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Forecast Disagreement about Long-run Macroeconomic Relationships*

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Abstract

Using survey forecast data, this paper studies whether professional forecasters utilize long-run cointegration relationships among macroeconomic variables to forecast the future, as postulated in stochastic growth models. Significant heterogeneity exists among forecasters. The majority of the forecasters do not use these long-run relationships. The results are robust across different groups, to addressing the multiple testing problem and to allowing for structural break.

Keywords: Survey expectation, Cointegration, Disagreement

JEL classifications: D84, G22, O41

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1 Introduction

Economic agents usually have heterogeneous expectations about macroeconomic outcomes. A large literature analyzes theoretical models with heterogeneous expectations. Heterogeneity in expectations can lead to non-fundamental driven business cycle fluctuations (Lorenzoni, 2009; Angeletos and La’O, 2013; Ilut and Schneider, 2014), extreme long-run wealth distribution (Blume and Easley, 2006), speculative bubbles in asset prices (Scheinkman and Xiong, 2003; Xiong and Yan, 2010), and can influence the effectiveness of monetary policy (Branch and McGough, 2009; Kurz et al., 2018). Empirically, heterogeneous expectations can be described as the disagreement among forecasters in the survey data which is shown to change over the business cycle (Dovern, Fritzsche and Slacalek, 2012) and can be used to measure economic uncertainty (Lahiri and Sheng, 2010; Beckmann, 2021).

An important source of heterogeneous expectations is the different information sets possessed by agents, such as in the cases presented in Mankiw, Reis and Wolfers (2003) and Coibion and Gorodnichenko (2012). Another source is to be found in the different subjective forecasting models adopted by agents, e.g., Andrade, Crump, Eusepi and Moench (2016), Angeletos, Huo and Sastry (2020), and Andre, Haaland, Roth and Wohlfart (2021). Recently, Andre, Pizzinelli, Roth, and Wohlfart (2021) studied the subjective models of the macroeconomy of households and experts using a hypothetical vignette approach. They found significant heterogeneity in subjective models within and between households and experts on the effects of several macroeconomic shocks on unemployment and inflation. This heterogeneity is partially explained by agents’ selective retrieval of different propagation mechanisms. They find experts tend to recall textbook models, while households with different personal experiences recall different propagation channels of the shocks.

Following the literature which suggests agents’ different subjective models as a source of disagreement, this paper empirically studies the heterogeneous expectations of professional forecasters or experts about macroeconomic variables (aggregate output, consumption and investment). Using forecasts data from the Survey of Professional Forecasters (SPF) managed

by the Federal Reserve Bank of Philadelphia, we formally test whether professional forecasters utilize textbook stochastic growth models which exhibit long-run equilibrium (or cointegration) relationships to forecast macroeconomic variables.¹ Stochastic growth models (e.g., King, Plosser and Rebelo, 1988; King, Plosser, Stock and Watson, 1991) have been widely used for analyzing economic growth, business cycles, and welfare in recent decades. A salient feature of these models (including their one-sector and multi-sector variants) is a balanced growth path along which aggregate output, consumption, and investment share a common trend. Our analysis sheds light on a crucial assumption underlying these models and improves understanding on the expectation formation about macroeconomic variables.

We firstly show that the median (or mean) survey forecasts of aggregate output are not cointegrated with median (or mean) forecasts of aggregate consumption and investment.² This result is robust to different statistical tests, different forecasting horizons (1-, 2-, 3-, and 4-quarter ahead forecasts), using data from output forecasts that is made at the same or different dates from consumption (or investment) forecasts, with or without imposing theoretical restrictions, and using a multivariate analysis.

Using the data of individual-level forecasts, the paper finds significant heterogeneity among professional forecasters in terms of utilizing cointegration relationships. The majority of forecasters do not appear to use the cointegration relation between aggregate output and consumption in forecasting, as their forecasts of aggregate output are not cointegrated with consumption forecasts.³ Similar results are found using forecasts of aggregate output and investment. The proportion of forecasters whose aggregate output forecasts are not cointegrated with investment forecasts is generally higher than the proportion of forecasters not utilizing the cointegration relation between output and consumption.

¹The paper studies the expectation formation of macroeconomic variables by professional forecasters, following a large literature on expert forecasts, such as Bordalo, Gennaioli, Ma, and Shleifer (2020) and Coibion and Gorodnichenko (2015). Some influential papers have modeled experts' forecasts as a crucial determinant of households' forecasts. For instance, Carroll (2003) shows that households' inflation expectations are well captured by a model in which households' views are derived from news reports of the views of professional forecasters. In Malmendier and Nagel (2015), households' inflation forecasts are influenced by knowledge gained from personal experience and from experts' forecasts.

²The same results were found for the US Federal Reserve's Greenbook forecasts.

³Again, this is robust to using different tests, using forecasts over different horizons, forecasts made at the same or different dates, with or without imposing the theory-implied cointegrating vector.

In the tests, we consider a number of econometric issues. For instance, testing many hypotheses separately and simultaneously may lead to false rejections of the null hypothesis (the multiple testing problem). In addition, potential structural breaks may lead to a non-rejection of the null hypothesis that output forecasts are not cointegrated with forecasts of consumption (or investment). After addressing these issues, we found that heterogeneity still exists in utilizing long-run equilibrium relationships for forecasting macroeconomic variables.

The paper is related to a strand of literature that studies economic agents' beliefs about the relationship between different macroeconomic variables. Carvalho and Necho (2014), Drager, Lamla, and Pfajfar (2016) and Kuchler and Zafar (2019) study the relationship between households' beliefs about unemployment, inflation, and interest rates. By employing a hypothetical vignette approach, Andre, Pizzinelli, Roth and Wohlfart (2019) study households' beliefs about how macroeconomic shocks impact unemployment and inflation. One of their findings is that experts tend to recall textbook models in forecasting. Connecting with this result, we find that some experts appear to utilize long-run equilibrium relationships present in stochastic growth models to forecast macroeconomic variables.⁴

A large literature utilizes survey expectations to test or discipline the modeling of agents' expectation formation process. Greenwood and Shleifer (2014) and Adam, Marcer, and Beutel (2017) reject the rational expectation (RE) hypothesis with the stock return forecast data. Branch (2004) and Coibion and Gorodnichenko (2015) find that full-information RE is hard to square with the inflation forecast updating mechanisms. The evidence provided in this paper contributes to this literature. On the one hand, realized output and consumption (or investment) are shown to be cointegrated. On the other hand, forecasts of output and forecasts of consumption (or investment) are to a large extent not cointegrated. The discrepancy between the cointegration among realized macroeconomic variables and the non-cointegration among forecasts of macroeconomic variables appears difficult to be explained by the RE hypothesis.⁵

⁴The remaining experts may recall other models which do not incorporate cointegration relationships in forecasting.

⁵Kuang, Tang, Zhang, and Zhang (2022) find that survey stock price expectations are not anchored by forecasts of fundamentals, as opposed to a wide range of asset pricing models with various informational assumptions.

The remainder of the paper is structured as follows. Section 2 outlines the survey evidence and tests cointegration between realized data. Section 3 presents evidence from median and mean survey forecasts. Section 4 reports evidence from individual forecast data and discusses potential mechanisms. Section 5 addresses several econometric and other issues. Section 6 concludes.

2 Survey evidence: roadmap

The (one-sector and two-sector) stochastic growth models typically include a balanced growth path along which aggregate output, consumption, and investment share a common trend; on-line Appendix A reviews some general stochastic growth models.⁶ Denote lowercase letters y, c, i the logarithm of aggregate output (Y), consumption (C) and investment (I). Aggregate consumption, investment and output contain a unit root. Thus, under the assumption of RE, forecasts of the three variables over an arbitrary forecasting horizon contain a unit root. Denote by $E_{t_1}x_{t_1+k_1}$ the conditional expectations of x made at date t_1 and over horizon k_1 . Moreover, we have the following results which summarize the cointegration relationships among forecasts of the three variables under RE:

Proposition 1 *For arbitrary $t_1, t_2, k_1, k_2 > 0$, (1) forecasts of aggregate output $E_{t_1}y_{t_1+k_1}$ are cointegrated with forecasts of aggregate consumption $E_{t_2}c_{t_2+k_2}$ with the cointegrating vector $(1, -1)$, i.e., $E_{t_1}y_{t_1+k_1} - E_{t_2}c_{t_2+k_2}$ are stationary; (2) forecasts of aggregate output $E_{t_1}y_{t_1+k_1}$ are cointegrated with forecasts of investment $E_{t_2}i_{t_2+k_2}$ with the cointegrating vector $(1, -1)$, i.e., $E_{t_1}y_{t_1+k_1} - E_{t_2}i_{t_2+k_2}$ are stationary.⁷*

Three special cases of Proposition 1 are worth mentioning. First, one special case is that forecasts of aggregate output and consumption (or investment) are made at the same date ($t_1 = t_2$) and over the same forecasting horizon ($k_1 = k_2$). Second, the cointegration relation

⁶Note that in some stochastic growth models with imperfect information (e.g., Beaudry and Portier, 2014), agents also have the knowledge of a balanced growth path and their forecasts of the three macroeconomic variables display corresponding cointegration relationships.

