different story, characterized by relative stability but with subtle fluctuations throughout the period. Beginning at *0.25* in *2011**,*** the probability for these firms is consistently lower, reflecting their better financial health and management practices. However, even at this lower level, the presence of a *0.25* baseline suggests that the wholesale trade sector carries inherent risks that can affect all firms, regardless of their initial stability. Changes in supply chain dynamics, market competition, and shifts in consumer demand play a role in maintaining this baseline level of restructuring risk.

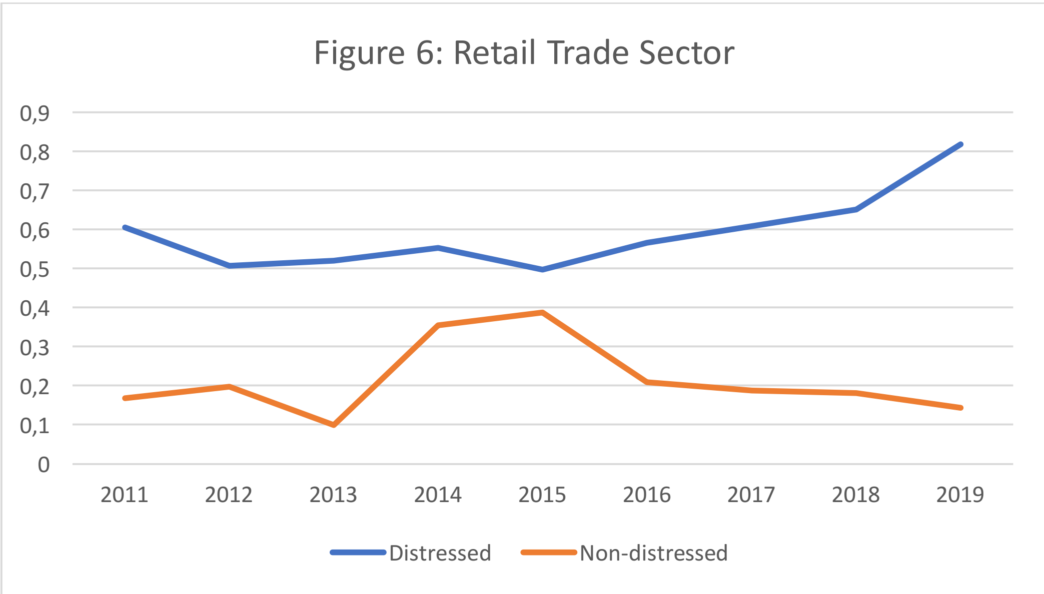
During *2014-2015*, non-distressed firms experience a dip in their TDR probability, which falls below *0.2****.*** This decline might indicate a period of more favorable market conditions, improved inventory management, or effective cost reductions that helped these firms maintain their financial health. Yet, this temporary improvement is followed by a moderate increase through *2016* and *2017*, where the probability rises back up to around *0.25*. This slight rise could be a response to emerging challenges such as market saturation or increasing operational costs, which put some pressure on even the more stable firms.

Towards the end of the period, *2018* and *2019* show a gradual decline in TDR probability for non-distressed firms, contrasting sharply with the rising trend observed among distressed firms. This decline suggests that non-distressed firms adapted better to the evolving market conditions, possibly through strategic adjustments like optimizing supply chains or leveraging technology to streamline operations. Their ability to reduce TDR probabilities during a period when distressed firms were struggling highlights the gap in resilience between the two groups, with non-distressed firms demonstrating a greater capacity to navigate the complexities of the sector.

Figure 5 captures the diverging paths of *distressed* and *non-distressed* firms within the *wholesale trade* sector, revealing how different levels of financial health influence a firm's ability to adapt to market conditions. For *distressed firms*, the volatility followed by a sharp increase in TDR probabilities underscores their vulnerability to external shocks and the limitations of internal adjustment efforts. In contrast, *non-distressed firms* manage to maintain a more stable outlook, despite facing some mid-period challenges. Their ability to rebound towards the end of the period emphasizes the importance of strategic flexibility and strong management practices in maintaining financial stability. This analysis provides valuable insights into the sector's dynamics, highlighting the need for targeted support and strategic intervention to help distressed firms stabilize, while also recognizing the strengths that allow non-distressed firms to sustain lower levels of restructuring risk amidst sectoral challenges.

f. TDR Probability Model for the Retail Trade Sector

Table 6 presents a detailed view of the *Troubled Debt Restructuring (TDR) probabilities* for both *distressed* and *non-distressed firms* in the *retail trade* sector over the period from *2011* to *2019*. This analysis delves into how the TDR probabilities evolved for each group, reflecting the broader economic conditions and sector-specific challenges that impacted these firms. The distinct trends seen between distressed and non-distressed firms offer valuable insights into the varying degrees of resilience and vulnerability within this competitive sector.



The *blue line* in the graph represents *distressed firms* and their respective TDR probabilities throughout the years. The trend for these firms starts at a relatively high level in *2011*, with a probability around *0.6***,** suggesting that even in the early years, these firms faced considerable financial stress. This initial high level could be attributed to the lingering effects of the global financial crisis of *2008-2009*, which had a prolonged impact on consumer spending and retail operations, particularly in markets where demand was slow to recover.

From *2012* to *2015*, the probability for distressed firms fluctuates slightly but remains around the *0.5* to *0.6* range. This period suggests a sustained struggle to manage costs, adapt to shifts in consumer behavior, and maintain profitability in a sector characterized by thin margins and intense competition. While some firms might have attempted restructuring or internal cost-saving measures during this time, the consistently high TDR probabilities indicate that these efforts did not significantly improve their financial position.

However, starting in *2016,* the trend for distressed firms shows a more marked upward trajectory. By *2019,* the TDR probability for these firms reaches close to *0.85*, highlighting a sharp increase in financial distress. This late-stage surge suggests that many of these distressed firms were unable to keep pace with the rapid changes in the retail environment, such as the shift to e-commerce, evolving consumer preferences, and increasing operational costs. The rise towards *0.85* indicates a critical juncture where the likelihood of these firms needing formal restructuring becomes almost inevitable, reflecting a deeper and possibly irreversible financial deterioration.

In contrast, the *red line* depicts the TDR probabilities for *non-distressed firms* in the retail sector. The trend starts significantly lower than that of distressed firms, around *0.15* in *2011,* indicating a much lower risk of restructuring. This difference reflects the stronger financial health and adaptability of non-distressed firms, which were better equipped to navigate the challenges of the early 2010s. These firms likely benefitted from more robust cash flows, effective inventory management, and a more strategic approach to market changes.

From *2012 to 2015*, non-distressed firms experience a slight dip followed by a peak around *2014-2015,* where the TDR probability temporarily rises to just above *0.3*. This peak could suggest that even relatively stable firms encountered challenges during this period, perhaps due to economic uncertainty or sector-specific pressures, such as shifts in consumer spending habits or the need to invest in digital capabilities. The peak indicates a momentary increase in vulnerability, where some non-distressed firms faced increased pressures that might have made restructuring more likely, albeit to a lesser extent than their distressed counterparts.

After *2015*, the TDR probability for non-distressed firms begins to decline gradually, dropping below *0.2* by *2019*. This downward trend suggests that many of these firms successfully adjusted to the evolving retail landscape, adopting measures such as enhancing their digital presence, optimizing supply chains, or focusing on customer experience to maintain competitiveness. The decline indicates a regained stability, where non-distressed firms managed to lower their restructuring risk, showing resilience in the face of ongoing sectoral transformations.

The contrasting trajectories of *distressed* and *non-distressed* firms in the retail trade sector reveal a clear divergence in financial outcomes. For *distressed firms*, the steady increase culminating in a near *0.85* probability by *2019* underscores their inability to adapt to the rapidly changing market conditions. This trend reflects deeper structural weaknesses, such as an over-reliance on traditional brick-and-mortar models, difficulty in managing debt levels, and an inability to respond effectively to the competitive pressures from online retail giants.

On the other hand, the trajectory of *non-distressed firms*, characterized by stability with a mid-period peak, highlights their relative strength. These firms managed to maintain lower restructuring risks despite the challenges posed by the shift in consumer behavior and the rise of e-commerce. Their declining probability towards the end of the period reflects successful strategic adaptations, suggesting that while they faced pressures similar to those of distressed firms, their stronger financial fundamentals allowed them to navigate these challenges more effectively.

The mid-period peak for non-distressed firms serves as a reminder that even financially healthy companies are not immune to broader economic trends. The temporary rise in their restructuring probability around *2014-2015* points to the sector-wide challenges that affected all players, such as increased competition, rising costs, and the need to adapt to technological changes. However, it is important to note that despite this rise, the TDR probability for non-distressed firms consistently remains *below the level* of distressed firms throughout the entire period. This difference underscores the stronger resilience of non-distressed firms, which managed to maintain a lower likelihood of restructuring even when faced with industry-wide pressures. Their position beneath the line of distressed firms indicates a relative financial stability and a greater ability to weather challenges compared to their struggling counterparts.

Yet, their ability to recover and reduce their risk by *2019* demonstrates a critical difference in their capacity for resilience.

The analysis of *Figure 6* provides valuable insights into the dynamics of financial risk within the *retail trade* sector. The steep increase in TDR probabilities for distressed firms indicates a sector that is unforgiving to those unable to adapt quickly to changes in market dynamics. For these firms, the escalating probability highlights the urgency of implementing more aggressive restructuring strategies or receiving external support to stave off failure. The near *0.85* probability by *2019* suggests that without significant changes, many of these firms may face irreversible financial outcomes.

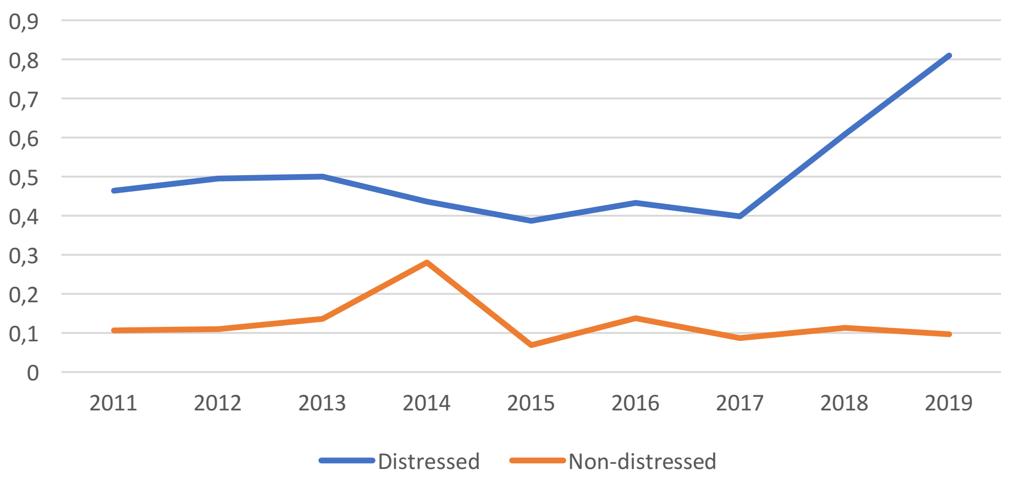
For *non-distressed firms*, the ability to maintain a lower probability of TDR, particularly as the sector continued to evolve, underscores the importance of strategic foresight and the ability to innovate. These firms were able to reduce their TDR probabilities over time, even as distressed firms faced increasing challenges, showing that success in the retail trade sector requires not just financial strength but also agility and a willingness to adapt to new consumer trends.

Overall, *Figure 5* captures the complex interplay between resilience and vulnerability within the *retail trade* sector. It emphasizes the critical role of adaptability in maintaining financial health and highlights how differences in strategic responses can result in dramatically different outcomes for firms operating within the same market environment. For stakeholders, such as investors and policymakers, these insights underline the importance of supporting firms in their efforts to innovate and adapt, while also recognizing the challenges faced by those struggling to keep pace with rapid industry changes.

g. *TDR Probability Model for the Finance, Insurance, and Real Estate Sector*

Table 7 provides a comprehensive view of the *Troubled Debt Restructuring (TDR) probabilities* for both *distressed* and *non-distressed* firms within the *finance, insurance, and real estate (FIRE)* sector from *2011* to *2019*. This visual representation allows us to understand how these firms, operating in a highly regulated and capital-intensive industry, have managed different levels of financial risk over time. The trends for each group, distressed and non-distressed, reveal much about their capacity to navigate economic changes, interest rate shifts, and regulatory adjustments.

Figure 7: Finance, Insurance, and Real Estate Sector



The *blue line* tracks the TDR probabilities for *distressed firms* in this sector, showing a pattern of sustained challenges, particularly in the later years. At the beginning of the period, from *2011* to *2013*, the probability remains consistently above *0.5,* suggesting that these firms faced persistent financial difficulties. This high baseline indicates that even after the immediate aftermath of the *2008-2009* financial crisis, many of these firms continued to struggle with issues like high debt levels, market volatility, and adapting to a changing regulatory environment. The consistently elevated risk level suggests that their recovery efforts were slow, and they continued to grapple with maintaining positive cash flow and stable balance sheets.

Between *2014* and *2016*, we observe a slight dip in the TDR probability for distressed firms, bringing the likelihood between 0.4 and *0.5***.** This dip might reflect a period of economic recovery, where improving market conditions provided some relief. During this time, firms may have attempted to stabilize through strategies such as asset sales or debt renegotiations. However, the relatively modest decline in probabilities indicates that while some financial pressures might have eased, many of the underlying challenges for these firms persisted. The relief was likely not sufficient to fundamentally change their financial outlook.

From *2017* onwards, the trend takes a dramatic turn upward, with the probability reaching *0.8* by *2019*. This sharp rise highlights a worsening financial situation for these distressed firms. It could reflect a combination of factors such as tighter credit conditions, increased regulatory demands, and greater competitive pressures in the sector, all of which might have further strained their already fragile financial health. This surge suggests that for many of these firms, the measures they had taken earlier were no longer enough to avoid restructuring, and they faced an increasing likelihood of needing external intervention to remain viable.

In contrast, the *red line* represents *non-distressed firms* in the sector, illustrating a more stable, though not entirely static, pattern. The TDR probabilities for these firms begin at a much lower level, around *0.1* in *2011*, reflecting their stronger financial health and resilience in the face of sector-wide challenges. This starting point suggests that these firms were better positioned to handle risks associated with credit, market shifts, and regulatory adjustments, thanks to more robust financial management practices and diversified sources of income.

However, from *2012* to 2*014*, there is a slight increase in the probability, rising to *0.3***.** This upward trend suggests that even firms in a relatively strong position faced some headwinds during this period. Factors such as evolving regulations, shifts in market demand, or rising operational costs could have contributed to the increased likelihood of restructuring, even if the risk remained lower compared to distressed firms.

A notable feature is the peak in *2015*, where the probability for non-distressed firms briefly rises above *0.3*. This suggests that during this time, even these more stable firms encountered significant pressures, perhaps due to market corrections or broader economic shifts that affected their liquidity and profitability. However, unlike their distressed counterparts, non-distressed firms were able to adapt quickly. From *2016* onwards, their TDR probability declines steadily, reaching back down to around *0.1* by *2019*. This downward trend highlights their ability to implement effective adjustments, such as refining their capital structures, optimizing operations, and taking advantage of favorable market trends, allowing them to mitigate risks.

The contrasting paths between *distressed* and *non-distressed* firms in the FIRE sector are striking and illustrate how different financial health levels shape a firm's ability to cope with challenges. For *distressed* firms**,** the persistently high TDR probabilities throughout the period reveal that they struggled to overcome deep-rooted financial issues. Even the slight relief seen between *2014* and *2016* was not enough to change their course significantly, as indicated by the sharp rise in risk towards *2019*. This suggests that the sector's challenges, such as increasing regulatory burdens and market fluctuations, had a particularly harsh impact on firms that were already financially weak.

On the other hand, *non-distressed* firms exhibit a much lower and more controlled trajectory of TDR probability, showing resilience in the face of similar pressures. Their ability to keep the probability below *0.3* for much of the period, despite experiencing a temporary rise around *2014***,** points to their capacity for strategic adjustments and effective risk management. This mid-period peak serves as a reminder that even healthier firms are not immune to sectoral shifts, yet their quicker recovery suggests a flexibility that allows them to maintain a strong position over time.

The continuous gap between the two groups highlights the importance of robust financial foundations and strategic agility in navigating a volatile sector like finance, insurance, and real estate. The fact that non-distressed firms managed to remain significantly below the TDR probabilities of their distressed peers throughout the period speaks to their ability to adapt to changes and maintain stability despite facing the same external environment.

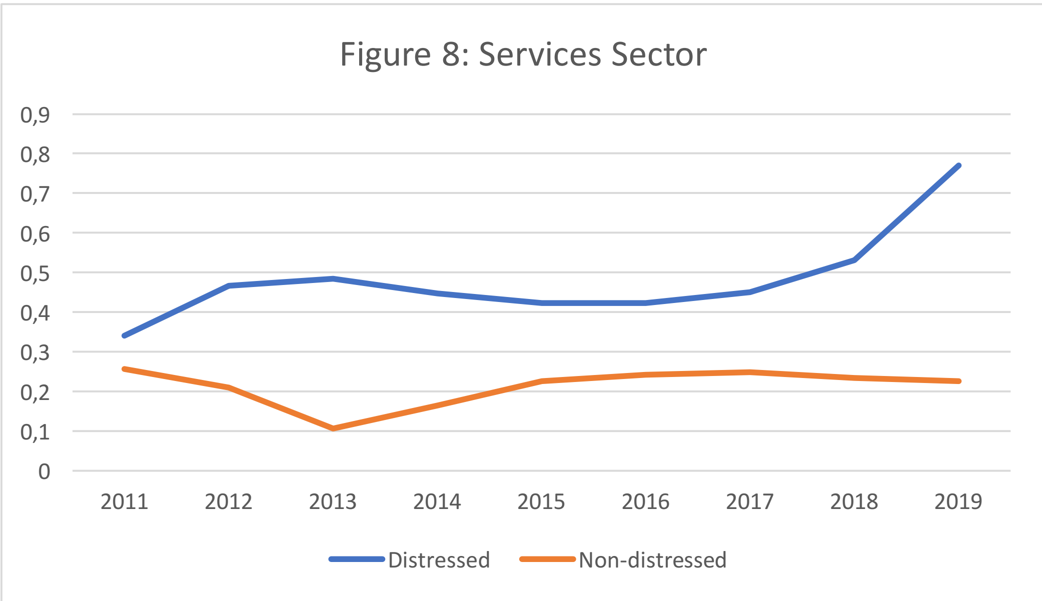
The analysis of *Table 7* reveals critical aspects of managing financial stability in the *finance, insurance, and real estate* sector. For *distressed firms*, the steady increase in TDR probabilities underscores the pressing need for deeper restructuring efforts or external support. The sharp rise in *2019* suggests that without such interventions, many of these firms may face severe outcomes. Their challenges highlight the importance of strategic planning, not only to recover from past economic crises but also to adapt to new regulatory and market conditions.

In contrast, *non-distressed firms* show that it is possible to navigate the sector's complexities successfully, provided that there is a strong foundation of sound financial management. Their ability to maintain low probabilities despite a challenging environment demonstrates the effectiveness of strategic foresight and adaptability. Their experience suggests that even in a sector as sensitive to external changes as finance and real estate, firms with proactive management can mitigate risks and maintain stability.

Overall, *Table 7* provides a detailed narrative of how different firms within the *FIRE* sector managed their risk of restructuring over time. It highlights the dual reality of the sector, where firms with strong management and financial practices can sustain stability, while those that are unable to adapt face an increasing likelihood of distress. These insights are invaluable for investors, regulators, and policymakers, as they underscore the areas where support is needed and emphasize the strategies that can help firms thrive amidst financial uncertainty.

h. TDR Probability Model for the Services Sector

Figure 8 illustrates the *Troubled Debt Restructuring (TDR) probabilities for distressed* and *non-distressed firms* within the *services sector* over the period from *2011* to *2019*. This sector, known for its diversity and the broad range of activities it encompasses - spanning from professional services to hospitality - faces unique financial dynamics. The graph provides a comprehensive view of how these firms have managed their financial challenges, reflecting both structural resilience and vulnerabilities



The *blue line* in the graph, representing *distressed firms*, reveals a trend of increasing financial difficulties over the years. The initial TDR probability in *2011* starts at just below *0.4*, indicating that even at the outset, a significant portion of these firms were at risk of needing restructuring due to underlying financial struggles. This initial position can be attributed to the residual impacts of economic downturns and the challenge of maintaining profitability in a sector that often faces fluctuations in demand and customer spending.

From *2011* to *2013*, the trend for distressed firms shows a gradual increase, peaking below *0.5.* This rise suggests that despite attempts at stabilization, many firms continued to face pressures such as rising operational costs, competition, and perhaps changes in customer preferences that strained their financial health. The relatively steady climb indicates that while there were no dramatic external shocks, the accumulation of smaller, persistent challenges made it increasingly difficult for these firms to avoid financial distress. Between *2014* and *2016*, the TDR probability for distressed firms appears to stabilize slightly, remaining around between 0.4 and *0.6* mark. This period might represent an attempt at internal adjustments or cost management strategies, as firms aimed to cope with their difficulties without fully resorting to restructuring.