⁷We have omitted the proof of the proposition. Lemma 2 and Theorem 4 in Kuang, Tang, Zhang, and Zhang (2022) provide the proof in a more general setting.

holds for forecasts of different variables made at the same date ($t_1 = t_2$) and over different horizons ($k_1 \neq k_2$). Third, the cointegration relation holds for forecasts of different variables made at different dates ($t_1 \neq t_2$).

Based on the proposition, the paper studies if professional forecasters indeed utilize the cointegration relationships in forecasting, and also examines the extent of heterogeneity among the forecasters. Specifically, Table 1 outlines the main survey evidence in this paper.

Table 1: **Main evidence from survey forecasts: roadmap**

Panel A: Analysis using median/mean forecasts	
Integration properties of the forecasts	Evidence 1
Cointegration between Y and C with imposing the vector $(1, -1)$	Evidence 2A
Cointegration between Y and I with imposing the vector $(1, -1)$	Evidence 2B
Cointegration between Y and C without imposing any vector	Evidence 3A
Cointegration between Y and I without imposing any vector	Evidence 3B
Multivariate analysis of a common trend shared by Y, C, and I	Evidence 4
Panel B: Analysis using individual-level forecasts	
Cointegration between Y and C over the same horizon with imposing the vector $(1, -1)$	Evidence 5A
Cointegration between Y and I over the same horizon with imposing the vector $(1, -1)$	Evidence 5B
Cointegration between Y and C over different horizons with imposing the vector $(1, -1)$	Evidence 6A
Cointegration between Y and I over different horizons with imposing the vector $(1, -1)$	Evidence 6B
Cointegration between Y and C over the same horizon without imposing any vector	Evidence 7A
Cointegration between Y and I over the same horizon without imposing any vector	Evidence 7B
Panel cointegration testing between Y and C	Evidence 8A
Panel cointegration testing between Y and I	Evidence 8B

Before presenting the evidence from the survey forecast data, we test for cointegration between realized output and consumption (or investment) with the cointegrating vector $(1, -1)$ implied by typical stochastic growth models. Data on aggregate output, consumption, and investment for the US are downloaded from the website of the Federal Reserve Bank of St. Louis. The sample covers the period between 1981:Q3 to 2018:Q4, which is the same as that of the survey forecast data. All data are at a quarterly frequency and seasonally adjusted. Real output is re-constructed by subtracting the real Government Consumption Expenditures from the real GDP.

Table 2 reports test results by applying the Dickey-Fuller GLS (DF-GLS) test to log output to consumption (or investment) ratios. Panel A shows that the DF-GLS test rejects the null

Table 2: **Tests of the cointegration with the vector $(1, -1)$ using realized data covering 1981:Q3 – 2018:Q4**

	Test stat.	5% critical	10% critical
Panel A: Realized (log) output-consumption ratio			
DF-GLS	-2.738**	-2.060	-1.749
Panel B: Realized (log) output-investment ratio			
DF-GLS	-1.757*	-2.040	-1.730

** and *: the test statistics with double asterisks or one asterisk indicate that the corresponding test rejects the null of unit root at the 5% and 10% significance level, respectively.

of unit root at the 5% level, in favor of cointegration between output and consumption. Panel B indicates that the test fails to reject the null of unit root at the 5% significance level, but rejects the null at the 10% level. Therefore, the realized data is broadly consistent with the model-implied cointegration between output and consumption (or investment), respectively.⁸

3 Evidence from median and mean forecast data

3.1 Data

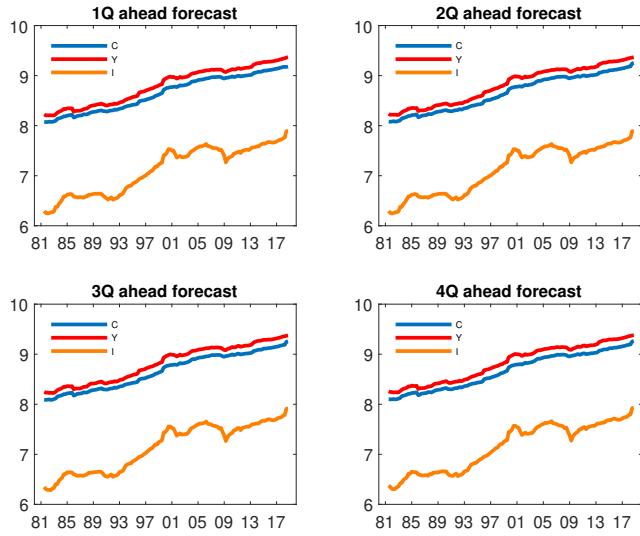
Survey forecast data are obtained from the SPF managed by the Federal Reserve Bank of Philadelphia. Consumption forecasts are measured by forecasts of chain-weighted real personal consumption expenditures (variable RCONSUM in the SPF documentation).⁹ Forecasts of real private output are measured by forecasts for the annual level of chain-weighted real GDP (RGDP), subtracting forecasts of the annual level of chain-weighted real federal government consumption and gross investment (RFEDGOV) and local government consumption and gross

⁸In addition, we conducted both Johansen trace and maximum-eigenvalue tests among realized output, consumption, and investment. Test results are in favour of one cointegrating equation at the 5% significance level.

⁹The SPF asks participants to predict real personal consumption expenditures (PCE), which include household expenditures on services and non-durable and durable goods. We cannot separate durable forecasts from non-durable goods and service forecasts. However, in macroeconomic models with durable and non-durable goods, such as Beaudry and Portier (2004) and Monacelli (2009), agents have knowledge of the balanced growth path and their forecasts of total consumption (including durable goods) and output are still cointegrated.

investment (RSLGOV).¹⁰ Forecasts of aggregate investment are measured as the sum of the forecasts of chain-weighted real non-residential fixed investment (RNRESIN) and residential fixed investment (RRESINV). All variables are available at a quarterly frequency from the third quarter (Q3) of 1981 onwards. Four forecasting horizons are available: 1-, 2-, 3-, and 4-quarter ahead.¹¹ Notably, SPF forecasts of the level of the three variables are provided with time-varying base years. Online Appendix B explains the rebasing of the forecast data.¹² Figure 1 plots the (normalized) rebased median forecasts of (log) output, consumption, and investment for all available forecasting horizons. Online Appendix C shows that the Greenbook forecasts from the Federal Reserve display similar results to those of the SPF and that there exists no cointegration between forecasts of output and forecasts of consumption (or investment).

Figure 1: Median forecasts of (log) output, consumption and investment



3.2 Integration properties of the forecasts

This section examines the integration properties of the median and mean SPF forecasts of aggregate consumption, output, and investment. Table 3 reports p-values from the Phillips-

¹⁰Before 1992, forecasts of real GNP are used.

¹¹Whelan (1993) discusses that consumption-output and investment-output ratios should be constructed using nominal data, given chain-weighted NIPAs. However, since forecasters in SPF are only asked to predict real variables and no nominal data are available, it is infeasible to construct ratios using nominal data. Therefore, in line with literature, we construct the forecast ratios using real data.

¹²Although forecasters do not directly predict (log) consumption-output and (log) investment-output ratios and the logarithm of an expectation value does not necessarily equal the expectation value of a logarithm (the Jensen's inequality), they are regarded equal when the first-order approximation is implemented.

Perron (PP) and Dickey–Fuller (DF) tests of median SPF forecasts of consumption, output, and investment for all four forecasting horizons.¹³

Table 3: **Integration properties of median SPF forecasts**

P values		1Q ahead	2Q ahead	3Q ahead	4Q ahead
Panel A: I(1) test					
<i>Median consumption forecasts</i>					
PP (Z_t test)	0.9054	0.9041	0.9019	0.9032	
Dickey–Fuller	0.9440	0.9425	0.9394	0.9388	
<i>Median output forecasts</i>					
PP (Z_t test)	0.7738	0.7873	0.7874	0.7907	
Dickey–Fuller	0.8942	0.9007	0.8968	0.8942	
<i>Median investment forecasts</i>					
PP (Z_t test)	0.7174	0.7174	0.7091	0.7106	
Dickey–Fuller	0.8877	0.8877	0.8835	0.8790	
Panel B: I(2) test					
<i>Median consumption forecasts</i>					
PP (Z_t test)	0.000	0.000	0.000	0.000	
Dickey–Fuller	0.000	0.000	0.000	0.000	
<i>Median output forecasts</i>					
PP (Z_t test)	0.000	0.000	0.000	0.000	
Dickey–Fuller	0.000	0.000	0.000	0.000	
<i>Median investment forecasts</i>					
PP (Z_t test)	0.000	0.000	0.000	0.000	
Dickey–Fuller	0.000	0.000	0.000	0.000	

Evidence 1: 1-, 2-, 3- and 4-quarter ahead median forecasts of aggregate consumption, output, and investment are I(1) but not I(2).