However, this stabilization is short-lived, as the probability begins to rise again from *2017* onward, culminating in a sharp increase by *2019,* reaching close to *0.8.* This escalation suggests that many distressed firms faced intensified pressures, possibly due to shifts in the market landscape or external economic factors that further limited their capacity for recovery. By *2019*, the near *0.8* probability highlights a critical point where these firms were highly likely to require external restructuring interventions, reflecting a deepening of their financial distress.

The *red line* traces the TDR probabilities for *non-distressed* firms and reveals a markedly different trajectory. Starting in *2011*, their probability is around *0.3*, substantially lower than that of their distressed counterparts. This initial difference underscores the stronger financial positions and management practices of non-distressed firms, allowing them to maintain a buffer against the types of pressures that drive distressed firms toward restructuring.

From *2011* to *2013*, the trend for non-distressed firms shows a gradual decline in TDR probability, dipping to just around *0.1* in *2013.* This downward trend suggests that during this period, non-distressed firms were able to benefit from a relatively stable economic environment, improving their cash flows and reinforcing their financial stability. The ability to reduce restructuring risk during this time may be attributed to effective cost management, strategic expansions, or better adaptation to changing customer needs within the diverse services sector.

However, in *2014*, the trend shifts as the TDR probability for non-distressed firms begins to rise slightly, approaching *0.3* again. This suggests that even the more stable firms faced renewed challenges around *2015*, possibly related to increased competition or changes in regulatory frameworks affecting service providers. Despite this increase, non-distressed firms maintain their probability levels below *0.3*, reflecting their capacity to absorb external pressures without significantly compromising their financial health.

After *2016*, the trend for non-distressed firms levels off, remaining steady at around *0.25* through *2019*. This stability indicates that these firms found a balance between managing external risks and maintaining their operational efficiency. Their ability to keep TDR probabilities consistently low throughout this period suggests that they adapted effectively to shifts in the market environment, whether through digital transformations, improved service offerings, or other strategic initiatives.

The differences between the trends of *distressed* and *non-distressed firms* in the services sector underscore the varying degrees of resilience among firms in this industry. For *distressed firms***,** the persistent increase in TDR probabilities highlights a struggle to adapt to changing market conditions and internal financial challenges. The sharp rise towards *0.8* by *2019* suggests that these firms were particularly vulnerable to cumulative pressures, such as wage increases, competitive pressures, and perhaps shifts in consumer behavior that they were unable to address through internal adjustments alone.

In contrast, *non-distressed firms* demonstrate a higher degree of stability, maintaining a significantly lower risk of restructuring throughout the period. Their ability to keep TDR probabilities below *0.3*, even during periods when distressed firms faced rising risks, indicates that these firms benefitted from stronger financial structures and strategic flexibility. This difference highlights how effective management practices, such as maintaining cash reserves, leveraging digital platforms, and focusing on customer retention, can help firms navigate the uncertainties of the services sector.

The trends shown in *Figure 8* reveal the complexities of financial management within the *services* sector, where firms must balance the challenges of a competitive market with the need to adapt to economic fluctuations. The increasing TDR probabilities for distressed firms emphasize the importance of early intervention and strategic adjustments to prevent the escalation of financial problems. For these firms, the sharp rise towards *2019* suggests that without significant changes, they faced an elevated risk of insolvency or the need for comprehensive restructuring.

Meanwhile, the relative stability of non-distressed firms illustrates the potential for resilience in the services sector, provided that companies are proactive in their risk management. Their ability to maintain low probabilities of TDR, even during challenging periods, suggests that strategies such as diversification of service offerings, cost control, and innovation can play a key role in maintaining financial health.

The clear gap between the distressed and non-distressed trajectories in *Figure 8* also serves as a reminder of the importance of financial preparedness in the services sector. While all firms in the sector face similar macroeconomic conditions, those with robust management practices and adaptive strategies are better positioned to mitigate risks and sustain their operations, even when broader market conditions become more volatile.

Figure 8 provides a detailed snapshot of how firms in the *services sector* have managed their financial risks over time, highlighting the critical role of adaptability and strategic foresight in maintaining stability. For stakeholders, these insights underscore the need to support distressed firms in their efforts to implement effective restructuring plans, while also recognizing the successful practices of non-distressed firms that have enabled them to navigate a dynamic market landscape.

In my analysis of the *Troubled Debt Restructuring (TDR) probabilities* across the entire sample and within individual sectors, a key observation is the upward trend in TDR probabilities that began around *2014***.** This trend becomes particularly significant as I consider the period from *2015* to *2020*, during which many firms filed for TDR. The underlying data that supports this trend is drawn from financial performance during *2014* to *2019*, providing a comprehensive view of how these firms' financial health evolved leading up to their TDR requests.

What stands out in the analysis is that *distressed* firms consistently exhibit *higher TDR probabilities* even *three years prior* to their formal request for TDR compared to their *non-distressed* counterparts**.** This finding is evident not only in the aggregated data from the overall sample but also holds true across various sectors, such as *manufacturing***,** *wholesale trade***,** *transportation*, and *services***.** These sectors display a similar pattern, where distressed firms show signs of financial distress well before they reach the point of needing formal restructuring. This early identification of distress is particularly valuable as it demonstrates the predictive strength of the *modified Z"-Score model* that I developed.

By capturing these heightened probabilities well in advance - whether three, two, or even one year before firms face significant financial challenges - this model proves its efficacy as an early warning tool. It allows stakeholders, such as creditors, investors, and managers, to recognize the signals of financial distress early and potentially take preemptive measures to stabilize the firm before it faces a severe liquidity crunch or insolvency. The ability to predict distress over such a time horizon is crucial, as it provides a window for strategic intervention that could include debt restructuring, cost management, or even operational overhauls to prevent the downward spiral into formal TDR procedures.

In contrast, the *non-distressed firms* show a very different pattern. Their *TDR probabilities* remain *lower and relatively constant* throughout the same period, indicating a consistent level of financial health. These firms did not file for TDR during the observed period, which aligns with their stable trend in the data. This stability is evident across most sectors, where non-distressed firms maintain their lower risk profiles despite the external economic fluctuations and sector-specific challenges. Their capacity to maintain stable financial ratios underlines their resilience and suggests robust internal financial controls and strategies that support long-term viability.

However, it is important to note that while the general trend aligns across various sectors, there are a few *outliers* that diverge from the overall pattern. For instance, the *construction* sector shows some deviations, particularly in the early years of *2011* and *2012*, where the data might not be fully reliable due to a smaller sample size. The *finance, insurance, and real estate (FIRE)* sector also presents a distinct trajectory with a sharper rise in TDR probabilities for distressed firms in later years, likely reflecting sector-specific shocks or regulatory changes. These sectoral deviations underscore the importance of contextualizing the model's predictions within the specific industry conditions that might influence the financial outcomes of firms.

Overall, the findings reinforce the value of the TDR probability model as a robust tool for *early detection* of financial distress. By signaling potential risks well before the critical point of TDR requests, this model allows for a proactive approach to financial management. While the general trends hold true across most sectors, acknowledging and understanding the sectoral nuances and outliers enhances the model’s applicability, ensuring that the insights are grounded in the specific realities of each industry. The ability to adapt the analysis to account for these differences further strengthens the model’s role in guiding financial strategy and decision-making across diverse business environments.

i. Strengthening Hypothesis H_4: Predicting TDR Probability

- H_4: a predictive model can be developed to accurately estimate the probability that a firm will initiate the troubled debt restructuring (TDR) procedure based on key financial indicators and firm-specific characteristics.

In my analysis, the findings strongly validate the choice of sample firms, underscoring the strategic approach I adopted in categorizing and analyzing these entities. Specifically, the sample comprises two distinct groups: *distressed firms*, which are companies that have encountered significant financial difficulties and subsequently sought *Troubled Debt Restructuring (TDR)*, and *non-distressed firms*, which are financially stable companies that have not needed to pursue TDR. This categorization allows for a clear differentiation between firms facing acute financial challenges and those maintaining relative financial stability, thereby facilitating a more precise assessment of the predictors of financial distress.

The decision to include both distressed and non-distressed firms in the sample enhances the robustness of the analysis, as it ensures that the model can distinguish between firms that are likely to seek restructuring and those that can sustain their operations without such interventions. This distinction is crucial for developing a predictive framework that can accurately anticipate financial distress based on specific financial indicators. The presence of non-distressed firms in the sample provides a benchmark, enabling a comparison against distressed firms and highlighting the financial ratios and trends that are more indicative of distress.

Additionally, the results also support the methodological adjustment made to the *Z"-Score model**,*** where I replaced one of the original ratios with an alternative financial indicator to improve the model's predictive power. This adjustment is detailed in the *equation*, where the new ratio - *cash and cash equivalents* to *current liabilities* - was introduced to better capture the liquidity conditions of firms. The original Z"-Score, while effective in many contexts, did not fully account for the nuances of liquidity management that play a crucial role in determining a firm's ability to avoid financial distress, particularly in the short-term. By introducing this modification, I aimed to create a more accurate tool for identifying early signs of financial instability.

The empirical findings affirm that this modification enhances the model's ability to predict financial distress with greater precision. Distressed firms, characterized by strained liquidity and cash flow management, are more likely to exhibit higher probabilities of TDR when evaluated with the modified ratio. This adjustment provides a sharper lens through which the financial vulnerabilities of firms can be identified, offering stakeholders a more reliable basis for decision-making. The success of this modification is further corroborated by its alignment with *Hypothesis 4 (H_4)***,** which posits that a tailored predictive model can accurately estimate the likelihood of firms initiating TDR based on specific financial indicators and firm characteristics.

Hypothesis H_4 focuses on the development of a predictive model that accurately estimates the probability of a firm initiating a troubled debt restructuring (TDR) procedure based on key financial indicators. The modified Z"-Score enhances the model's capability to estimate this probability by including a ratio that better captures the liquidity pressures leading up to a TDR decision. This is particularly relevant, as liquidity constraints are often a primary trigger for firms entering TDR processes when they are unable to meet debt obligations.

The results demonstrate that the modified model provides a more accurate estimation of the likelihood of firms initiating TDR, especially in cases where liquidity constraints are a leading factor. By replacing the original ratio with one that directly measures cash availability relative to short-term liabilities, the model can better anticipate scenarios where a firm may need to restructure its debt. This adjustment supports *H4* by highlighting the modified Z"-Score's role in capturing the financial dynamics that precede a TDR decision, making it a more effective tool for predicting the onset of debt restructuring.