Panel A shows that for all forecasting horizons considered, both tests fail to reject the null that median forecasts of consumption, output, and investment are integrated of order 1, i.e. I(1), at conventional significance levels. Panel B indicates that all median forecasts over different forecasting horizons are not integrated of order 2, i.e. I(2). Tests on mean forecasts

¹³We use PP tests to examine integration properties of forecasts with Newey-West optimal lags imposed and DF tests without any lag included. The critical values of PP and DF tests are computed by Cheung and Lai (1995b).

reach the same conclusion; see online Appendix D.

3.3 No cointegration with imposing theoretical restrictions

This section tests the cointegration relation between output forecasts and consumption (or investment) forecasts with the cointegrating vector $(1, -1)$ implied by stochastic growth models. Many papers criticized the power of the standard DF class of unit root tests in the 1980s and 1990s. This paper uses some of the most powerful tests, such as the DF-GLS test. Another way to mitigate this concern is by applying the KPSS test, which tests the null hypothesis of a stationary process against the alternative of a unit root. The robustness of the results is also demonstrated by analyzing the effects of sample size on testing outcomes in Section 3.5 and 4.2.1.

Table 4 reports the results of testing cointegration between median forecasts of output and consumption (or investment) with the cointegrating vector $(1, -1)$ using the PP, DF-GLS, and KPSS tests.¹⁴ The first four columns display test statistics for forecasts made at the same date and over the same forecasting horizon. The last column, labeled as “4Q Y & 1Q C” in Panel A (or “4Q Y & 1Q I” in Panel B), tests cointegration between 4-quarter ahead output forecasts and 1-quarter ahead consumption (or investment) forecasts.

¹⁴We report test results assuming that there exists no trend component in the consumption-output ratios or investment-output ratios. The cointegration test results are robust when the trend component is added. The critical values of the DF-GLS test have been computed by Cheung and Lai (1995a).

Table 4: **Cointegration tests for median SPF forecasts with the cointegrating vector $(1, -1)$**

Panel A: no cointegration between forecasts of output and consumption					
Median	1Q ahead	2Q ahead	3Q ahead	4Q ahead	4Q Y & 1Q C
PP (Z_t test)	-1.586	-1.672	-1.713	-1.720	-1.779
10% critical value	-2.577	-2.577	-2.577	-2.577	-2.577
DF-GLS	-1.378	-1.478	-1.526	-1.508	-1.588
10% critical value	-1.749	-1.749	-1.749	-1.749	-1.749
KPSS	1.885	1.862	1.847	1.861	1.685
5% critical value	0.463	0.463	0.463	0.463	0.463

Panel B: no cointegration between forecasts of output and investment					
Median	1Q ahead	2Q ahead	3Q ahead	4Q ahead	4Q Y & 1Q I
PP (Z_t test)	-1.705	-1.694	-1.559	-1.479	-1.719
10% critical value	-2.577	-2.577	-2.577	-2.577	-2.577
DF-GLS	-0.209	-0.182	-0.193	-0.242	-0.155
10% critical value	-1.737	-1.737	-1.743	-1.743	-1.737
KPSS	2.630	2.641	2.773	2.843	2.645
5% critical value	0.463	0.463	0.463	0.463	0.463

Evidence 2A (Panel A): Median aggregate consumption forecasts and output forecasts are not cointegrated with the vector $(1, -1)$;

Evidence 2B (Panel B): Median aggregate investment forecasts and output forecasts are not cointegrated with the vector $(1, -1)$.

In Panel A, the PP and DF-GLS tests fail to reject the null hypothesis that median forecasts of output are not cointegrated with median forecasts of consumption over the same or different forecasting horizons with the vector $(1, -1)$ at the 10% significance level. Meanwhile, the KPSS tests reject the null hypothesis that forecasts of output are cointegrated with forecasts of consumption for all forecasting horizons at the 5% significance level. Thus, all tests yield the same conclusion, which is that forecasts of output are not cointegrated with forecasts of consumption with the vector $(1, -1)$ over the same or different horizons.

Panel B of Table 4 reports the results of testing cointegration between median forecasts of output and investment with the vector $(1, -1)$. All tests produce the same conclusion that forecasts of output are not cointegrated with median forecasts of investment with the vector $(1, -1)$ over the same or different horizons. Similar results are found using mean forecasts; see online Appendix E.1.

We tested overidentifying restrictions on the cointegration relation among forecasts using the likelihood-ratio test. We imposed two cointegration relations implied by stochastic growth models when estimating a Vector Error Correction Model (VECM); the two cointegration vectors are $(1, -1, 0)$ and $(1, 0, -1)$ for the forecasts of output, consumption and investment. The optimal number of lags is selected by the Akaike information criterion (AIC). Table 5 reports the p values of tests for median forecasts and mean forecasts, respectively. The null hypothesis is rejected at the 5% significance level, implying that the theoretical cointegration relation among the forecasts is not supported by the data.

Table 5: Likelihood ratio test of over-identifying restrictions

	1Q ahead	2Q ahead	3Q ahead	4Q ahead
P value (median)	0.00	0.00	0.00	0.00
P value (mean)	0.00	0.00	0.00	0.00

Note: Lag selection is based on the AIC criterion. Results are robust to different lag selections.

3.4 No cointegration without imposing theoretical restrictions

This section tests cointegration between forecasts of macroeconomic variables without imposing the theory-implied cointegrating restriction. Using the Engle–Granger test, Panel A (or B) of Table 6 reports the results of testing the cointegration between output forecasts and consumption (or investment) forecasts over the same or different horizons and the corresponding 10% critical values.¹⁵ Again, the last column is the testing results of using 4-quarter-ahead output forecasts and 1-quarter-ahead consumption (or investment) forecasts.

¹⁵We report the Engle–Granger test results without incorporating the trend component. Our test results are robust when the trend is included. The Engle–Granger test’s critical values have been calculated by MacKinnon (1996).

Table 6: **Cointegration test for median SPF forecasts without imposing the cointegrating vector $(1, -1)$**

Engle-Granger test					
Panel A: cointegration between forecasts of output and consumption					
Median	1Q ahead	2Q ahead	3Q ahead	4Q ahead	4Q Y & 1Q C
Test stats.	-2.315	-2.378	-2.424	-2.451	-2.475
10% critical value	-3.073	-3.073	-3.073	-3.073	-3.073
Panel B: cointegration between forecasts of output and investment					
Median	1Q ahead	2Q ahead	3Q ahead	4Q ahead	4Q Y & 1Q I
Test stats.	-2.257	-2.270	-2.251	-2.250	-2.306
10% critical value	-3.073	-3.073	-3.073	-3.073	-3.073

Evidence 3A (Panel A): Median consumption forecasts are not cointegrated with output forecasts without imposing any cointegrating vector;

Evidence 3B (Panel B): Median investment forecasts are not cointegrated with output forecasts without imposing any cointegrating vector.

The test results uniformly suggest that we cannot reject the null hypothesis that output forecasts are not cointegrated with consumption (or investment) forecasts at the 10% significance level and over the same or different forecasting horizons, as all test statistics are greater than the corresponding 10% critical values. The test results are robust to using mean forecasts; see online Appendix E.2.

3.5 Multivariate analysis: testing for common trends among forecasts of output, consumption, and investment

This section tests if forecasts of output, consumption, and investment share a common trend. We tested the number of cointegrating vectors among median forecasts of output, consumption, and investment over a 1- to 4-quarter ahead forecasting horizon, using the Johansen trace and maximum-eigenvalue tests. The Johansen trace test examines the multivariate cointegrating relation with the null of no more than r cointegrating vector(s) (rank of r), against the alternative hypothesis that the number of the cointegrating vector(s) is strictly greater than r . Meanwhile, the Johansen maximum-eigenvalue test evaluates the null of exact r cointegrating

vector(s) against the alternative of $r + 1$ cointegrating vectors.

Table 7: **Johansen trace and maximum-eigenvalue tests for the number of common trends among median forecasts**

Johansen test				
Trace test: $J^{trace}(r)$, $r = \text{rank}$				
null:	no more than r cointegration vector			
Alternative:	number of the cointegrating vector $> r$			
Median	$r=0$	5% critical	$r=1$	5% critical
1Q ahead	28.5*	29.7	9.6	15.4
2Q ahead	27.6*	29.7	9.7	15.4
3Q ahead	26.2*	29.7	9.2	15.4
4Q ahead	25.2*	29.7	7.6	15.4
Maximum-eigenvalue test: $\max(r)$, $r = \text{rank}$				
null:	no more than r cointegration vector			
Alternative:	number of the cointegrating vector $= r+1$			
Median	$r=0$	5% critical	$r=1$	5% critical
1Q ahead	18.9*	21.0	9.5	14.1
2Q ahead	17.9*	21.0	9.7	14.1
3Q ahead	17.2*	21.0	9.1	14.1
4Q ahead	17.5*	21.0	8.4	14.1

*: $J^{trace}(r)$ or $\max(r)$ test statistics with an asterisk indicate that the corresponding rank r is the lowest rank, for which the trace test fails to reject its null number of the cointegration equation and is accepted as the number of the cointegrating vector among the three forecast variables.