Furthermore, the findings for *H4* are consistent with existing literature, particularly the work of *De Luca F. and Meschieri E. (2017) and De Luca F. and Mehmood A. (2023)***.** Their research emphasized the importance of adjusting traditional financial models to account for sector-specific dynamics and liquidity constraints. My adaptation of the Z"-Score model follows this rationale, as it aims to refine the prediction of financial distress by focusing on the most relevant financial indicators for the sample of firms under study. By aligning with established scholarly perspectives, this study not only reinforces the validity of my methodological choices but also contributes to the ongoing discourse on improving distress prediction models.

In summary, the results underscore the effectiveness of both the sample selection strategy and the methodological adjustments made to the predictive model. The clear distinction between distressed and non-distressed firms enables a targeted analysis of financial distress, while the modification to the Z"-Score model ensures that the prediction of distress is grounded in the most relevant financial dynamics. This alignment with both practical observations and theoretical frameworks positions the study as a meaningful contribution to the field of *financial distress prediction***,** offering insights that are both empirically grounded and theoretically sound.

CHAPTER 5
FINDING & CONCLUSION

1. *DIFFERENTIATING FINANCIAL DISTRESS from FAILURE: INSIGHTS and IMPLICATIONS*

In this thesis, I have explored the critical distinction between financial distress and corporate failure, two concepts that, while often treated as synonymous in the literature, are fundamentally distinct in both their nature and implications. My focus has been on predicting financial distress, defined specifically as a firm's likelihood of filing for *Troubled Debt Restructuring (TDR)* in order to navigate a temporary state of crisis, rather than descending into outright failure. This distinction is pivotal, as financial distress represents a phase where firms are still capable of recovery, provided they undertake timely and strategic interventions. The prediction of such events is particularly relevant for both management and investors, as it allows for earlier identification of risks and the implementation of corrective actions that can avert total collapse *(Altman E.I., 1968; Ohlson J.A., 1980).*

Through a comprehensive empirical analysis, this study has demonstrated that financial distress is not a singular event but rather a gradual, multifaceted process influenced by a range of financial variables. Traditional financial indicators, including liquidity ratios, leverage, and profitability, have shown to be reliable predictors of distress, corroborating the findings of foundational works such as those by *Altman E.I. (1968) and Zmijewski M.E. (1984)*. For instance, declining liquidity and increasing leverage ratios are often early warning signs of a firm's inability to meet its short-term obligations, which can eventually lead to the need for TDR.

Moreover, this research emphasizes that financial distress is typically a slow-developing condition rather than an abrupt failure. This gradual progression, often marked by deteriorating financial performance over time, supports the arguments made by scholars like *Ohlson J.A. (1980)*, who noted that distress signals can be detected well before a firm reaches the point of no return. The models constructed in this study highlight how early intervention - through strategic refinancing, asset restructuring, or managerial changes - can significantly alter the outcomes for distressed firms, preventing their decline into insolvency.

Another key insight from this work is the potential impact of industry-specific factors on financial distress. Firms operating in highly cyclical industries, such as real estate, construction, or consumer goods, may face greater exposure to financial volatility, as their fortunes are closely tied to economic cycles. This suggests that sectoral analysis, alongside traditional financial metrics, could provide additional layers of accuracy in predicting distress, particularly for industries with high fixed costs and variable revenues. Future studies may benefit from developing industry-specific distress models to account for these nuances.

Moreover, the role of external stakeholders - particularly creditors and investors - has emerged as a critical factor in the distress process. Firms that maintain transparent communication with their creditors and demonstrate a commitment to restructuring are more likely to secure favorable terms in TDR processes. This reaffirms the idea that financial distress is not purely an internal problem but a negotiation process that involves multiple parties *(Jensen M.C. & Meckling W.H., 1976).* Effective stakeholder management, therefore, becomes a key determinant of a firm's ability to emerge from distress, making it a crucial area for future research and managerial focus.

Thus, the findings of this thesis contribute to a deeper understanding of financial distress as a distinct process, offering both theoretical and practical implications. On a theoretical level, they challenge the conventional view that distress and failure are interchangeable, highlighting the importance of developing more refined models for distress prediction. Practically, these insights provide a valuable tool for corporate management, investors, and regulators, enabling them to identify at-risk firms and take preemptive action to safeguard against potential failures.

2. *FINANCIAL DISTRESS ACROSS KEY SECTORS*

To address the challenges associated with predicting financial distress, I developed a model grounded in financial ratios, building on and adapting established frameworks. Specifically, I modified the well-known Z"-Score model, originally proposed by *Altman E.I. (1983),* which has long been used to predict corporate bankruptcy. In my adaptation, I replaced the ratio of working capital to total assets with a more liquidity-focused metric: the ratio of cash and cash equivalents to current liabilities *(De Luca F. & Meschieri E., 2017; De Luca F. & Mehmood A., 2023).* This modification was motivated by the recognition that, particularly in times of financial distress, liquidity plays a critical role in a firm’s ability to meet its short-term obligations and avoid insolvency. Given the heightened importance of liquidity management, especially in industries characterized by high volatility or significant working capital requirements, this adjustment allows for a more precise evaluation of a firm’s immediate financial health.

The empirical scope of my study focused on large and medium-sized privately-held firms operating within Italy, selected for their relevance to the national economic landscape and their unique financial profiles. For the purposes of this analysis, I concentrated on seven key industries, classified according to the Standard Industrial Classification (SIC) system: *construction,* *manufacturing, transportation-communications-electric-gas-sanitary services, wholesale trade, retail trade, finance-insurance-real estate,* and *services.* These sectors were not only chosen for their substantial contribution to Italy's economy but also for the distinct financial and operational challenges they face within their respective SIC classifications. By analyzing firms within these diverse industries, this study aims to uncover the sector-specific dynamics of financial distress, providing a detailed examination of how particular financial and operational characteristics can influence a firm's vulnerability. The classification by SIC allows for a systematic approach to understanding how different industries react to financial pressures, and how these pressures may necessitate different predictive models for distress.

The choice of these sectors was not arbitrary. Each represents a significant component of the economies in question and faces varying levels of exposure to financial distress.

The *construction* sector, for example, is highly capital-intensive and cyclical, with large investments in fixed assets and reliance on both short- and long-term debt. As such, the sector is highly sensitive to changes in economic conditions, particularly interest rates and credit availability. Firms in this sector showed a higher dependency on liquidity for their operational continuity, with distressed firms often demonstrating lower cash reserves relative to their short-term liabilities, a key factor that my modified model successfully captured.

In *manufacturing*, another capital-intensive industry, firms tend to operate on narrow margins with significant reliance on inventory and receivables management. Financial distress in this sector often arises from disruptions in the supply chain or declining demand for products. My model was able to identify these early distress signals by incorporating liquidity measures that highlight a firm's ability to cover immediate liabilities, which is critical in industries with heavy working capital requirements.

The *transportation, communications, electric, gas* and *sanitary services* sector represents an industry cluster where firms face high operational costs and fixed asset investments, but with relatively stable demand. However, firms in this sector are still prone to distress due to fluctuating fuel costs, regulatory changes, and infrastructure expenditures. My model's emphasis on cash and liquidity ratios allowed for a clearer assessment of short-term financial health, especially in periods of rising operational expenses.

For *wholesale trade* and *retail trade*, cash flow management is paramount due to the fast-moving nature of inventory and the volatility in consumer demand. Firms in these sectors often experience financial distress when sales slow or inventory accumulates, leading to increased short-term liabilities. By focusing on liquidity metrics, the model was able to accurately predict which firms were more likely to face distress and resort to TDR, even when other financial ratios appeared stable.

In the *finance, insurance,* and *real estate (FIRE)* sector, the risk factors are different, with firms being more vulnerable to credit market fluctuations, interest rate changes, and asset devaluations. The reliance on financial instruments and real estate market conditions means that liquidity is critical to their survival during periods of distress. The inclusion of cash-to-liabilities ratios in the model provided a more nuanced picture of financial distress within this sector, offering insights into firms' abilities to meet short-term obligations amidst volatile market conditions.

Finally, the *services* sector, which includes a wide range of businesses from professional services to hospitality, is highly sensitive to economic cycles and consumer spending patterns. The model showed that firms in this sector often struggled with liquidity, particularly during downturns when revenues dropped but fixed costs remained constant. By highlighting the importance of liquidity, the model was able to accurately predict distress in this sector, providing a key advantage over traditional financial ratios.

My findings indicate that the modified model significantly improves the accuracy of distress prediction across all seven sectors compared to the traditional Z"-Score model. The enhanced performance of the model, particularly in capital-intensive sectors like manufacturing and real estate, highlights the importance of incorporating cash liquidity measures in predicting distress. By focusing on cash and cash equivalents relative to current liabilities, the model better captures the short-term financial pressures that firms face when approaching distress, particularly in industries where cash flow management is critical to operational stability.

Furthermore, the analysis revealed notable differences between distressed and non-distressed firms within these sectors concerning their probabilities of filing for TDR. In industries such as manufacturing, construction, and wholesale trade, distressed firms exhibited significantly lower liquidity ratios and higher leverage compared to their non-distressed counterparts, suggesting that these firms were more prone to short-term liquidity crises that pushed them toward restructuring. In contrast, sectors like *transportation, communications, electric, gas* and *sanitary services*, where financial performance is often more volatile, showed that even non-distressed firms could exhibit elevated probabilities of distress, albeit for different reasons, such as innovation risks or market demand fluctuations.

Overall, these findings underscore the importance of sector-specific dynamics in the prediction of financial distress. The ability of firms in different industries to manage their liquidity and navigate through financial challenges varies significantly, and this variability must be taken into account when assessing their risk of distress. By adapting the Z"-Score model and tailoring it to these specific industry characteristics, my research provides a more accurate and industry-sensitive tool for predicting financial distress, offering valuable insights for both corporate management and investors seeking to mitigate risk.

3. METHODOLOGY EMPLOYED

In this study, a comprehensive methodology was developed to predict financial distress and the likelihood of firms resorting to *TDR (Troubled Debt Restructuring)*. The approach combined traditional statistical techniques, namely *Linear Discriminant Analysis (LDA)* and *logistic regression*, to provide a solid framework for identifying distressed firms and estimating their probability of undergoing restructuring. These methods were carefully selected not only for their effectiveness but also for their balance between interpretability and practical utility, which are crucial when applying predictive models in real-world corporate finance settings.

The decision to use *LDA* was based on its proven ability to classify firms into distinct categories - those at risk of financial distress and those in stable financial condition. *LDA* works by creating a linear combination of financial ratios that maximizes the variance between distressed and non-distressed firms. In this study, key financial indicators such as *liquidity*, *leverage,* and *profitability* ratios were used as input variables. The method's strength lies in its ability to simplify complex financial data, creating a straightforward classification framework. This made it highly applicable for identifying companies that were either on the verge of financial instability or at a healthier stage, thus offering valuable insights for early intervention.

However, *LDA* has some inherent limitations, such as assumptions regarding multivariate normality and equal covariance among groups, which could affect the accuracy of the model if not carefully managed. To address these potential issues, the dataset was meticulously prepared. *Outliers* were identified and treated appropriately, and any *missing values* were handled to ensure the integrity of the dataset. Furthermore, to validate the robustness of the model, *cross-validation techniques* were employed. This involved splitting the data into training and testing sets, which allowed for a thorough evaluation of the model's predictive power across different segments of the dataset. By doing so, the model's reliability was strengthened, and its ability to generalize to new, unseen data was enhanced.

In addition to LDA, *logistic regression* was employed to further improve the model's predictive power. Logistic regression is particularly useful when the outcome of interest is binary—in this case, whether a firm would or would not enter financial distress and eventually seek TDR. The strength of logistic regression lies in its flexibility, as it can handle both continuous and categorical variables, allowing for the inclusion of a broader set of financial indicators. In this study, variables such as *cash flow ratios,* *leverage,* and other financial health measures were incorporated into the regression model, providing a comprehensive view of the factors contributing to a firm's likelihood of distress. Logistic regression also offered a practical advantage: its results are easily interpretable. By examining the *odds ratios,* it became possible to quantify the impact of each financial indicator on the likelihood of distress, making the findings more accessible to decision-makers.

One of the most significant innovations in this study was the modification of the traditional *Z"-Score model*, a cornerstone in financial distress prediction. This model, while effective, often places significant weight on general measures of financial health, such as working capital to total assets. To make the model more sensitive to short-term liquidity pressures—which are often the most immediate cause of financial distress—the study replaced the *working capital* to *total assets* ratio with a more liquidity-focused metric: *cash and cash equivalents* to *current liabilities.* This adjustment allowed for a sharper focus on a firm's ability to meet its short-term obligations, which is critical in predicting whether a firm is likely to seek TDR. This liquidity-sensitive metric provided a more accurate reflection of a company's short-term financial health, enabling the model to capture early warning signs of financial instability that might not be evident through more traditional ratios.

The data used in this study were sourced from the *Bureau Van Dijk ORBIS database*, a highly respected and comprehensive resource that contains financial and operational data on firms from a wide range of industries. For this research, the focus was on large and medium-sized private firms within *Italy*, classified into seven key industrial sectors: *construction, manufacturing, transportation-communications-electric-gas-sanitary services, wholesale trade, retail trade, finance-insurance-real estate,* and *services.* These sectors were selected for their economic significance and their exposure to financial volatility, making them ideal candidates for studying financial distress. The use of the *Standard Industrial Classification (SIC)* system further ensured that the model accounted for sector-specific characteristics, thus enhancing its applicability across different industries.

An important methodological decision in this study was the exclusion of smaller firms. Smaller companies often exhibit greater financial volatility and are subject to different risk factors compared to larger enterprises. Including these firms in the sample could have introduced noise into the analysis, potentially distorting the results. By focusing on larger and medium-sized firms, the study was able to provide a clearer picture of financial health in more stable companies, ultimately enhancing the accuracy of the predictive model in assessing distress and the likelihood of TDR.

To distinguish between distressed and non-distressed firms, a range of financial ratios was analyzed over a three-year period leading up to TDR events. This *longitudinal approach* allowed for the identification of trends in a firm's financial health, capturing gradual declines that might not be immediately apparent in shorter-term data. By analyzing the evolution of key financial ratios over time, the model was able to pinpoint the early signs of distress, offering firms the opportunity to take preemptive action before their financial situation deteriorated to a critical level.

In choosing *LDA* and *logistic regression* as the core analytical tools, this study made a deliberate decision to prioritize *interpretability* and *practical utility* over the use of more complex methods, such as *Artificial Intelligence (AI)* and *Machine Learning (ML)***.** While AI and ML have demonstrated strong predictive capabilities in some applications, they often lack the transparency needed for practical use in business and financial decision-making. Corporate managers, financial institutions, and policymakers require models that not only deliver accurate predictions but also provide insights that are easy to understand and interpret. By using LDA and logistic regression, this study ensured that the model's results were both actionable and interpretable, making them more useful in real-world scenarios where clarity is paramount.

Moreover, the relative simplicity of these methods allowed for the development of a model that could be implemented with relatively low computational cost, making it accessible to a wide range of firms and stakeholders. This aligns with the broader goal of creating a tool that can be widely used across industries without requiring extensive technical expertise. Firms, investors, and creditors can use this model to assess a company's financial health, identify early warning signs of distress, and take proactive steps to mitigate the risk of bankruptcy or financial failure.

The combination of traditional statistical techniques and carefully selected financial indicators resulted in a model that was both *robust* and *practical*. The model effectively captured the early signs of financial distress and provided valuable insights for firms seeking to avoid insolvency. Furthermore, it offered external stakeholders—such as creditors and investors—a reliable tool for assessing the financial health of firms, supporting more informed decision-making, and mitigating the risks associated with financial distress.

4. *SECTOR-SPECIFIC and OVERALL SAMPLE RESULTS*

The findings from this study have demonstrated clear and significant distinctions between distressed and non-distressed firms, not only across the various sectors analyzed but also within the overall sample. These results provide a detailed and nuanced understanding of how companies in different industries approach and manage their financial health, and they bring to light the specific financial stressors that are more likely to lead to financial distress in each sector. By examining key financial indicators, the study has uncovered crucial factors that influence a firm's likelihood of experiencing distress or seeking Troubled Debt Restructuring (TDR).

This analysis enables us to appreciate the complex financial dynamics at play within each sector, as well as the broader trends that apply across the full sample. It highlights the different ways in which firms handle profitability, liquidity, and debt management, all of which are critical to their financial stability. In the sections that follow, the findings for each industry are discussed in greater detail, with particular attention given to the financial ratios that proved to be essential in predicting distress and the need for TDR.

4.1. Linear Discriminant Analysis (LDA) results

4.1.1. Overall Sample

In the analysis of the overall sample, it became evident that certain financial ratios played a pivotal role in distinguishing distressed firms from non-distressed ones. Across all sectors, the key financial indicators - *Retained Earnings to Total Assets (RE_TA)*, *Earnings Before Interest and Taxes to Total Assets (EBIT_TA)*, *Book Value of Equity to Total Liabilities (BVOE_TL*), and *Cash and Cash Equivalents to Current Liabilities (CAR*) - consistently emerged as critical in predicting financial distress.

Non-distressed firms across the sample demonstrated higher *RE_TA* ratios, indicating their ability to accumulate retained earnings over time, which allowed them to reinvest in operations and maintain financial stability. This accumulation was a crucial differentiator, as distressed firms typically showed lower or negative *RE_TA* values, reflecting an inability to generate profits and build reserves, leaving them vulnerable to financial difficulties

Similarly, the *EBIT_TA* ratio, which measures operational profitability, further separated distressed firms from non-distressed ones. Non-distressed firms consistently demonstrated positive *EBIT_TA* ratios, efficiently utilizing their assets to generate profits. Conversely, distressed firms struggled with negative *EBIT_TA* values, highlighting operational inefficiencies that exacerbated their financial challenges.

Additionally, the *BVOE_TL* ratio was another key factor. Non-distressed firms managed their equity better, maintaining a strong balance between equity and liabilities, which reduced their dependence on debt. Distressed firms, by contrast, exhibited much lower *BVOE_TL* ratios, signaling a heavier reliance on debt, which increased their financial vulnerability. Lastly, liquidity, measured by the *CAR* ratio, proved critical. Non-distressed firms had higher liquidity, allowing them to meet short-term obligations with ease, while distressed firms faced liquidity constraints, reflected in their significantly lower *CAR* ratios.

4.1.2. Construction Sector

In the *construction* sector, the results reflected the unique challenges faced by firms in this capital-intensive industry. Construction firms often deal with long project timelines, substantial upfront investments, and cyclical demand patterns, making financial management crucial.

Non-distressed construction firms exhibited strong *RE_TA* ratios, meaning they were able to retain a significant portion of their earnings. This allowed them to reinvest in ongoing projects and maintain a financial buffer to manage project delays or downturns in demand. The ability to generate and retain earnings was critical in this sector because it provided the stability necessary for long-term projects that often have delayed returns on investment.

Distressed construction firms, however, had much lower or negative *RE_TA* ratios, indicating that they were unable to generate consistent profits, which left them financially exposed. Operational profitability, as reflected in *EBIT_TA*, was also a significant differentiator. Non-distressed construction firms maintained positive *EBIT_TA* values, showing that they managed their assets efficiently to generate profits. On the other hand, distressed firms struggled with negative *EBIT_TA* ratios, suggesting poor project management, cost overruns, or declining demand, all of which contributed to their financial difficulties.

Moreover, capital structure was a key factor in the construction sector. Non-distressed firms maintained stronger *BVOE_TL* ratios, indicating a healthier reliance on equity rather than debt. This allowed them to manage the high capital demands of the industry without becoming overly reliant on external financing. Distressed firms, by contrast, were more dependent on debt, making them more vulnerable to financial shocks. Liquidity, measured by the *CAR* ratio, was another critical issue. Non-distressed firms managed their liquidity well, ensuring they had the cash reserves necessary to meet short-term obligations, such as paying suppliers and contractors.

Distressed firms, however, faced significant liquidity shortages, making it difficult for them to manage day-to-day operations and putting them at higher risk of failure.

4.1.3. Manufacturing Sector

In the *manufacturing* sector, operational efficiency and asset management were the key drivers of financial health. Non-distressed manufacturing firms consistently demonstrated strong *EBIT_TA* ratios, indicating their ability to efficiently convert assets into profits. This operational profitability was essential in an industry characterized by competitive pressures and narrow profit margins. Manufacturing firms that could maximize the use of their assets to generate profits were better positioned to avoid financial distress.

Distressed manufacturing firms, on the other hand, often exhibited negative *EBIT_TA* ratios, which reflected significant operational inefficiencies. These firms struggled with high production costs, inefficiencies in supply chain management, or declining demand for their products, all of which contributed to their negative profitability. As a result, these firms were more likely to experience financial distress.