Evidence 4: Median forecasts of output, consumption, and investment do not share a common trend.

Table 7 reports test results using the median forecasts of output, consumption, and investment.¹⁶ The trace test statistics reported in the upper part of Table 7 suggest that tests for the median forecasts of output, consumption, and investment over the period of 1- to 4- quarter ahead uniformly fail to reject the null of no cointegrating vector against the alternative of the existence of cointegration. Moreover, the maximum-eigenvalue test statistics reported in the lower part of Table 7 also uniformly fail to reject the null of no cointegrating vector against the

¹⁶For the Johansen trace and maximum-eigenvalue tests, we report the outcomes with 1 plus the number of lags selected by the AIC (robust to other selection criteria, like HQIC and SBIC). This is because the Johansen test uses first difference and, to compensate for the one lag loss, we incorporated one additional lag.

alternative of exactly one cointegrating vector. Thus, the results show that any linear combination of forecasts of output, consumption and investment is not stationary. Similar results are found using the mean forecasts; see online Appendix E.3.

A concern is that tests of unit roots or of cointegration may have low power. We have used several most powerful and state-of-the-art tests. Two additional ways are used to address this concern, by analyzing the effect of sample size on testing outcomes. We report the recursive Johansen trace test statistics from testing the common trends shared by forecasts of output, consumption and investment. The initial subsample of median or mean forecasts covers 1981:Q3 to 1986:Q2 (20 quarters). By adding a new observation, the sample gradually expands to the full sample (from 1981:Q3 to 2018:Q4). Figure A.15 in online Appendix M plots the test statistics (red lines) and the corresponding 5% critical values (blue lines) of recursive Johansen trace tests with rank = 0 for median forecasts of output, consumption, and investment. All test statistics are below the corresponding critical values, indicating that recursive trace tests fails to reject the null of no cointegrating vector against the alternative of the existence of at least one cointegrating vector.¹⁷ In Section 4.2.1 where individual-level forecasts are used, individual forecasters' are split into different groups with different sample sizes, and we find that there is no monotonic relation between the proportions of rejecting the null hypothesis and the sample size.

4 Evidence from individual forecasts

4.1 Data

Individual forecasts from the SPF are used for testing. To obtain reliable testing results, we selected individual forecasters who have answered more than 60 quarters between 1981:Q3 and 2018:Q4; this resulted in 21 forecasters.¹⁸ Individual-level forecasts of real output, consumption,

¹⁷Similar results are obtained with mean forecasts; see Figure A.16 in online Appendix M. Moreover, this appendix shows the recursive trace statistics when testing the null hypothesis of 1 cointegrating vector against more than 1 cointegrating vector.

¹⁸We found a similar result signifying the existence of significant heterogeneity in utilizing long-run macroeconomic relationships in forecasting when we used a different cutoff value than 60 quarters, such as 70 or 58.

and investment are measured and rebased in the same way as the median forecasts in Section 3.1.

Figure 2: Number of SPF forecasts reported by each selected forecaster for 1- to 4-quarter ahead forecasting horizons

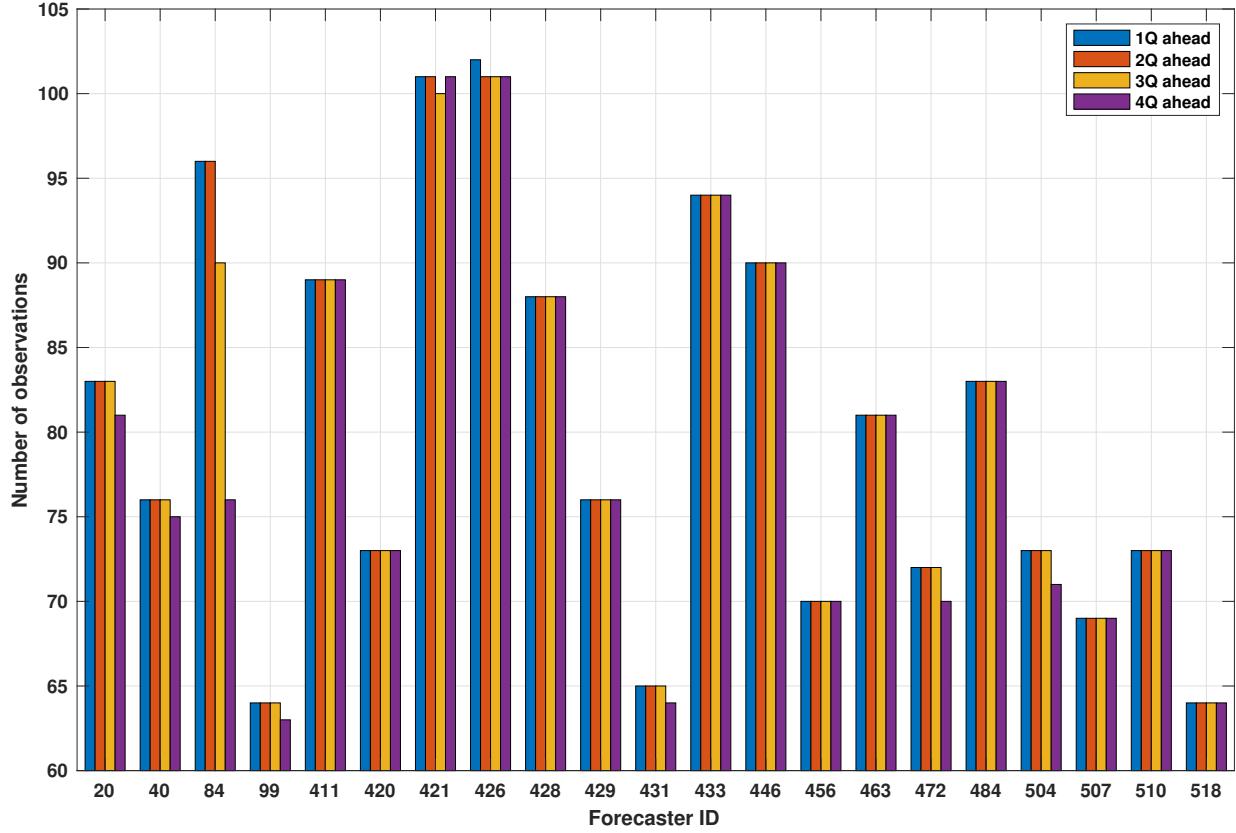


Figure 2 plots the number of valid forecasts of output-consumption ratio (which are also numbers of valid output-investment ratio) for each of the 21 selected individual forecasters, over 1- to 4-quarter ahead. The X-axis denotes each forecaster's ID in the SPF dataset, while the Y-axis denotes the number of forecast observations over different forecasting horizons. In total, we have 84 time series of forecasts for each individual variable (output, consumption, or investment). There is a small number of missing values in the dataset, and the procedure for addressing this is detailed in online Appendix F.

4.2 Heterogeneity in utilizing cointegration relationships in forecasting: testing using forecasts made over the same horizon

This section tests cointegration between the individuals' forecasts of output and consumption (or between forecasts of output and investment) over the *same* forecasting horizon, with the theory-implied $(1, -1)$ restriction imposed. Table 8 reports the results that emerged from the PP, DF-GLS, and KPSS tests.

Table 8: Cointegration tests using individual forecasts over same forecasting horizon

No. of forecasters:	21 forecasters and 4 horizons (1 – 4Q-ahead)	
With $(1, -1)$ restriction	No. of no cointegration detected out of 84 forecasts	Proportion of no cointegration detected
Panel A: cointegration between forecasts of output and consumption		
PP Z_t test (10% crit. value)	63	75.0%
DF-GLS (10% crit. value)	66	78.6%
KPSS (5% crit. value)	56	66.7%
Panel B: cointegration between forecasts of output and investment		
PP Z_t test (10% crit. value)	77	91.7%
DF-GLS (10% crit. value)	76	90.5%
KPSS (5% crit. value)	62	73.8%

Evidence 5A (Panel A): In 67% – 79% of the 84 cases (21 forecasters and 4 forecasting horizons), forecasts of output and consumption are not cointegrated with vector $(1, -1)$;

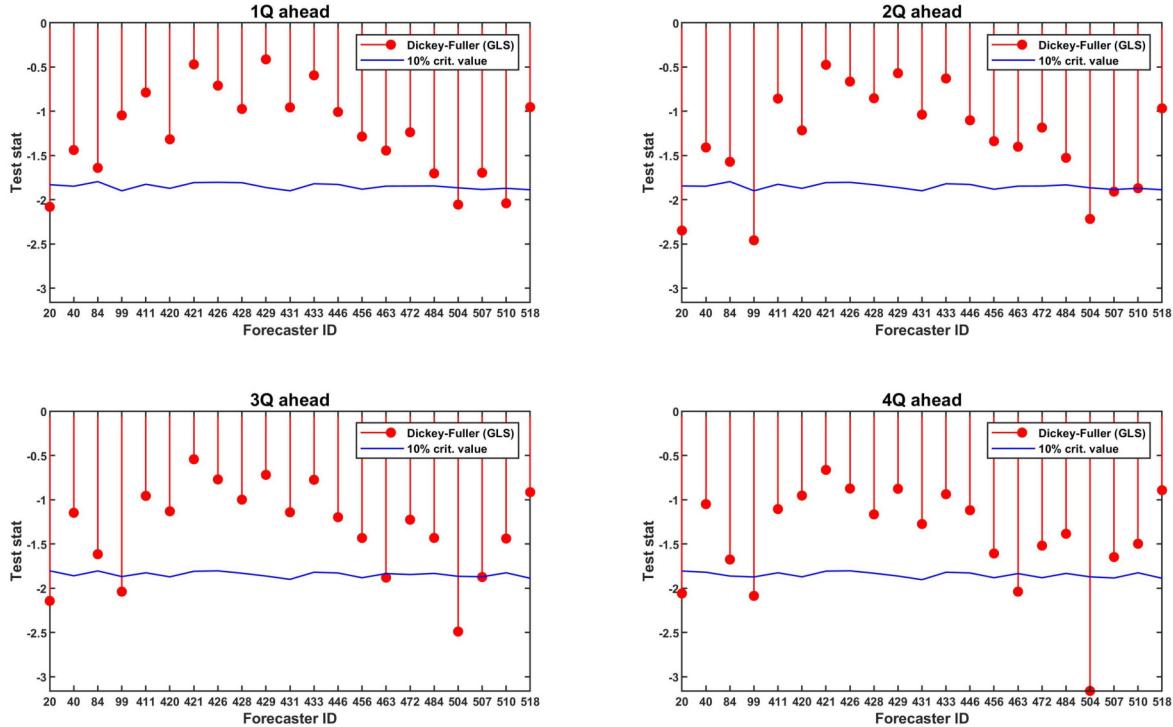
Evidence 5B (Panel B): In 74% – 92% of the 84 cases (depending on the test), forecasts of output and investment are not cointegrated with vector $(1, -1)$.

The testing results suggest a significant disagreement among individual forecasters in terms of utilizing the cointegration relation between output and consumption (or investment) in forecasting. Panel A shows that, using the PP test, in 75% of the 84 cases, output forecasts are not cointegrated with consumption forecasts, while they are cointegrated in the remaining 25% of cases.¹⁹ The DF-GLS test suggests that for about 21% of the 84 cases, forecasts of output and consumption are cointegrated; the corresponding number is about 1/3 based on the KPSS test.

¹⁹Recall that there are 84 time series of forecasts for each variable (21 forecasters times four forecasting horizons).

Figure 3 plots the DF-GLS test statistics and the associated critical values from Panel A of Table 8. The test statistics are pinned down by the circle at the end of each red stem, while the corresponding critical value is located on the blue line.²⁰ For the majority of forecasters, the test statistics are above the blue line of critical values, suggesting no cointegration between output forecasts and consumption forecasts. The DF-GLS test indicates that three forecasters (with IDs of 20, 99, and 504) form cointegrated output and consumption forecasts for at least 3 forecasting horizons. In addition, the forecaster with ID 463 (or ID 507) forms cointegrated output and consumption forecasts over 3- and 4-quarter ahead (or over 2- and 3-quarter ahead).

Figure 3: **DF-GLS test statistics vs. critical values using individual-level output and consumption forecasts**



Panel B of Table 8 tests the cointegration between output forecasts and investment forecasts over the same forecasting horizon and with the $(1, -1)$ restriction. In 92% of the 84 cases, the PP tests suggest that output forecasts are not cointegrated with investment forecasts at the 10% significance level, while the two forecasts are cointegrated for the remaining 8% of the cases. The DF-GLS test fails to reject the null of no cointegration between forecasts of output and

²⁰For the DF-GLS test, since we report the test statistics and critical values with the number of lags that minimizes the modified AIC criterion, the critical value for different tests differs. See Ng and Perron (2000).

investment for 91% of the cases, while the KPSS test rejects the null of cointegration between forecasts of output and investment for around 74% of the cases (and fails to reject the null for 26% of the cases).²¹

Overall, the majority of forecasters do not utilize the cointegration relation between output and consumption (or investment) with the cointegrating vector $(1, -1)$ in forecasting. Moreover, for all three tests, the proportion of forecasters who do not utilize the cointegration relation between output and investment in forecasting is higher than the proportion of forecasters who do not utilize the cointegration relation between output and consumption.²²

Using individual-level forecasts, we tested overidentifying restrictions by imposing two cointegration relations implied by stochastic growth models when estimating a VECM. The two cointegration vectors are $(1, -1, 0)$ and $(1, 0, -1)$ for forecasts of output, consumption, and investment. The optimal number of lags was selected by the AIC criterion. Table A.13 in online Appendix G.3 reports outcomes of the likelihood-ratio test. The test rejected the cointegrating restrictions at the 5% significance level for the majority of cases, while the null hypothesis was not rejected for a handful of cases. Thus, there still exists heterogeneity in the utilization of the long-run relationships in forecasting macroeconomic variables.

We also recursively applied Johansen trace test to examine if a common trend exists in forecasts of consumption, output, and investment for several forecasters IDs. Figure A.17 in online Appendix M displays the test statistics against the corresponding 5% critical values. The null hypothesis of a common trend was persistently rejected for some IDs (ID 20 and 510) as the sample became longer, and was not rejected for some other IDs (ID 421 and 429).

4.2.1 Test results by groups with different sample size and characteristics

We examined how the proportion of rejecting the null hypothesis of no cointegration depends on the sample size. Forecasters were divided into three groups according to their sample sizes (top, middle, and bottom one-third). Table A.20 in online Appendix N reports the proportions

²¹Figure A.4 in online Appendix plots the DF-GLS test statistics against the corresponding critical values associated with Panel B of Table 8.

²²PP and KPSS test statistics are illustrated by plots in online Appendix H.

of no cointegration between output forecasts and forecasts of consumption (or investment) in each group. There is no monotonic relation between the proportions and the sample size.

Moreover, we examined how the testing results depend on the forecasters' characteristics. The SPF provides limited information regarding the participants' characteristics. It contains information on the types of institutions to which the forecasters belong, which are either financial service providers or non-financial service providers. One may wonder if the test results are robust across different groups or if there exists heterogeneity across the two groups.

Table A.21 in online Appendix N reports the proportion of forecasters within each group whose forecasts of consumption (or investment) are not cointegrated with their own output forecasts with the cointegration vector $(1, -1)$. Within each group (financial or non-financial service providers), heterogeneity still exists in utilizing the cointegration relation between output and consumption (or investment) in forecasting. Across the two groups, it appears that a higher proportion of forecasters from financial service institutions make use of the cointegration relations between macroeconomic variables in their forecasts.²³

4.3 Heterogeneity in utilizing cointegration relationships in forecasting: testing using forecasts made over different horizons

Using individual forecasts data, this section tests the cointegration between forecasts of output and consumption (or investment) over *different* forecasting horizons and with imposing the theory-implied $(1, -1)$ cointegrating vector. Panel A (or B) of Table 9 reports the test results produced by the PP, DF-GLS and KPSS tests, using 4-quarter ahead output forecasts and the 1-quarter ahead consumption (or investment) forecasts of 21 forecasters.

Panel A reports that there is no cointegration between 4Q-ahead output forecasts and 1Q-ahead consumption forecasts in 13 out of 21 cases based on the PP test in 17 out of 21 cases based on the DF-GLS test, and in 16 out of 21 cases based on the KPSS test. Thus, all three tests suggest that in over 62% of the cases, 4-quarter ahead forecasts of output and 1-quarter

²³A potential reason for this is that financial institutions may have more demanding requirements on practitioners' economic and quantitative skills and therefore the experts employed in this type of institutions are more likely to recall stochastic growth models in forecasting.