The *RE_TA* ratio, while still important in the manufacturing sector, played a secondary role compared to *EBIT_TA*. Non-distressed firms that retained earnings had an advantage, as they could reinvest in technology, innovation, and production capacity. Distressed firms, however, struggled to retain earnings, leaving them financially vulnerable. Liquidity was also a key factor, as firms in the manufacturing sector need to manage working capital effectively. Non-distressed firms typically had stronger *BVOE_TL* and *CAR* ratios, allowing them to manage short-term liabilities and maintain operational efficiency, while distressed firms faced liquidity constraints that hampered their ability to meet immediate financial obligations.

4.1.4. Transportation, Communications, Electric, Gas, and Sanitary Services Sector

Firms in the *transportation, communications, electric, gas,* and *sanitary services sector* operate in industries characterized by high fixed costs and long-term capital investments. As such, liquidity and long-term capital management were crucial differentiators between distressed and non-distressed firms in this sector.

Non-distressed firms maintained higher *BVOE_TL* ratios, indicating that they relied on equity rather than debt to finance their operations. This reliance on equity allowed these firms to absorb financial shocks and continue making necessary investments in infrastructure. By maintaining a healthy balance between equity and liabilities, these firms were able to manage the capital-intensive nature of their industries without overextending themselves financially.

Distressed firms, however, exhibited weaker equity positions and were more heavily reliant on debt, which made them more vulnerable to financial instability. Liquidity, as measured by the *CAR* ratio, also played a critical role in this sector. Non-distressed firms maintained higher liquidity levels, ensuring that they could meet operational and maintenance costs, while distressed firms struggled with lower liquidity, making it difficult for them to manage day-to-day operations. This lack of liquidity often led to operational disruptions and increased the likelihood of financial distress.

4.1.5. Wholesale Trade Sector

The *wholesale trade* sector presented some unique findings compared to other sectors. In this sector, operational profitability, as measured by *EBIT_TA*, emerged as the most important financial indicator. Non-distressed wholesale firms demonstrated strong *EBIT_TA* ratios, reflecting their ability to manage inventory, supply chains, and distribution efficiently.

The *wholesale* sector operates on tight margins, and firms that could efficiently convert their assets into profits were more likely to maintain financial stability. Distressed firms in the sector, however, often struggled with operational inefficiencies, leading to negative *EBIT_TA* values. These firms were less able to manage the complexities of supply chain logistics and inventory turnover, which contributed to their financial difficulties.

Liquidity, measured by the *CAR* ratio, also played a key role in the wholesale sector. Non-distressed firms managed their liquidity effectively, ensuring they had sufficient cash to cover short-term financing needs, while distressed firms faced liquidity shortages, making it difficult for them to manage inventory and operational costs.

4.1.6. Retail Trade Sector

In the *retail trade* sector, liquidity management was critical due to the sector's reliance on short-term financing and exposure to fluctuations in consumer demand. Non-distressed retail firms consistently reported higher *CAR* ratios, indicating their ability to manage liquidity effectively and meet short-term obligations. This ability to maintain liquidity was crucial for managing inventory fluctuations and responding to changes in consumer behavior.

Profitability, as measured by *EBIT_TA*, was also a strong differentiator in the retail sector. Non-distressed firms demonstrated higher operational profitability, reflecting their ability to convert inventory into sales and generate profits. Distressed firms, by contrast, struggled with declining sales, poor asset utilization, and negative *EBIT_TA* values, which contributed to their financial difficulties.

4.1.7. Finance, Insurance, and Real Estate Sector

In the *finance, insurance, and real estate* sector**,** retained earnings ***(RE_TA)*** and liquidity ***(CAR)*** were the most critical indicators of financial health. Non-distressed firms in this sector maintained strong reserves to meet regulatory requirements and manage long-term risks. These firms consistently reported higher *RE_TA* ratios, allowing them to reinvest in operations and absorb financial shocks.

Liquidity, as measured by the *CAR* ratio, was also crucial in this sector. Non-distressed firms maintained higher liquidity levels, ensuring they had sufficient cash to meet short-term obligations and weather potential financial shocks. Distressed firms, on the other hand, struggled with liquidity shortages and weaker equity positions, making them more vulnerable to financial distress.

4.1.8. Services Sector

In the *services* sector, retained earnings *(RE_TA)* play a significant role in a firm's ability to reinvest in operations and maintain financial stability. Non-distressed firms show stronger *RE_TA* ratios, which allows them to reinvest in technology and human capital, enhancing their long-term growth potential. Distressed firms, however, face liquidity and operational challenges, as reflected in their lower *CAR* and *EBIT_TA* ratios, leading to financial distress.

Across all sectors, non-distressed firms consistently demonstrate stronger financial health through higher retained earnings, better profitability, and more effective liquidity management. These firms are able to manage their capital structures efficiently, balancing equity and debt to maintain stability. On the other hand, distressed firms face common challenges, such as poor liquidity, weaker equity positions, and operational inefficiencies, which increase their risk of financial distress. Understanding these sector-specific dynamics is crucial for developing effective financial distress prediction models and for helping firms, investors, and creditors make more informed decisions.

4.2. TDR Probabilities and Predictive Trens: 2014-2020

The analysis of *Troubled Debt Restructuring (TDR)* probabilities across both the overall sample and individual sectors revealed a clear and notable trend beginning in 2014 and continuing through the observed period, particularly from 2015 to 2020. During this timeframe, the upward trajectory in TDR probabilities became increasingly pronounced, especially as a growing number of firms filed for formal TDR. This trend, which is supported by financial data spanning from 2014 to 2019, provides a comprehensive view of the deterioration in firms' financial health leading up to their TDR requests, offering valuable insights into the model's ability to predict distress well in advance of formal filings.

A key observation that emerged from the analysis is the consistent pattern displayed by distressed firms. These firms, across various sectors, demonstrated significantly higher TDR probabilities even *three years prior* to their formal requests for TDR. This finding is evident not only in the aggregated data from the overall sample but also within specific industries such as *manufacturing, wholesale trade, transportation,* and *services*. In these sectors, the model successfully identified early signs of financial distress well before these firms reached the critical point of requiring restructuring, underscoring the predictive power of the model developed in this study.

A particularly important aspect of the findings is the early detection of distress through the *modified Z"-Score model*. The model, which placed a stronger emphasis on liquidity - especially through the substitution of the traditional *working capital* to *total assets* ratio with *cash and cash equivalents* to *current liabilities* - proved to be an effective early warning tool. By capturing heightened TDR probabilities up to three years before firms faced significant financial challenges, the model demonstrated its utility as a *predictive measure*. This early identification is crucial as it offers stakeholders, including creditors, investors, and corporate management, the opportunity to take proactive measures to stabilize firms before they enter a severe liquidity crisis or formal insolvency proceedings.

The model's ability to predict distress over multiple time horizons - whether *three, two, or even one year* prior to financial difficulties - significantly enhances its practical value. The extended lead time provided by these predictions gives firms and stakeholders a critical window to intervene with strategic actions such as *debt restructuring, cost reduction initiatives,* or *operational restructuring.* This period allows for a more measured response to emerging financial distress, potentially preventing the need for formal TDR proceedings altogether. Such early intervention could involve renegotiating debt terms, managing cash flow more efficiently, or streamlining operations to preserve the firm's financial health and long-term viability.

The predictive power of the model was not confined to the overall sample; it also proved effective across various sectors, revealing industry-specific trends that align with the broader results. For instance, in sectors like *manufacturing, wholesale trade, transportation,* and *services,* distressed firms consistently exhibited an upward trend in TDR probabilities as early as three years before their formal TDR requests. This trend indicates that the financial difficulties experienced by these firms were not sudden but rather the result of a gradual decline, which the model was able to track and predict accurately.

Notably, non-distressed firms across these sectors showed a starkly different pattern. Their TDR probabilities remained *low* and *stable* throughout the observed period, reflecting a consistent level of financial health. These firms did not file for TDR during the period of analysis, and their stability in the face of sector-specific challenges underscores the robustness of their financial strategies and internal controls. This contrast between distressed and non-distressed firms provides further validation of the model's ability to differentiate between firms at risk of financial distress and those that are well-positioned to weather economic fluctuations.

Despite the overall alignment of trends across most sectors, a few notable outliers emerged, highlighting the importance of contextualizing the model's predictions within specific industry dynamics. For example, the *construction sector* exhibited some deviations from the broader trends, particularly in the early years of the analysis (2011-2012), where the data showed some volatility due to a smaller sample size. This suggests that firms in the construction industry may have experienced unique challenges, such as project delays, fluctuating material costs, or shifts in market demand, which could have distorted the TDR probabilities during those years.

Similarly, the *finance, insurance,* and *real estate (FIRE)* sector displayed a distinct trajectory, with a sharper rise in TDR probabilities for distressed firms during the later years of the analysis. This deviation likely reflects sector-specific shocks, such as regulatory changes, shifts in interest rates, or broader market volatility, which may have disproportionately affected firms in this sector. These sectoral deviations are significant as they underscore the need to interpret the model's predictions with an understanding of the specific economic and regulatory conditions that influence financial outcomes within each industry.

The findings from this study have important implications for financial management and strategic decision-making. By signaling potential financial distress well in advance of formal TDR requests, the model offers firms and stakeholders the ability to adopt a *proactive approach* to financial management. The ability to anticipate financial distress over a multi-year horizon provides a valuable opportunity for *strategic intervention*. For example, firms could implement cost-cutting measures, restructure their debt, or overhaul their operations to address inefficiencies before financial problems become unmanageable. Additionally, creditors and investors can use the model's insights to make informed decisions about lending and investment, adjusting their strategies to mitigate risk in firms showing signs of impending distress.

Furthermore, the sectoral nuances observed in the analysis emphasize the importance of tailoring financial strategies to the specific realities of each industry. While the general trends in TDR probabilities hold across most sectors, understanding the unique challenges and risks faced by each industry - whether it is the capital-intensive nature of construction or the regulatory environment of the FIRE sector - can enhance the model's applicability. By adapting financial strategies to account for these industry-specific factors, firms and stakeholders can ensure that their responses to potential financial distress are both effective and contextually appropriate.

Overall, the findings of this study reinforce the value of the *TDR probability model* as a robust tool for the *early detection of financial distress*. By identifying elevated TDR probabilities well before firms reach the critical point of needing formal restructuring, the model empowers stakeholders to take timely, strategic actions that can prevent further deterioration in financial health. The general trends observed across sectors validate the model's predictive accuracy, while the identification of sectoral outliers highlights the importance of contextualizing predictions within specific industry conditions.

This ability to foresee financial distress and provide a *proactive window for intervention* significantly enhances the model's role in guiding financial strategy and decision-making. Firms that use this model as part of their financial management toolkit can better position themselves to address emerging challenges, safeguard their long-term viability, and mitigate the risk of insolvency.