Table 9: Testing using individual forecasts over different forecasting horizons

With $(1, -1)$ restriction	No. of no cointegration detected out of 21	Proportion of no cointegration detected
Panel A: cointegration between 4Q ahead Y forecasts and 1Q ahead C forecasts		
PP Z_t test (10% crit. value)	13	61.9%
DF-GLS (10% crit. value)	17	81.0%
KPSS (5% crit. value)	15	71.4%
Panel B: cointegration between 4Q ahead Y forecasts and 1Q ahead I forecasts		
PP Z_t test (10% crit. value)	20	95.2%
DF-GLS (10% crit. value)	19	90.5%
KPSS (5% crit. value)	16	76.2%

Evidence 6A (Panel A): In 62% – 81% of 21 cases (depending on the test used), 4-quarter ahead output forecasts and 1-quarter ahead consumption forecasts are not cointegrated, when the $(1, -1)$ cointegrating vector is imposed.

Evidence 6B (Panel B): In 77% – 95% of 21 cases, 4-quarter ahead output forecasts and 1-quarter ahead investment forecasts are not cointegrated, when the $(1, -1)$ cointegrating vector is imposed.

ahead forecasts of consumption are not cointegrated. Panel B shows that, compared with the results in Table 8, a larger proportion of no cointegration is detected between 4-quarter ahead output forecasts and 1-quarter ahead investment forecasts by all three tests. Overall, the testing results suggest that heterogeneity exists in terms of utilizing cointegration relations in making forecasts among individual forecasters and that the majority of forecasters do not make use of the long-run relationships.

4.4 Individual level cointegration tests without imposing theoretical restrictions

Table 10: **Cointegration tests without imposing $(1, -1)$ restriction, using individual-level forecasts over the same forecasting horizon**

Total individual forecasters:	21, with 4 forecasts each (1-, 2-, 3- & 4Q ahead)	
Engle–Granger test (10% crit. value) over the same horizons	No. of no cointegration detected out of 84 forecasts	Proportion of no cointegration detected
Forecasts of Y and C	69	82.1%
Forecasts of Y and I	82	97.6%

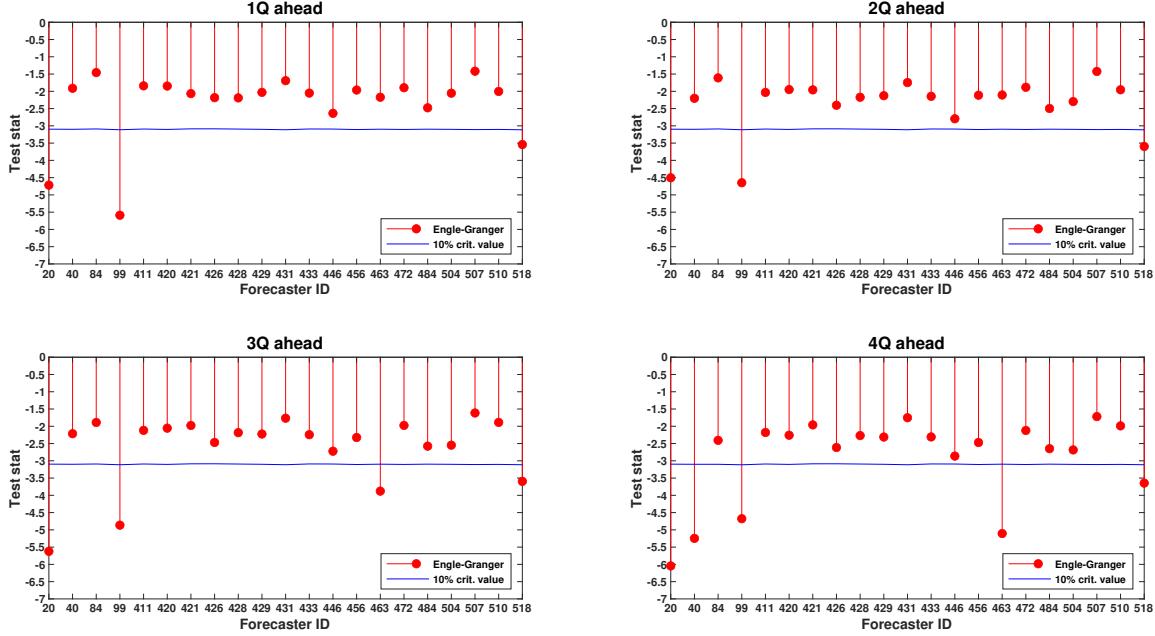
Evidence 7A (Panel A): In 82% of 84 cases, the forecasts of Y and C are not cointegrated, when no cointegrating vector is imposed.

Evidence 7B (Panel B): In 98% of 84 cases, the forecasts of Y and I are not cointegrated, when no cointegrating vector is imposed.

Table 10 provides results of the Engle–Granger test of cointegration between forecasts of output and consumption (or investment), without imposing the theoretical cointegration restrictions. In 61 out of 84 (or 82 out of 84) cases, the test suggests no cointegration between individual output forecasts and consumption (or investment) forecasts at the 10% significance level.

Figure 4 presents the test statistics and critical values, when the test is applied to output and consumption forecasts. For the majority of forecasters, forecasts of output and consumption are not cointegrated. They are cointegrated for three forecasters (those labelled as ID 20, 99, and 518) over all four horizons. Moreover, the forecasts of output and consumption made by the forecaster with ID 463 are cointegrated over 3- and 4-quarter ahead, respectively. The 4-quarter ahead forecast of output and consumption made by the forecaster designated as ID 40 are cointegrated. Figure A.9 in online Appendix I displays the test statistics and critical values, when the test is applied to output and investment forecasts.

Figure 4: Engle–Granger test statistics vs. critical values for testing cointegration between output and consumption forecasts and without imposing the $(1, -1)$ restriction



4.5 Discussion on potential mechanisms

The empirical analysis reveals heterogeneity in utilizing the cointegration relationships present in stochastic growth models in the forecasting of macroeconomic variables. Some experts (or professional forecasters) make use of these relationships, while the majority of the experts do not. Why do the forecasts made by most forecasters not make use of cointegration relationships?

A special survey, SPF Panelists' Forecasting Methods, conducted by the Federal Reserve Bank of Philadelphia in 2009, explicitly asked participants for their forecasting methods.²⁴ Most respondents – 21 out of 25 – responded that they adopt models to produce their forecasts. Among them, the vast majority (20 out of 21) add subjective adjustments (i.e., based on their judgment) on top of the forecasts produced by the models. This result may not be surprising, as add-factoring is a common practice in the forecasting community, where forecasters incorporate their personal judgements on top of the statistical models. Moreover, the vast majority (20 out of 21) of forecasters acknowledge that a model is a prominent part in producing forecasts

²⁴The exact questions and answers can be seen from p.10 of a summary of survey available at <https://www.philadelphiafed.org/-/media/frbp/assets/surveys-and-data/survey-of-professional-forecasters/spf-special-survey-on-forecast-methods.pdf?la=en&hash=DA9492A3DE5E3BF70D40F807B7278C83>.

over up to two years ahead. The forecasting horizons (1-, 2-, 3- and 4-quarter ahead) examined in our empirical evidence fall within the 2-year ahead horizon, indicating that most of these forecasts are produced by a method which consists of a forecasting model alongside the inclusion of forecasters' judgements.

There are two potential causes for the documented evidence that most forecasters' forecasts of consumption (or investment) are not cointegrated with forecasts of output. First, the component of subjective judgments that these SPF forecasters add to their forecasts of macroeconomic variables do not display a cointegration relation. Various studies have examined the implications of employing subjective judgements in macroeconomics and policy making. For instance, Svensson and Tetlow (2005) empirically demonstrate the impact of judgment on forecasting by the Federal Reserve Board.²⁵ The second possible cause of no cointegration between these macroeconomic forecasts is that, the forecasting models (the components before adding adjustments) of most SPF forecasters do not impose long-run equilibrium relationships.

5 Econometric issues and discussions

5.1 Addressing the multiple testing problem

In the previous section, we studied the cointegration relation between individual-level forecasts of output and consumption (or investment) by performing multiple unit root (or cointegration) tests simultaneously. A concern is that the testing outcomes might be subject to the multiple testing problem. This issue was addressed in two ways. First, we used Anderson's sharpened False Discovery Rate (FDR) q-values, which is a corrected version of p-values and has greater power than many other methods. Second, we performed a panel cointegration test that considers cross-sectional dependence, which utilizes a larger sample size and has higher power.

Online Appendix J reports test outcomes using Anderson's sharpened q-values for both forecasts of output-consumption ratios and forecasts of output-investment ratios. Compared

²⁵Svensson (2003, 2005) formally illustrates that economic performance can be improved if monetary policymakers explicitly incorporate subjective adjustments to the forecasts of key variables. Bullard, Evans, and Honkapohja (2007) show the inclusion of the add-factoring judgmental adjustment in forecasting can lead to self-fulfilling fluctuations.

with the results in Table 8, the proportions of rejecting the null hypothesis of no cointegration become lower. However, the testing results suggest that heterogeneity still exists in terms of utilizing cointegration relations in making forecasts among individual forecasters, and that the majority of forecasters do not make use of long-run relationships.

Since these professional forecasters are exposed to common aggregate shocks, their forecasts may be highly correlated. Online Appendix J.1 tests and confirms the cross-sectional dependence for forecasts of output-to-consumption ratios (or output-to-investment ratios) for 1- to 4-quarter forecasting horizons. First-generation panel unit root tests such as the Fisher-type panel tests generally ignore such instances of dependence and suffer from size distortions. To address this issue, we adopted a version of second-generation panel unit root tests, the cross-sectionally augmented Dickey–Fuller (CADF) test proposed by Pesaran (2003). It tests the unit root in heterogeneous panels with the null hypothesis that all panels are non-stationary, against the alternative hypothesis that at least one panel is stationary. The CADF test eliminates cross-sectional dependence by augmenting the standard Dickey–Fuller (DF) or the Augmented Dickey–Fuller (ADF) regression with cross-sectional average lagged levels and the first differences of the individual data series.²⁶ We examine if 1- to 4-quarter ahead forecasts of output are cointegrated with forecasts of consumption (or investment) with the $(1, -1)$ restriction.

Panel A and B of Table 11 report the p-values of the panel unit root tests on forecasts of output-consumption ratios and output-investment ratios over 1- to 4-quarter ahead forecasting horizons, respectively. In Panel A, the CADF panel unit root tests uniformly reject the null that all panels are non-stationary at any conventional significance level, indicating that at least one forecaster imposes the $(1, -1)$ cointegration relation when forecasting output and consumption. Similar results are obtained in Panel B where the CADF tests uniformly reject the null, suggesting that at least one forecaster imposes the cointegrated relation between output and investment in forecasting.

²⁶All panel unit root tests are performed with Newey-West optimal lags. Test results are robust to different numbers of lags incorporated.

Table 11: **P-values of Pesaran panel cointegration test with the cointegrating vector $(1, -1)$**

CADF panel unit root test		Number of panels: 21 in each test			
H_0 : All panels are non-stationary;					
H_1 : At least one panel is stationary					
		1Q ahead	2Q ahead	3Q ahead	4Q ahead
Panel A: cointegration between forecasts of output and consumption					
P-values		0.000	0.000	0.000	0.000
Panel B: cointegration between forecasts of output and investment					
P-values		0.001	0.001	0.001	0.007

Evidence 8A (Panel A): The CADF panel cointegration tests suggest that at least one forecaster's output forecasts are cointegrated with consumption forecasts over 1-, 2-, 3- and 4-quarter ahead with the $(1, -1)$ restriction.

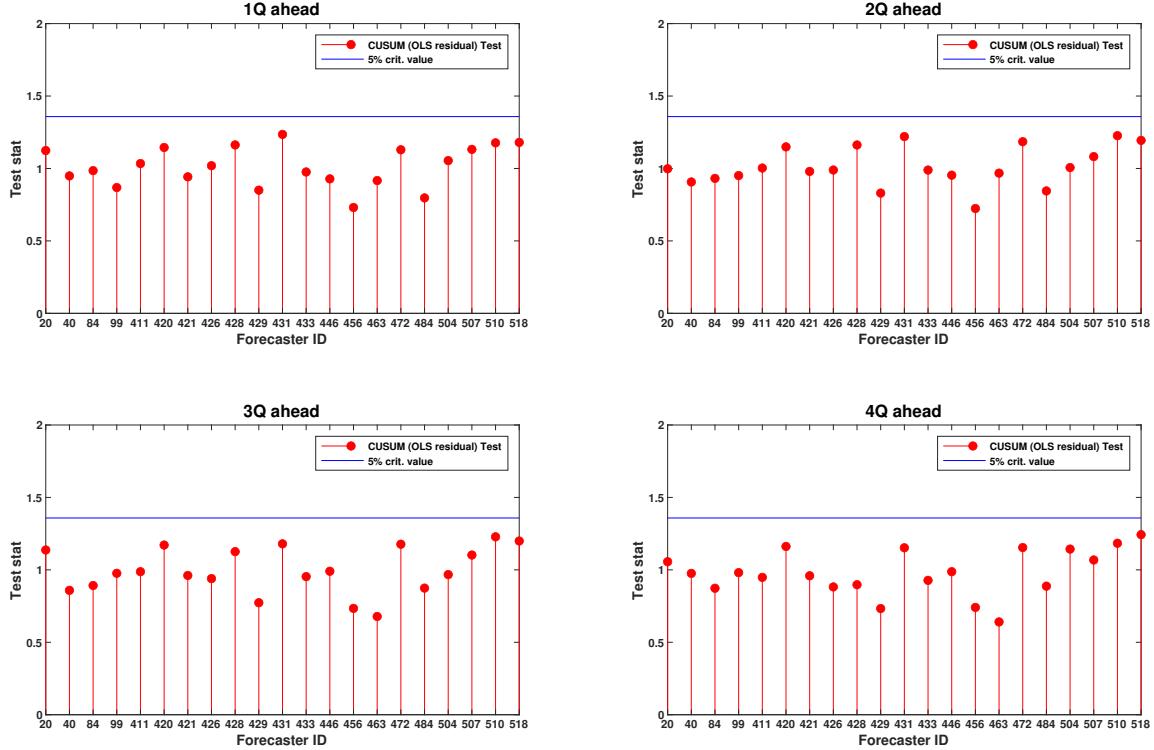
Evidence 8B (Panel B): The CADF panel cointegration tests suggest that at least one forecaster's output forecasts are cointegrated with investment forecasts over 1-, 2-, 3- and 4-quarter ahead with the $(1, -1)$ restriction.

5.2 Structural breaks

The results so far have not considered potential structural breaks in the sample. A concern is that a structural break may lead to the non-rejection of no cointegration between the forecasts. To address this concern, this section employs the Recursive Cusum test (Krämer and Ploberger, 1992; Brown, Durbin, and Evans, 1975) and the Gregory and Hansen (1996) cointegration test.

The Recursive Cusum test investigates parameter stability with the null hypothesis being no structural break. The test statistics are based on whether the time series abruptly changes in a way not predicted by the model across rolling samples. Figure 5 displays the individual-level Recursive Cusum test statistics and the corresponding 5% critical value, assuming OLS residuals for forecasts of output-consumption ratios. The test statistics (red dots) are all below the corresponding critical values (blue lines). This implies that for all individual forecasts, the Recursive Cusum tests indicate that no structural break is found in the estimated coefficients from the augmented DF regression. Similar results are obtained with recursive residuals. Online Appendix L shows that the same results are obtained for individual-level forecasts of output-investment ratios and for median and mean forecasts.

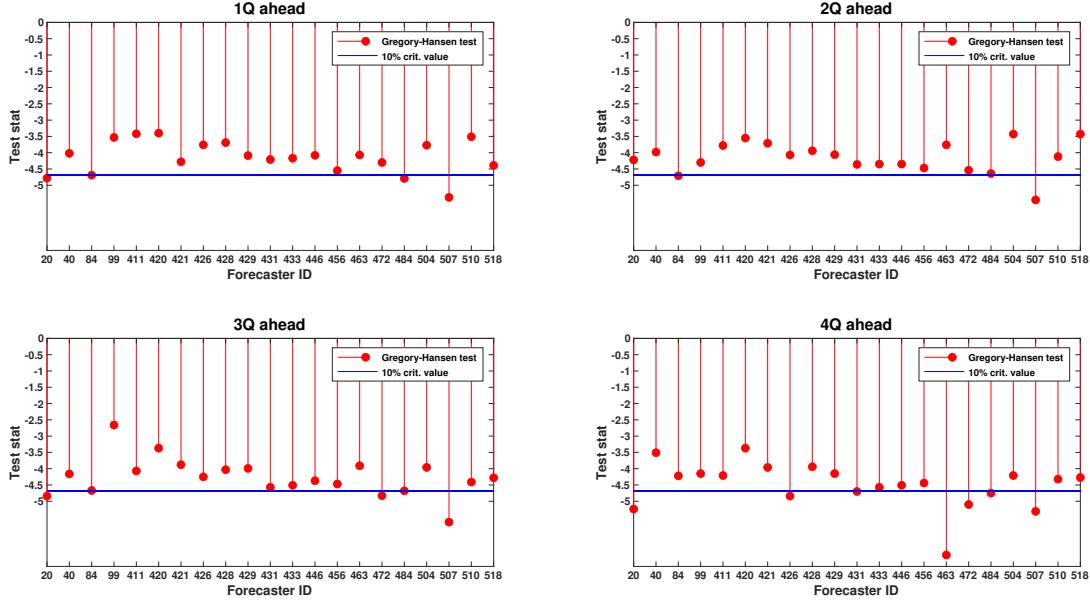
Figure 5: Illustration of individual-level cumulative sum test (OLS residuals) for coefficient stability, forecasts of output-consumption ratios



Now turn to the Gregory-Hansen test. The null hypothesis of the test assumes the existence of a structural break and no cointegration at the break point. Rejecting the null hypothesis implies the existence of a cointegration relation with a structural break. Figure 6 plots the Gregory-Hansen test statistics and the corresponding critical values for individual-level forecasts of output-to-consumption ratios. Each red dot stands for the test statistics for an individual forecaster, while the blue horizontal line corresponds to the 10% critical value. The null hypothesis is not rejected for the vast majority of cases. This implies that there is no cointegration between the two forecasts, taking into account structural breaks. In a small proportion of cases, the null hypothesis is rejected, implying the existence of cointegration between the two forecasts with a structural break. Overall, the results suggest heterogeneity in utilizing long-run equilibrium relationships in forecasting among the forecasters.²⁷

²⁷Online Appendix L shows that the same results are obtained when using forecasts of output and investment ratios. Moreover, it shows that the Gregory-Hansen test indicates no cointegration in median and mean forecasts.