5. *ANALYSIS PERIOD and CONSIDERATION of the COVID-19 IMPACT*

The analysis conducted in this study covers the period from *2011* to *2019*, incorporating financial data up until 2020, where publicly available. However, it is important to note that the data for the year 2020 were not fully integrated into the primary analysis due to the onset of the *COVID-19 pandemic,* which had unprecedented and wide-reaching economic impacts. The decision to exclude the *COVID-19 period* from the core analysis was made to avoid potential distortions that could arise from the extraordinary and highly abnormal financial conditions brought about by the pandemic.

The global health crisis of 2020 created a unique economic environment characterized by sudden demand shocks, government-mandated shutdowns, supply chain disruptions, and widespread liquidity shortages. Many firms faced financial challenges not as a result of structural weaknesses or poor financial management but due to external and unforeseen factors. Including data from this period could have skewed the model's results, leading to overestimated TDR probabilities that do not reflect the firms' long-term financial health or the typical dynamics of financial distress in more stable periods. As such, the exclusion of 2020 and the immediate COVID-19 years was deemed necessary to maintain the *integrity* of the model's predictive power.

While this study has successfully demonstrated the model's predictive strength in the years leading up to 2020, it is essential to recognize that the financial landscape post-2020 will likely evolve in ways that reflect the *long-term effects of the pandemic*. The pandemic created significant changes in how businesses operate, particularly with regard to liquidity management, debt structures, and overall financial resilience. Therefore, it is recommended that the same predictive model developed in this study be re-applied and reassessed in the future, once sufficient post-COVID financial data becomes available.

For future analysis, the same time frame used in this study - covering a period of *10 years* - should be employed, with 2020 serving as a key demarcation point between pre-COVID and post-COVID economic realities. By maintaining this consistent time frame, the model can be tested to see whether the trends and predictive indicators observed during the pre-pandemic period remain valid, or if they have shifted in response to the structural changes introduced by the pandemic.

Reapplying the model in the post-COVID era will allow for a more comprehensive understanding of how firms have adapted to the new economic landscape and whether the early warning signals identified in this study remain robust in the face of new challenges. The model's focus on *liquidity, debt management,* and *operational efficiency* - all critical indicators of financial distress—will be particularly important to assess in light of how firms have restructured or evolved their financial strategies in response to the pandemic.

Moreover, revisiting the model after a few years will offer insight into whether the same financial patterns of distress, as identified in the 2011-2019 period, persist, or if the pandemic has fundamentally altered the financial trajectories of firms. By applying the model over a post-COVID period of 10 years, similar to the pre-COVID analysis, stakeholders will be able to observe any deviations from established patterns and assess whether firms' financial resilience has improved or weakened in the wake of the pandemic.

In addition, this future analysis will allow researchers and practitioners to examine the sector-specific impacts of COVID-19. Certain industries, such as retail, hospitality, and transportation, were disproportionately affected by the pandemic and may exhibit new financial stress patterns that were not present during the pre-pandemic years. Understanding how these sectors have recovered - or failed to recover - will provide critical insights into the long-term financial health of firms and the effectiveness of the model's predictions in these evolving contexts.

The decision to exclude the COVID-19 period from the core analysis of this study was grounded in the need to avoid data distortions and to ensure that the model accurately reflects standard financial distress indicators rather than one-off shocks caused by the pandemic. However, it is clear that the COVID-19 pandemic has brought about long-lasting changes in the financial environment, and as such, revisiting this analysis in the coming years will be crucial for understanding how these changes have influenced the predictive power of the model.

By reapplying the same time frame and model to a post-COVID context - while using 2020 as a dividing line - future studies will be able to offer a more nuanced understanding of financial distress in the new economic landscape. This approach will provide key stakeholders with valuable insights into how the early warning signs of distress identified in this study hold up in a post-pandemic world, thereby enhancing the model's utility as a tool for strategic financial management and risk mitigation across industries.

6. REFLECTIONS on the THEORETICAL and SYSTEMIC SCOPE

As we draw the conclusions of this research, it is crucial to dedicate a moment to a broader reflection on the approach adopted and on the underlying intentions that guided the work. Although the empirical model developed in this study does not directly incorporate certain variables - such as behavioural dimensions, governance mechanisms, or institutional responses - these elements were nevertheless carefully considered throughout the research in a conceptual and global framework.

The choice not to include such variables in the formal quantitative model stems from the inherent complexity and heterogeneity of these factors, which often require different methodologies to be properly captured. However, they have not been ignored. On the contrary, they were purposefully analysed from a theoretical standpoint to highlight how financial distress cannot be fully understood through accounting indicators alone. The crisis of a company often emerges from the convergence of internal inefficiencies, managerial behaviours, and broader systemic vulnerabilities - elements that are deeply interconnected and mutually reinforcing *(La Porta R. et al., 1998; Gennaioli N. et al., 2015).*

This theoretical exploration embraced a global and multidisciplinary perspective, drawing on contributions from corporate governance theory, behavioural economics, and crisis management studies. It aimed to show that the firm must be interpreted not as an isolated entity, but as a node within a complex system shaped by internal dynamics and external influences - ranging from market conditions to institutional frameworks, from investor confidence to regulatory environments *(Zingales L., 2000; Reinhart C.M. & Rogoff K.S., 2009).*

From this standpoint, the study emphasizes the necessity of moving beyond reductionist analyses and adopting an integrated view of corporate vulnerability. The company in crisis should be assessed within a broader ecosystem where financial, cognitive, and structural factors converge. While the empirical model focused on measurable macro-indicators, these were chosen and interpreted with the awareness that they are proxies of deeper processes, which include psychological, strategic, and political dimensions as well.

In short, the research sought to promote a conceptual shift: from the firm as a mere economic unit, to the firm as an adaptive system embedded in a wider institutional and economic environment. This reflection opens the door for future investigations that may develop more holistic models, integrating both quantitative and qualitative variables to better explain corporate distress and resilience.

7. *PRACTICAL IMPLICATIONS and APPLICATIONS*

This study offers several important practical implications, particularly for firms and policymakers within the European Union. One of the central contributions of this research is its response to the European Union's ongoing call for enhanced mechanisms to predict financial distress and encourage timely interventions. By developing a robust financial distress prediction model, this study provides a valuable tool that could enable firms experiencing financial difficulties to identify early warning signs and take proactive steps toward restructuring their debt.

Timely debt restructuring is crucial for firms on the brink of financial distress, as it allows them to avoid further deterioration of their financial position. The model developed in this study, which incorporates key financial ratios and sector-specific variables, equips businesses with the ability to forecast distress with greater accuracy. This foresight can be instrumental in facilitating early negotiations with creditors and implementing debt restructuring strategies before financial difficulties become insurmountable.

Moreover, the practical significance extends beyond individual firms. At the policy level, this predictive tool aligns with broader EU objectives to promote financial stability and ensure the sustainability of enterprises within the single market. By encouraging firms to utilize such predictive models, policymakers can help mitigate systemic risks, reduce the likelihood of widespread corporate failures, and contribute to overall economic resilience. In doing so, the model addresses not only the microeconomic challenges faced by individual businesses but also the macroeconomic stability of the region.

Additionally, firms that are better equipped to predict financial distress and restructure their obligations in a timely manner are more likely to preserve jobs, protect stakeholder interests, and maintain operations. This outcome aligns with the EU's broader goals of fostering sustainable growth and safeguarding employment across member states. In this context, the model developed in this research could become an integral part of corporate governance practices, particularly in industries prone to cyclical downturns or external shocks, allowing firms to better weather financial storms.

The practical application of this model has the potential to transform how firms across the EU manage financial risk. By providing an empirically validated framework for identifying distress early on, the study offers a path for companies to make informed decisions about debt restructuring, improving their chances of recovery and long-term viability.

Another key implication of this study is that the financial distress prediction model developed can be applied not only in Italy but also in France and Spain, due to the significant similarities in their legal frameworks concerning debt restructuring, specifically in the context of Trouble Debt Restructuring (TDR). These countries share a common approach that prioritizes the reorganization of financially distressed firms over immediate liquidation, and their TDR mechanisms are aligned in both purpose and procedure, making the model highly transferable across borders.

In *Italy*, the debt restructuring process is governed by the *concordato preventivo*, a legal tool that allows firms facing financial difficulties to propose a restructuring plan to their creditors under court supervision. This process provides legal protection from creditors during negotiations, offering the company time to reorganize its debt while avoiding bankruptcy. The goal is to give firms an opportunity to recover from temporary financial distress, maintaining business continuity while restructuring their liabilities.

Similarly, *France* employs the *redressement judiciaire*, a judicial procedure designed to help companies in distress reorganize their debt under court protection. Like in Italy, this mechanism allows for the suspension of creditor claims during the restructuring process, ensuring that the company can continue operating while working out new terms with its creditors. The focus is on facilitating recovery through negotiated debt adjustments rather than forcing insolvency.

In *Spain*, the *concurso de acreedores* functions in a similar manner, providing distressed firms with a legal framework to renegotiate their debt obligations with creditors while under judicial supervision. As with Italy and France, the primary aim of this process is to restructure debt in a way that allows the company to remain operational, avoiding the more severe consequences of liquidation.

Across these three countries, the legal frameworks for TDR share several important similarities. First, they all emphasize reorganization over liquidation, offering companies the chance to recover from financial distress rather than forcing immediate bankruptcy. Second, in all three countries, the TDR process is judicially supervised, ensuring that companies are granted protection from creditor claims during the restructuring process. This court involvement is crucial in providing the necessary legal protection and time for firms to renegotiate their debts. Finally, in each of these jurisdictions, the objective of TDR is to preserve employment and business continuity, recognizing that the reorganization of a financially distressed firm can benefit not only the company but also its employees, creditors, and the broader economy.

Given these strong legislative similarities, the financial distress prediction model developed in this study can be effectively applied in Italy, France, and Spain. Firms in all three countries operate under comparable legal environments that encourage proactive restructuring of debt during periods of financial distress. The model's ability to predict financial distress early on can assist companies in initiating TDR processes in a timely manner, thereby maximizing the chances of successful restructuring.

Furthermore, the harmonized approach to TDR across these countries means that this model can serve as a valuable tool not only for individual firms but also for policymakers and regulators who are seeking to strengthen corporate recovery frameworks in the EU. By utilizing a model that aligns with the legal realities of multiple jurisdictions, stakeholders can enhance the effectiveness of financial distress interventions, ensuring that companies are able to leverage their legal options to restructure debt and avoid insolvency.

The shared characteristics of TDR legislation in Italy, France, and Spain - judicial supervision, protection from creditors, and a focus on reorganization - make the model developed in this study applicable across these countries. This interoperability presents a significant practical implication, as firms can use the model to detect early signs of financial distress and promptly initiate restructuring processes, ultimately contributing to the broader goal of maintaining economic stability and corporate sustainability.

Another important implication of this study lies in the practical application of the developed model for monitoring financial health, which is not only beneficial for firms but also for a range of stakeholders involved in corporate governance and financial decision-making. For management, the primary goal is to prevent the firm from reaching a point of bankruptcy or liquidation. With the predictive model established in this study, firms now have a tool that allows them to regularly assess their financial status and detect early signs of financial distress. This capability is crucial, as it provides management with the opportunity to take preemptive action before financial problems escalate into more serious threats to the company's survival.

One key action enabled by early detection is the possibility of negotiating a Trouble Debt Restructuring (TDR) agreement. By identifying distress at an early stage, management can initiate discussions with creditors to restructure the company's debt and restore financial balance without having to resort to bankruptcy proceedings. The proactive use of this model ensures that companies have the time and legal framework necessary to address their financial issues in a structured and organized manner, thus avoiding a crisis situation where liquidation might become the only option. This not only preserves the firm's operational capacity but also protects employees, stakeholders, and the firm's reputation in the marketplace.

Beyond the internal benefits for firms, this model also has significant implications for banks and other creditors. Creditors, particularly those exposed to high levels of corporate lending, need to continuously evaluate the financial health of their debtors. The model developed here provides a systematic approach for these institutions to assess the creditworthiness of companies and to anticipate potential risks before they manifest fully. Early identification of distress allows banks to adjust their lending strategies, perhaps renegotiating terms or requiring additional collateral, thus minimizing the financial exposure that could result from a company's failure to meet its debt obligations. As such, the model not only helps companies to survive periods of distress but also supports creditors in protecting their interests and managing risk effectively.

In addition, investors - who have a vested interest in the financial health of the firms in which they hold equity or bonds - can also benefit from the model's predictive power. The financial market environment is highly dynamic, and the ability to forecast potential distress in a firm provides investors with valuable insights into the risk associated with their investments. The model allows investors to make more informed decisions, reducing the likelihood of engaging with firms that may be on the brink of financial collapse. Furthermore, for institutional investors with large portfolios, this tool could be integrated into broader risk management frameworks, offering a data-driven method to assess portfolio risk and make timely adjustments.

Moreover, the utility of the model is not limited to firms and private financial stakeholders; it can also serve a critical function for regulatory bodies and policymakers. In economies where systemic financial risks can arise from widespread corporate distress - especially in interconnected industries - the ability to monitor the financial health of firms on a large scale is vital for maintaining economic stability. Regulators could use the model to track distress across different sectors, allowing for early interventions and the implementation of policies that could prevent a domino effect of corporate failures. This would contribute to broader financial stability, helping to safeguard the economy from potential crises triggered by corporate insolvency.

In sum, the model developed in this study serves as a comprehensive tool that offers significant value to firms, creditors, investors, and regulators. By providing an early indication of financial distress, it enables firms to take proactive measures, such as entering into TDR agreements, that can prevent the need for more drastic solutions like bankruptcy or liquidation. Simultaneously, it supports external stakeholders in managing credit risk, making informed investment decisions, and fostering financial stability within the broader economy. The model's flexibility and applicability across various sectors and stakeholder groups highlight its wide-reaching practical implications, making it a valuable addition to the tools available for managing financial health and corporate risk.

8. EXPANDING PREDICTIVE MODELS: KEY STRUCTURAL and CONTEXTUAL DRIVERS

Although the current study presents an in-depth exploration of corporate distress, it is important to acknowledge that macroeconomic variables - while theoretically addressed and contextually discussed - have not been empirically included in the proposed model**.** This methodological choice stems from a desire to first develop a more focused financial-structural framework, with the intent of setting a foundation that can be expanded in future research.

Nevertheless, the study embraces the view that corporate performance and vulnerability cannot be fully understood without considering the broader economic, institutional, and financial environment in which firms operate. Indeed, the probability of corporate distress is often a reflection not only of internal inefficiencies but also of systemic pressures and exogenous shocks, such as financial instability, credit contractions, and weak legal infrastructure. These insights are strongly supported in the existing literature. For instance, *La*

Porta R. et al. (1998) demonstrate how legal and institutional frameworks shape financial development and corporate governance. Similarly, *Reinhart C. M., and Rogoff K. S. (2009)* show that financial crises often originate from macro-level imbalances and inadequate regulatory responses. *Gennaioli N., Shleifer A., and Vishny R. (2015)* also highlight the critical role played by state and financial institutions - so-called "money doctors" - in managing systemic risk.

Based on these considerations, it becomes evident that the model developed in this study, though not integrating macroeconomic variables directly, offers a conceptual basis for subsequent expansion. Future research should aim to incorporate macro-level indicators**,** such as inflation rates, GDP growth, credit access, or sovereign risk indices, to assess how these dimensions interact with firm-level data in predicting financial distress.

Moreover, attention should also be given to non-financial micro-level variables such as managerial behaviour, behavioural biases, and corporate governance structures. As discussed in Section 6.1, factors such as CEO overconfidence, escalation of commitment, and board ineffectiveness have been empirically linked to delayed responses to financial decline and increased vulnerability. The integration of these dimensions – though methodologically challenging – would enable the development of more comprehensive predictive models that account for the full complexity of corporate distress, especially in turbulent or high-uncertainty environments.

From a policy-making perspective, this study also underscores the need for public institutions to adopt more comprehensive and anticipatory tools**.** By systematically including macro-financial variables in early warning systems, regulators and policymakers can better identify vulnerabilities in the business sector, facilitate timely interventions, and design more targeted crisis-prevention strategies. As *Zingales L. (2000)* notes, understanding the interplay between markets and institutions is vital to developing robust economic systems.

In conclusion, while the present contribution offers a detailed firm-level analysis, its broader significance lies in advocating for a multidimensional and systemic approach to corporate crisis analysis - one that merges micro-level data with macroeconomic context. This dual-level integration is not only methodologically sound, but also essential for generating meaningful implications for both academic research and public policy.

9. *DIRECTIONS for FUTURE RESEARCH*

While the findings of this study offer significant contributions to the understanding of financial distress prediction and the probabilities of Trouble Debt Restructuring (TDR), there remain several avenues for further investigation and enhancement. One of the most evident limitations of this study lies in the relatively small sample size used in the empirical analysis. Although the results derived from this sample are promising, they may not be fully representative of the broader population of firms. Future research should aim to apply the model to larger and more diverse samples to verify its generalizability and robustness across different sectors, industries, and geographical regions.

Expanding the sample size is crucial for several reasons. Firstly, it would improve the statistical power of the model, enabling a more accurate estimation of financial distress and TDR probabilities. A larger dataset would allow researchers to capture a broader spectrum of financial health conditions, from stable firms to those on the verge of distress, thereby improving the precision of the model's predictive capabilities. Additionally, a more extensive sample would allow for a deeper analysis of sector-specific dynamics. For instance, firms in capital-intensive industries such as construction or manufacturing may exhibit different distress patterns compared to those in service-oriented sectors. By examining a more diverse set of firms, future studies could explore these industry-specific variations and refine the model accordingly.

Furthermore, incorporating larger datasets could also provide valuable insights into the performance of the model across varying economic conditions. Financial distress is often exacerbated by macroeconomic downturns, and testing the model in periods of both economic stability and volatility would offer a more comprehensive understanding of its effectiveness. For example, during times of financial crisis, firms may experience heightened liquidity pressures or sudden changes in market demand, which could impact the predictive accuracy of the model. Understanding how the model performs in such contexts is critical for its application in real-world decision-making processes.

In addition to addressing the sample size limitation, future research should explore the integration of more advanced methodologies to improve prediction accuracy. In recent years, there has been a growing interest in the application of artificial intelligence (AI) and machine learning (ML) techniques in financial forecasting. These methods have the potential to significantly enhance the model's predictive power by identifying complex, non-linear relationships within the data that traditional statistical approaches may overlook. Unlike traditional models, which rely on predefined assumptions and linear relationships, AI and ML algorithms can adapt to changing data patterns and automatically adjust their predictions based on new information.

For instance, machine learning models such as random forests, support vector machines, or neural networks could be used to analyze large datasets with high dimensionality, allowing for the detection of subtle signals of financial distress that may not be captured by simpler models. These models could also incorporate alternative data sources, such as market sentiment, news reports, or even social media data, providing a more holistic view of the factors influencing a firm's financial health. The inclusion of these non-traditional data sources could greatly improve the model's ability to predict distress in volatile or rapidly changing environments.

Moreover, AI- and ML-based models offer the potential for real-time prediction capabilities. Future research could focus on developing models that continuously update predictions based on new financial data, providing firms and stakeholders with timely warnings of impending distress. This real-time analysis would be particularly valuable in dynamic market environments, where delays in identifying financial difficulties could lead to missed opportunities for intervention. By integrating these advanced technologies into the predictive framework, companies could better anticipate financial challenges and implement corrective actions before the situation deteriorates.

Another promising direction for future research lies in the development of user-friendly, "ready-to-use" tools that integrate these advanced predictive models. While AI and ML techniques hold great potential, their complexity may be a barrier to widespread adoption by non-specialists. To bridge this gap, future studies could focus on translating these sophisticated models into practical, easy-to-use tools that firms and financial practitioners can incorporate into their decision-making processes without requiring extensive technical knowledge. These tools could be developed as software applications that provide real-time financial health assessments, offering companies immediate feedback on their risk of distress and potential need for TDR agreements. Such tools would democratize access to advanced financial analysis, making it more accessible to small and medium-sized enterprises (SMEs) that may not have the resources to develop or implement complex predictive models on their own.

Additionally, the implications of such advancements extend beyond corporate management. Banks, investors, and regulatory bodies would also benefit from the application of these tools. Banks, in particular, could use these models to improve their credit risk assessments, enabling them to better evaluate the financial health of their debtors and adjust their lending practices accordingly. Investors could leverage these models to enhance their portfolio management strategies, identifying firms that are at risk of distress and making more informed investment decisions. For regulators, the ability to monitor the financial health of large segments of the corporate sector in real time could provide critical insights for maintaining financial stability and preventing systemic risks.

In conclusion, while the current study represents a significant step forward in the prediction of financial distress and TDR probabilities, there is considerable scope for future research to refine and extend these findings. Expanding the sample size, incorporating advanced AI and ML techniques, and developing practical tools for widespread use would not only improve the accuracy and applicability of the model but also provide valuable resources for a wide range of stakeholders. As financial markets continue to evolve and become more complex, the ability to accurately predict financial distress and take timely action will be increasingly important for ensuring corporate resilience and economic stability.

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