Figure 6: Gregory–Hansen cointegration test statistics with forecasts of output-consumption ratios



5.3 Testing for forecasters' beliefs about mean reversion

One issue of interest is whether the forecasters believe in mean reversion to the long-run equilibrium implied by the full-information RE stochastic growth models. This section tests this question using the data of individual-level forecasts and following Armona, Fuster, and Zafar (2019). They conducted a survey experiment in which participants were provided information regarding recent local house price growth rates. They found that participants were more likely to believe in momentum in house price growth rather than mean reversion in house price growth.

First, note that for the majority of forecasters, forecasts of output are not cointegrated with forecasts of consumption (or investment). This implies that they do not believe in mean reversion to the long-run equilibrium. Second, for the forecasters whose output forecasts are cointegrated with forecasts of consumption (or investment), we tested if they believe in mean reversion to the long-run equilibrium in the following way. We regressed the revisions to forecasts of output to consumption (or investment) ratios on the forecast errors of the ratios. The SPF provides participants information about the latest data in the survey questionnaire and

elicits their macroeconomic expectations. Specifically, the regression is as follows:

$$(E_t^i V_{t+1} - E_{t-1}^i V_t) = \alpha^i + \beta^i (V_t - E_{t-1}^i V_t) + u_t^i, \quad (1)$$

where i is an individual, t is time (quarter), and E_t^i is the expectation operator for individual i based on the information available at t . V represents output to consumption (or investment) ratios. α^i is an intercept term, and the slope coefficient β^i represents how strongly, and in what direction, the individual i revises their expectation in response to the forecast errors of the ratios. Table 12 reports the estimation results. The regressions yield positive regression coefficients β^i , suggesting that the forecasters are extrapolating (or believe in momentum) instead of believing in mean reversion.

Table 12: **Regression results, extrapolating v.s. mean reversion**

	Estimated coefficient (β^i)	P value
Panel A: output to consumption ratios		
ID 20	1.09*** (0.10)	0.00
ID 504	0.92*** (0.11)	0.00
ID 510	0.82*** (0.09)	0.00
Panel B: output to investment ratios		
ID 504	0.91*** (0.09)	0.00
ID 510	0.78*** (0.09)	0.00

Note: ***means significant at 1% level.

5.4 Cointegration among other macroeconomic variables

We examined the integration and cointegration properties of other macroeconomic variables, such as inflation, unemployment, and nominal interest rates. We briefly summarize the results and relegate the detailed testing results in online Appendix O.

First, the mean and median forecasts of inflation and unemployment are I(0). Online Ap-

pendix O also shows that the Johansen trace and maximum-eigenvalue tests confirm the existence of multiple cointegrating vectors between inflation forecasts and unemployment forecasts. All individual forecasts of inflation are also $I(0)$. Most individual unemployment forecasts are $I(0)$ and a small proportion of unemployment forecasts are $I(1)$. Thus, it is not meaningful to analyze the cointegration between inflation forecasts and unemployment forecasts.

Second, if the real interest rate is stationary, the nominal interest rate and the inflation rate are cointegrated with the $(1, -1)$ vector, according to the Fisher equation. King, Plosser, Stock, and Watson (1991) provide some evidence for the co-integration between realized nominal interest rates and inflation. Median and mean forecasts of nominal interest rate are $I(1)$. As mean, median, and individual inflation forecasts are $I(0)$, there exists no coegration vector that yields cointegration between the non-stationary forecasts of nominal interest rate and the stationary inflation forecasts. The recursive trace test is utilized to illustrate the non-existence of cointegrating vectors between the forecasts of the inflation rate and the nominal interest rate in online Appendix O.

5.5 Fitting forecasting models

Macroeconomists may be interested in how to model the expectation formation process of forecasters who use or do not use long-run relationships in forecasting. We explore one way to approximate the expectation formation of the two types of forecasters by fitting parsimonious recursive forecasting models (constant gain learning algorithms) to the survey forecast data, following Branch and Evans (2006). Specifically, the expectation formation process of the forecasters who utilize cointegration relationships is approximated by recursive estimation and forecasting using a cointegrated Vector Autoregressive (VAR) model which imposes cointegration among output, consumption, and investment. The expectation formation process of the forecasters who do not utilize cointegration relationships is approximated by recursive estimation and forecasting using a simple univariate autoregressive process. Online Appendix P.2 provides the details of fitting the models to survey forecasts data.

5.6 Forecast accuracy

Another interesting issue concerns the accuracy of forecasts made by the forecasters who utilize (or do not utilize) long-run relationships. To evaluate this, individual forecasters from the SPF are divided into two groups: those who utilize a long-run relationship in forecasting and those who do not. Forecasts made by each individual from each group are evaluated against the corresponding realized values. Table A.28 in online Appendix reports the accuracy of forecasts, measured by root-mean-square errors (RMSEs) over 1-, 2-, 3-, and 4-quarter horizons. The forecasters who do not use long-run cointegration relationships are generally more accurate, as they have slightly smaller average root-mean-square errors. However, these differences are unimportant economically and it remains an open question that whether utilizing the long-run relationships improves the accuracy of forecasting.

6 Conclusion

This paper studies the expectation formation of macroeconomic variables (aggregate output, consumption, and investment) by professional forecasters. We focused on analyzing whether professional forecasters utilize long-run cointegration relationships among the aforesaid macroeconomic variables postulated by stochastic growth models to make forecasts. The median (or mean) survey forecasts of aggregate output are not cointegrated with the median (or mean) forecasts of aggregate consumption and investment. Significant heterogeneity exists among forecasters in terms of utilizing long-run relationships. The majority of the forecasters do not appear to utilize these relationships in forecasting. The results are robust to using different tests, using forecasts over different horizons, forecasts made at the same or different dates, with or without imposing the theory-implied cointegrating vector, and considering the multiple testing problem and structural breaks.

The findings have implications for policymakers.²⁸ First, not utilizing long-run equilibrium relationships in forecasting or estimations by the policymakers may lead to mis-measurements in the output gap, which is an important input for monetary and fiscal policy. For instance, fiscal policymakers in the EU target a certain level of structural fiscal balance, which is crucially determined by estimates of the output gap. Kuang and Mitra (2020) examine the interaction between policymakers' mis-measurements of the output gap, fiscal policy decisions, and the prolonged recession in the aftermath of the 2007–08 financial crisis in European countries. Kuang, Mitra and Tang (2021) study monetary policy design when policymakers do not make use of knowledge of the balanced growth path and use various detrending methods to learn about the output gap. They find that the methods that intrinsically produce larger and more volatile output gap estimates are more prone to self-reinforcing deflation spirals and large welfare losses, and that the optimal response to the output gap estimates from these methods is smaller.

Second, households and firms who do not make use of long-run equilibrium relationships in forecasting may hold overly optimistic or pessimistic beliefs about macroeconomic variables. This may lead to inefficient fluctuations in economic decisions in areas such as consumption and investment. As an example, Kuang and Mitra (2016) develop a quantitative business cycle model where agents do not have knowledge of the balanced growth path and learn about the long-run growth rates of economic variables. The model replicates pro-cyclical forecasts of long-run growth rates of output and productivity in the data and suggests that optimism and pessimism about long-run growth rates is crucial to understanding business cycle fluctuations.

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²⁸Like the majority of experts, the long-run relationships are not utilized by policymakers either; online Appendix C shows the evidence from the US Federal Reserves' Greenbook forecasts. One caveat is that households' expectations may deviate substantially from those of experts, as in Coibion, Gorodnichenko and Kumar (2018), and Andre, Pizzinelli, Roth, and Wohlfart (2021). Additionally, forecasts of firms often deviate significantly from those of households as well (Candia, Coibion and Gorodnichenko 2021; Link, Peichl, Roth and Wohlfart 2021). It is worthwhile to test households' or firms' expectations on the long-run relationships. For our purpose, unfortunately we do not have an appropriate dataset of survey forecasts for households or firms. Nevertheless, some influential papers modeled experts' forecasts as a crucial determinant of households' forecasts, such as Carroll (2003) and Malmendier and Nagel (2015).

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